Adding Some Remarks Concerning the Dzahanibekov Effect

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Abstract. The paper further investigates the Dzhanibekov effect. This effect was noticed for the first time in 1985 aboard the Salyut 7 orbital station. Several different rigid bodies, each and every one of them with its main moments of inertia having close values, are analysed. Rigid bodies, each and every one of them with significant differences between the values of maximum moment of inertia and minimum moment of inertia, are also analysed. The effects of the main rotational motion angular velocity on the time of occurrence of the Dzhanibekov effect are analysed. Furthermore, the effects of perturbations on the time period of movement of a rigid body experiencing Dzhanibekov effect are studied.

Keywords: Free Rigid Body · Moments of Inertia · Dzhanibekov effect

1 Preliminaries

A particular case in the study of the dynamics of a free rigid body is the one of the resulting zero moment, relative to the centre of mass, of all the external forces and torques acting on it. Previous studies have been presented about Euler-Poinsot case. Although it has been studied in detail by Euler [1], and Poinsot gave a representation of body movement, it seems that some phenomena have not been highlighted. One such phenomenon was discovered by the astronaut Vladimir Dzhanibekov in 1985 during his journey aboard the Salyut Orbital Station.

During the unpacking process it became necessary to unscrew several nuts so they had to be rotated several times. To simplify this process, Vladimir Dzhanibekov applied significant torque to the wing of the nut, which led to the initiation of a rapid rotation of the nut and, simultaneously, to its translation along the threaded rod. The momentum was enough for the nut to complete the unscrewing process on its own and then leave the rod. From this point on, the nut continued its free flight, traveling along the rod axis while still rotating around this axis. After a traveling by translation for a distance of about 40 cm, in which the motion was apparently stable, it suddenly changed its axial orientation by 180 degrees, simultaneously changing its rotating direction. This moving pattern was periodically repeated, without any external force being applied. This spectacular behaviour, in

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the weightless environment, of a rotating rigid body around a principal axis of rotation, has been called the "Dzhanibekov effect" or "Dzhanibekov phenomenon". Subsequently, the Dzhanibekov phenomenon was reproduced for study purposes during numerous demonstrations aboard the International Space Station. Although initially perceived by some people as counter-intuitive or even mysterious, Dzhanibekov's phenomenon has been explained, based on Euler's equations [2–4], in various publications.

Although it has been highlighted quite recently, possible applications of this phenomenon have already appeared. For example, in [5] a new concept of using the Dzhanibekov effect for inertial navigation was proposed and developed by changing the distribution of spacecraft components without using classical gyroscopes. To implement this change, two main conceptual solutions are proposed, which involve system modifications requiring for the intermediate moment of inertia to become the lowest or highest principal moment of inertia.

2 Theoretical Approach

Typically, two reference systems (Cartesian frames) are, formally, attached to the rigid body: an external reference system (space frame) that is considered fixed and a reference system (body frame) that sticks with the rigid body. To simplify the equations of motion, the body frame is chosen with the origin O into the center of mass of the body and the axes Ox_1, Ox_2 and Ox_3 will be the principal axes of inertia. The equations of motion of the rigid body, projected onto the axes of the body frame (scalar equations), are [6–8]:

$$\begin{aligned} J_1\dot{\omega}_1 + (J_3 - J_2)\omega_2\omega_3 &= 0 \\ J_2\dot{\omega}_2 + (J_1 - J_3)\omega_1\omega_3 &= 0 \\ J_3\dot{\omega}_3 + (J_2 - J_1)\omega_1\omega_2 &= 0 \end{aligned} \tag{1}$$

where J_1, J_2 and J_3 are, respectively, the moments of inertia with respect to the before mentioned axes and, ω_1, ω_2 and ω_3 are the components of the angular velocity with respect to the body frame. We shall assume that $J_1 > J_2 > J_3$.

The system of Eqs. (1) describes the rotating motion of the rigid body around its mass centre. This system owns the following particular solutions:

- $\omega_1 = \omega_1(0) = ct; \omega_2 = \omega_3 = 0$, meaning the rigid body does make an uniform rotating motion around the Ox_1 axis;
- $\omega_2 = \omega_2(0) = ct; \omega_1 = \omega_3 = 0$, meaning the rigid body does make an uniform rotating motion around the Ox_2 axis;
- $\omega_3 = \omega_3(0) = ct; \omega_1 = \omega_2 = 0$, meaning the rigid body does make an uniform rotating motion around the Ox_3 axis.

