Profile of Students' Geometric Thinking Ability in Terms of Van Hiele Level

Venissa Dian Mawarsari[1](✉), St. Budi Waluya[2], and Nuriana Rachmani Dewi[2]

[1] Universitas Muhammadiyah Semarang, Semarang, Indonesia
venissa@unimus.ac.id
[2] Universitas Negeri Semarang, Semarang, Indonesia

Abstract. The ability to think geometrically is very important to be improved in students at all levels of school, from an early age to college. In the ability to think geometry, there is a level which of course an individual will have according to age level and school level. This includes universities, where students who can think abstractly and logically will be at level 3 (informal deduction) or 4 (rigor). But of course, it is necessary to study more deeply the characteristics at each level of thinking possessed by students. Given this need, the purpose of this study is to determine the profile of students' geometric thinking abilities based on Leve Van Hiele. This research is a qualitative descriptive study by describing the ability to think geometry based on the van Hiele level. The research sample is the 3rd-semester students of the mathematics education study program at one of the private universities in Semarang who have received the spatial geometry course. Data were obtained from van Hiele-level evaluation tests, observations, and interviews. The results showed that the ability to think geometrically is level 4, requires time to think about how to solve the problem of proof, needs to dig deeper and recall the concepts involved in solving it. The ability to think at level 3, students actually solve it in a different way. The ability to think at level 2, students can complete a good process and use the concepts they already have and interviews. The results showed that the ability to think geometrically level 4, requires time to think about how to solve the problem of proof, and needs to dig deeper and recall the concepts involved in solving it. The ability to think at level 3, students actually solve it in a different way. The ability to think at level 2, students can complete a good process and use the concepts they already have and interviews. The results showed that the ability to think geometrically level 4, requires time to think about how to solve the problem of proof, and needs to dig deeper and recall the concepts involved in solving it. The ability to think at level 3, students actually solve it in a different way. The ability to think at level 2, students can complete it a good process and use the concepts they already have.

Keywords: Geometric thinking · Van Hiele Level · Characteristic of Student'

1 Introduction

Learning mathematics is crucial for developing numeracy, communication, problem-solving, critical thinking and collaboration [1]. An instrument for developing thought processes is mathematics. This is crucial to deal with the advancement of science and

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technology [2]. Basic knowledge such as mathematics is necessary for the development of science and technology because it is a thinking tool for developing systematic reasoning and critical thinking. We unconsciously apply the ideas and abilities we gain from solving math problems because everything is governed by mathematical laws [3]. Geometry is an important math topic in the curriculum [4, 5]. We frequently come across the mathematical discipline of geometry in our daily lives. Galileo believed that geometry was the key to comprehending nature [6]. Many objects around us are shaped like a geometric plane or solid.

Geometric thinking is one of the branches of mathematics that can help students develop critical thinking skills [7] and be able to relate geometry to real life [3] for example determining numbers, measuring numbers, soil and earth, and making maps [8]. Curriculum and Evaluation Standards for School Mathematics requires an emphasis on geometry at all levels [9–12]. Learning mathematics must be able to emphasize the thinking process of students, one of which is geometric thinking [3]. Through learning geometry, students can practice problem-solving skills and make it easier to learn various topics in mathematics and other sciences [13, 14].

The purpose of studying geometry is to provide students with critical thinking and problem solving skills and to better understand mathematical concepts by acquiring higher level geometric thinking skills [15]. Learning geometry helps enhance visual thinking abilities [16]. The need for math is growing quickly, as is our ability to understand and use it in this crucial aspect of our daily lives [17]. The relevance of enhancing students' geometric thinking in the teaching and learning process stems from the fact that, in addition to mathematics, geometric thinking is crucial in many other scientific, technological, and vocational disciplines [16, 17].