Because, in all the before mentioned cases, the body rotates uniformly about an axis which does not change its position with respect to either the body frame or the space frame, that axis is called "permanent axis of rotation". It can be proven that the rotational movements around the axes with respect to which the inertia moments have maximum or minimum values are stable and the rotational movements around an axis with respect to which the inertia moment has an intermediate value are unstable.

It can be immediately proven that the system of Eqs. (1) admits two prime integrals [6–8]:

$$J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2 = 2E = ct \tag{2}$$

$$J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2 = K_c^2 = ct \tag{3}$$

Also, it can be immediately proven that:

$$\begin{aligned} K_c^2 - 2J_3E > 0 \\ 2J_1E - K_c^2 > 0 \end{aligned} \tag{4}$$

From the equalities (2) and (3) ω_1 and ω_3 are extracted as functions of ω_3. It comes out [7, 8]:

$$\omega_1 = \pm\sqrt{\frac{\left(K_c^2 - 2J_3E\right) - J_2(J_2 - J_3)\omega_2^2}{J_1(J_1 - J_3)}} \tag{5}$$

$$\omega_3 = \pm\sqrt{\frac{\left(2J_1E - K_c^2\right) - J_2(J_1 - J_2)\omega_2^2}{J_3(J_1 - J_3)}} \tag{6}$$

It comes out that solving the equations of motion simply consists in solving a one single differential equation. The Dzhanibekov effect occurs when the rigid body rotates around the principal axis of inertia with respect to which the intermediate moment of inertia is obtained and there is a disturbance with respect to one of the other two axes.

The first studied case considers a rigid body with: $J_1 = 5\ \text{Kg}\cdot\text{m}^2$, $J_2 = 4\ \text{Kg}\cdot\text{m}^2$ and $J_3 = 3\ \text{Kg}\cdot\text{m}^2$. In this specific case we assume that all the three principal moments of inertia do not have close values.

In Fig. 1 is shown the behaviour of each and every component of the angular velocity for the initial conditions. These conditions are referring to a disturbance with respect to the axis corresponding to the minimum moment of inertia.

In Fig. 2 is shown the behaviour of each and every component of the angular velocity for the initial conditions. These conditions are referring to a disturbance with respect to the axis corresponding to the maximum moment of inertia.

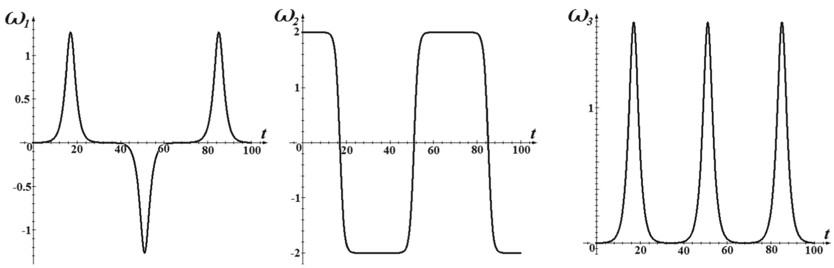

Fig. 1. The behaviour of each and every component of the angular velocity for the initial conditions $\omega_1(0) = 0\ \text{rad/s}$, $\omega_2(0) = 2\ \text{rad/s}$ and $\omega_3(0) = 0.001\ \text{rad/s}$ (for the minimum moment of inertia - first case studied)

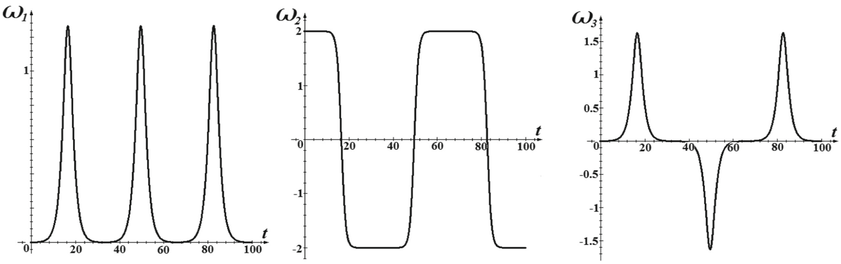

Fig. 2. The behaviour of each and every component of the angular velocity for the initial condition $\omega_1(0) = 0.001$ rad/s, $\omega_2(0) = 0$ rad/s and $\omega_3(0) = 0$ rad/s (for the maximum moment of inertia - first case studied)

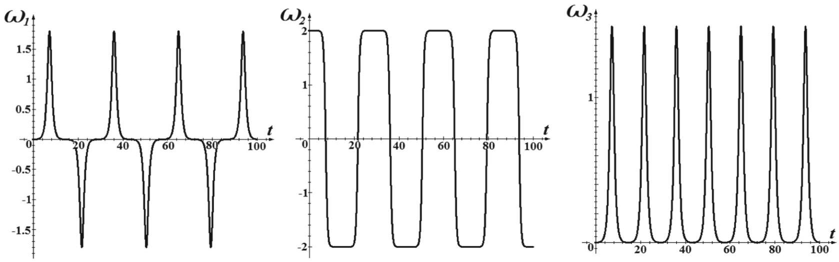

Fig. 3. The behaviour of each and every component of the angular velocity for the initial conditions $\omega_1(0) = 0$ rad/s, $\omega_2(0) = 2$ rad/s and $\omega_3(0) = 0.001$ rad/s (second case studied)

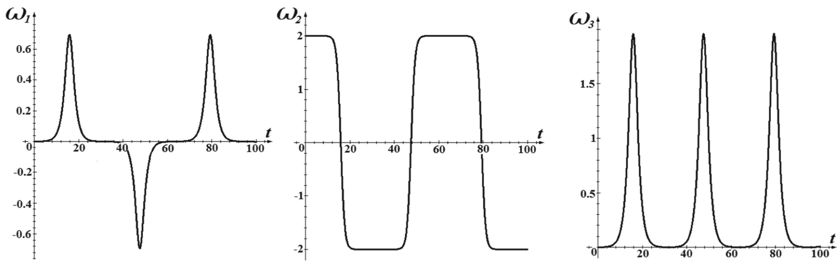

Fig. 4. The behaviour of each and every component of the angular velocity for the initial conditions $\omega_1(0) = 0$ rad/s, $\omega_2(0) = 2$ rad/s and $\omega_3(0) = 0.001$ rad/s (third case studied)

The second case under study is referring to a rigid body with $J_1 = 11$ Kg $\cdot$ m^2, $J_2 = 10$ Kg $\cdot$ m^2 and $J_3 = 2$ Kg $\cdot$ m^2. In this specific case, the biggest principal moments of inertia are considered as having close values. In Fig. 3 is shown the behaviour of each and every component of the angular velocity for the initial conditions.

The third case under study considers a rigid body with $J_1 = 10$ Kg$\cdot$m^2, $J_2 = 6$ Kg$\cdot$m^2 and $J_3 = 5$ Kg $\cdot$ m^2. In this specific case the lowest moments of inertia are considered to have close values. In Fig. 4 is shown the behaviour of each and every component of the angular velocity for the initial conditions.

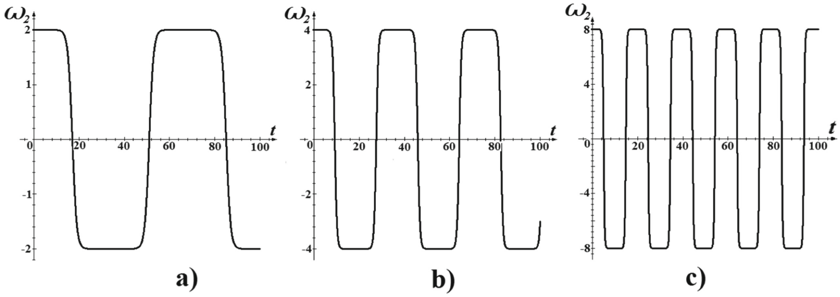

Fig. 5. The behaviour of the angular velocity component ω_2 for three of its initial values: a) $\omega_2(0) = 2$ rad/s, b) $\omega_2(0) = 4$ rad/s, c) $\omega_2(0) = 8$ rad/s (first case studied)