Evidence from the classroom shows that students struggle to understand geometry issues [13]. Research has been carried out in various countries, including Turkey [15, 18], the Czech Republic [19], South Africa [20], Ghana [21], Malaysia [9, 22–25]resulted in the conclusion that students have low Geometry performance. Indonesian pupils likewise struggle with the same low performance in geometry learning that is faced by students around the world [3, 6, 13, 18, 26–28].

Problems related to the level of geometric thinking ability are also experienced at the university level. Based on observations in the Mathematics Education Study Program, there are still many difficulties experienced by students. Students still have difficulty in studying geometry courses and their learning outcomes are also lacking. This can be seen from the problem-solving process given by the lecturer. Students should have reached Van Hiele's level of geometric thinking, namely formal deductive and Rigor. The reality in the field is that many students still reach the informal deductive level. Based on this, it is necessary to analyze the level of thinking that exists in students.

At the University of Utrecht in the Netherlands, Dina van Hiele-Geldof and Pierre van Hiele originally put forth their model of geometric reasoning in 1957. Many math educators credit van Hiele's model with helping children develop their geometric reasoning [27]. Three factors were used by Van Hiele to describe a geometric thinking model: the existence of levels, the characteristics of levels, and the progression from one level to the next [29]. [30] define the level as Level 0 (Visualization) where pupils recognize all visual presentations without considering shape components; Level 1 (Analysis) students can distinguish between shapes by noticing qualities that are not thought to be relevant; Level 2 (Informal Deductive) – students gain knowledge of, and practice using, relationships between form features; Level 3 (Formal Deductive) – students are able to logically create evidence regarding forms and classify forms in meaningful ways; and Level 4 (Rigor) – students are able to prove axiomatic theorems and evaluate the results of axiomatic manipulation [7]. The development of the geometric thinking process is described by the Van Hiele Model in five interconnected stages, which is its most significant aspect [10]. These steps each specify the mental procedures required to identify geometric relationships.

Van Hiele's model's definition of the evolution of geometric cognition at five interconnected levels is its most significant aspect. Each of these five levels focuses on geometric context-specific cognitive processes. Instead of emphasizing the amount of material learned, these levels describe the manner of thinking and the types of geometric concepts addressed. The main difference between the two levels is the object of thought; those are concepts that can be understood geometrically [31]. Instead than demonstrating how much information is known, this level describes different ways of thinking and sorts of geometric ideas. The level of students' geometric thinking affects their mathematical ability in general and geometric thinking skills in particular. Van Hiele's high degree of geometric reasoning is more likely to be attained by students who are highly proficient in mathematics and geometry. The students' geometric thinking level will progress sequentially from Level 0 to the highest level [26].

Two main research topics based on van Hiele theory at present are using dynamic geometry to obtain higher van Hiele levels and extending van Hiele theory to other branches of mathematics (such as boolean algebra, functional analysis, and calculus) [29]. In this study, we will look at the level of student geometry skills based on the Van Hiele level. The purpose of this research is to know the characteristics of geometric thinking ability based on van Hiele level in Mathematics Education students.

2 Research Method

This research is a qualitative descriptive study by describing the ability to think geometry based on the van Hiele level. The research sample is 3rd-semester students in mathematics education study program at one of the private universities in Semarang who have received spatial geometry courses. Collecting data by data triangulation, namely evaluation tests, observations, and in-depth interviews. The evaluation test is made based on the van Hiele level, namely level 0 (visual), level 1 (analysis), level 2 (informal deduction), level 3 (formal deduction), and level 4 (rigor).[30]. The following is a grid of the Van Hiele-level evaluation tests used in the study (Table 1).

The evaluation test used has been analyzed for validity and reliability. Data analysis consists of presentation and validation of data reduction [32, 33]. Data were reduced using interview text coding. The initial stage in this research is to test students' geometric thinking skills, then analyze the students' geometric thinking levels to find out the distribution. The next step is to conduct in-depth interviews at all levels of candidates. The final step is to triangulate the data to arrive at a conclusion. Data analysis uses induction to draw descriptive conclusions and present the big picture from small case studies [34].