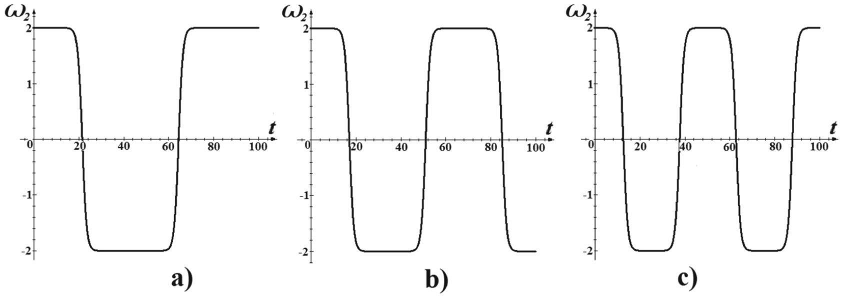

Fig. 6. The behaviour of the angular velocity component ω_2 for three of its initial values: a) $\omega_3(0) = 0.0001$ rad/s, b) $\omega_3(0) = 0.001$ rad/s, c) $\omega_3(0) = 0.01$ rad/s (first case studied)

In order to assess how the initial angular velocity does influence the behaviour of the rigid body, for the first case ($J_1 = 5$ Kg $\cdot$ m^2, $J_1 = 4$ Kg $\cdot$ m^2 and $J_1 = 3$ Kg $\cdot$ m^2) the body motion for the disturbing conditions $\omega_1(0) = 0$ rad/s and $\omega_3(0) = 0.001$ rad/s is studied. In Fig. 5 is shown the behaviour of the angular velocity component ω_2 for three of its initial values. In Fig. 5.a is shown the behaviour of the angular velocity component ω_2 for the initial condition $\omega_2(0) = 2$ rad/s, in Fig. 5.b for the initial condition $\omega_2(0) = 4$ rad/s and in Fig. 5.c for the initial condition $\omega_2(0) = 8$ rad/s.

It is to be noticed that the time period of motion decreases as the angular velocity of rotation of the rigid body around the axis corresponding to the intermediate moment of inertia increases.

Another issue is related to the influence the values of the initial perturbations does have on the behaviour of the angular velocity ω_2. Figure 6 shows how the angular velocity ω_2 is behaving for three values of the initial perturbation with respect to the axis corresponding to the minimum moment of inertia. Figure 6.a shows the behaviour of the angular velocity ω_2 for the initial disturbance $\omega_3(0) = 0.0001$ rad/s, Fig. 6.b for the initial disturbance $\omega_3(0) = 0.001$ rad/s and Fig. 6.c for the initial disturbance $\omega_3(0) = 0.01$ rad/s.

It is to be noticed that the time period of movement decreases as the angular velocity of disturbance with respect to the axis of minimum moment of inertia increases. Similar results are obtained when the angular velocity of the disturbance has the direction of the axis of maximum moment of inertia.

3 Conclusions

The Dzhanibekov effect occurs when the rotational motion of the rigid body around the axis with respect to which the intermediate moment of inertia is obtained is, in fact, the main motion of the body, and, moreover, there is a disturbance with respect to one of the other two axes.

If the disturbance does occur by the appearance of a component of the angular velocity onto the axis of minimal moment of inertia, then, throughout the movement, the component of the angular velocity on this axis keeps its orientation and the component onto the axis of maximum moment of inertia alternately changes its orientation. On the contrary, if the disturbance occurs due to the appearance of a component of the angular velocity onto the axis of maximum moment of inertia, then, throughout the movement, the component of the angular velocity on this axis keeps its orientation and the component onto the axis of minimal moment of inertia changes alternatively its orientation.

The movement of the rigid solid is periodic and the time period of the movement depends on the values of the moments of inertia with respect to all the three axes but also on the initial conditions of the movement. Thus, if we consider as landmark the time period of motion for the case when the three moments of inertia have close values, we find out that the time period of motion is shorter if the maximum and intermediate moment of inertia have close values and much higher than the value of minimum moment of inertia. There are no significant differences if the minimum and intermediate moments of inertia have close values and significantly lower than the maximum moment of inertia. Also, the time period of motion decreases as the angular velocity of the main rotational motion around the axis of intermediate moment of inertia increases. The time period of movement decreases even if the value of the angular velocity of disturbance increases.

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