Table 1. Indicator of geometric thinking based on van Thiele level

Van Hiele's Level	Indicator	No Question
Level 0 (Visual)	An evaluation test was used for validity and reliability analysis.	1
Level 1 (Analysis)	Students can identify the shape of the wedge formed by showing it based on the characteristics of the plane.	2
Level 2 (Formal Deduction)	Students can calculate the cross-sectional area based on student ideas or check other methods.	3
Level 3 (Informal Deduction)	Students can try out the location statement.	4
Level 4 (Rigor)	Students will be able to proportionally analyze and create proofs in various engineering systems.	5

3 Results and Discussion

The results of the geometric assessment test according to the Van Hiele scale gave the following data.

Figure 1 shows that among the 44 research topics most students entered Level 2 (informal deduction) and 7 entered Level 1 (analysis). Based on these findings I then conducted in-depth interviews with students who entered each Van Hiele stage to complete S1 (Level 1) S2 (Level 2) and S3 (Level 3). The purpose of the in-depth interviews was to capture the students characteristics in the process of completing and completing assessment tests based on the Van Hiele levels of geometric thinking skills.

The geometric thinking skill results at level 1 showed that students could only answer the assessment test at level 1 and the test at level 2. Following are the mistakes made by students in completing level 2 exam.

Based on Fig. 2 it was found that students could not determine the value of the side length of the wedge formed. To determine the ability to think geometrically conducted interviews with S1.

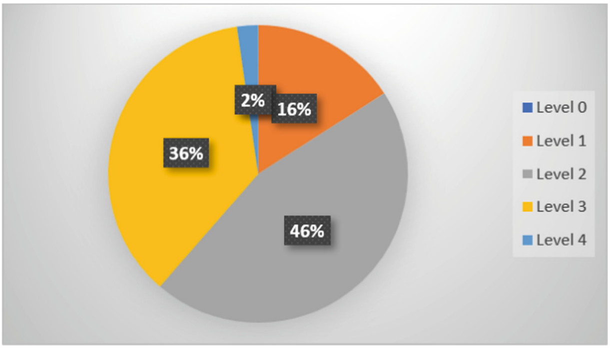


Fig. 1. Percentage of Van Hiele's level of geometric thinking

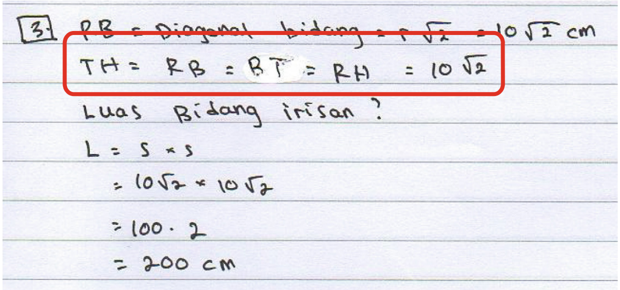


Fig. 2. Level 2 test results

The results of the S1 conference concluded that students have the ability to describe geometric shapes and know which parts belong and can correctly state the shape of these parts according to the characteristics of the images made. For example students cannot calculate the area of shaped disks because they have the wrong idea of finding the side lengths of flat disks. Students assume that the length of one side of the intersection is the length of one side of the diagonal of the cube but the two are different. The concept of side length of a constructed plane is the imaginary concept of the side length of a cube and the opposite half of a cube [35]. it's just that they have weaknesses in analyzing problem-solving problems, because visual analysis has not been carried out at level 2. Before students move on to non-visual components and aspects of logical order, conclusions, and proofs, drawing geometric objects is crucial [36]. The student's inability is due to the association of concepts in geometry in the problem [37].

A geometric thinking skill score at level 2 indicates that students can answer only at level 2 and make errors at level 3. The consequences for a student who does not complete step 3 are as follows:

Based on Fig. 3, it was found that students could not complete the level 3 test. Students could not prove existing statements, namely in this level 3 test students were asked to prove "The volume of the cube ABCD.EFGH equals 6 times the volume of the F.ABC pyramid". Students can only search for the volume of the F.ABC pyramid, but not related to the volume of the ABCD.EFGH cube. Another error is the area of the base

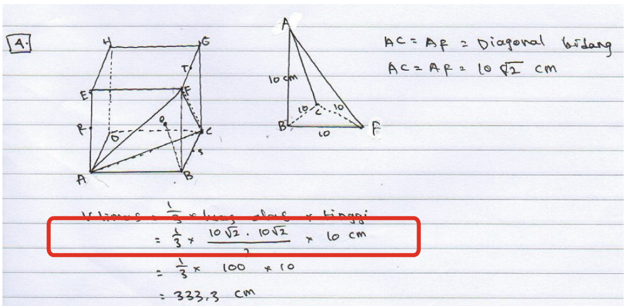
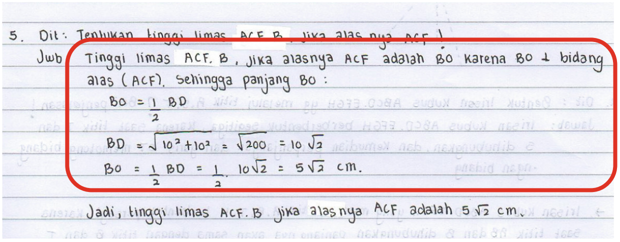


Fig. 3. Level 3 test results


5. Dit: Tentukan tinggi limas ACF.B, jika alas nya ACF!

Jwb: Tinggi limas ACF.B, jika alasnya ACF adalah BO karena BO ⊥ bidang alas (ACF). Sehingga panjang BO :

$BO = \frac{1}{2} BD$

$BD = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$

$BO = \frac{1}{2} BD = \frac{1}{2} . 10\sqrt{2} = 5\sqrt{2}$ cm.

Jadi, tinggi limas ACF.B jika alasnya ACF adalah $5\sqrt{2}$ cm.

Fig. 4. Level 4 test results

of the F.ABC pyramid, which is an error in determining the length of the sides AB and BC, which should be 10 cm.

Interviews with professors concluded that students still struggle to understand well-known statements in questions because of their abstract concepts. Errors also occur when students enter the value of the length of the sides that are perpendicular to each other. This is because students have weak concepts and cannot remember the concept of volume building. This shows that there is a need for creativity in connecting geometric concepts to problem-solving [38].

The results of students' geometric thinking skills at level 3 fail at the level 4 test. Below are the results for students who failed to complete Level 4.

Based on Fig. 4, an error was obtained that the student could not prove the existing statement. Students make mistakes in identifying abstract shapes from shapes and cannot explore their memories about the concept of volumetric shapes.

The result of the interview with S3 is that the students know the formula for the size of geometric shapes but they forget that the position of a rectangular pyramid can be changed so that the base and height change but the size of a rectangular pyramid does not change. However, students can complete level 3 because students can prove the concept of spatial geometry based on existing formal concepts. Geometry tests require strong conceptual understanding and the ability to make connections between concepts [39]. Yet the proof of geometry still requires the ability to show statements as geometric pictures [40].

4 Conclusion

The conclusion of this study is that there are three levels of students' geometric thinking skills: Level 1 (Analysis) Level 2 (Formal Reasoning) and Level 3 (Informal Reasoning). Most students are at level 3. Each level has different characteristics of geometric thinking abilities, namely level 1 (analysis) can visually describe abstract forms, but in analyzing problem-solving problems it still has weaknesses. The causative factor is that in solving geometric problems it is also necessary to describe it first to be able to explore existing concepts. At level 2 (formal deduction), students' geometric thinking ability allows students to visualize geometric shapes and perform geometric calculations, but in proving the need for creativity in connecting geometric concepts to problem-solving. At level 3 (informal deduction), students can prove the concept of spatial geometry based on existing formal concepts, but the obstacle obtained is complex proof of geometry.

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