Hierarchical Teaching Practice Based on Majors and Applications of Mathematica in Advanced Mathematics

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Abstract. To better reflect the principles of individualized teaching, apply computer technology, adopt individualized education for students, and cultivate students' innovative thinking ability, we explore the reform of advanced mathematics education. We firstly point out the importance of hierarchical teaching of advanced mathematics in different majors and applications of Mathematica, then arrange corresponding teaching according to the requirements of different majors for students. Students with similar knowledge foundations and abilities are grouped into the same level, and students at the same level can choose courses freely. Besides, in order to stimulate students' interest in learning and improve their ability to solve problems, we analyze and explore the applications of Mathematica in advanced mathematics. We present a practical problem solved with the aid of Mathematica for example. Furthermore, during our practice, multimedia technology and online learning (Rain Classroom) have been used in our teaching.

Keywords: advanced mathematics · hierarchical teaching · applications of Mathematica

1 Introduction

As we know, computer technology continues developing, its applications have been used widely and bring great change in our life and work. Due to growing requirement of education reforms in teaching quality of advanced mathematics, teachers need to ask related computer technology for help. Applications of computer technology is necessary to modern teaching methods. Computer technology can provide rich content and various teaching means, add novelty and interest to class, cater to the needs of contemporary college students, and finally, teaching quality and effects will be enhanced.

At the same time, there have been extensive teaching reforms in higher education emerging. As one of the basic compulsory courses in science and engineering schools, the teaching reform of higher mathematics has attracted much attention from teachers and students. We can find fruitful results in the reform of higher mathematics teaching in China. For example, the author in [1] explored the application of model teaching method in advanced mathematics teaching. The "Internet+" background of advanced

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mathematics flipped course teaching model was proposed in [2] and hierarchical teaching in advanced mathematics was discussed in [3–10]. Furthermore, combination of the ideological and political courses in advanced mathematics has been explored. Based on information technology, the experimental course teaching mode and modular design method for advanced mathematics have been proposed. Online teaching quality promotion strategies were given. There has already been extensive research, yet there are still a few shortcomings. For example, theory and practice are not closely linked, and teachers cannot teach students according to their aptitude.

Due to rigorous logic, abstraction, and complex reasoning, it is not easy to learn higher mathematics well. In addition, different majors have different requirements for higher mathematics, and there are also significant differences in students' mathematical foundations. Students' ability to apply mathematical knowledge to solve practical problems needs to be improved. Under the traditional teaching mode, the teaching quality of higher mathematics courses is not satisfactory. It is necessary to carry out hierarchical teaching based on higher mathematics majors. To solve these problems, this paper tries to adopt the hierarchical practice teaching of different majors. We incorporate mathematical modeling into the teaching of higher mathematics to achieve the following three goals:

(1) Teaching is guided by requirements and needs from majors and reflects the characteristics of the majors;
(2) Teaching activities are student-centered, considering individual differences, respecting students' choices, and teaching students of different majors according to their aptitude.
(3) We cultivate innovative talents and improve students' abstract logical thinking ability and the ability to solve practical problems.

In this paper, we introduce the reasons and details of teaching by majors in Sect. 2. Theoretical basis and details of hierarchical teaching practice are described in Sect. 3. In Sect. 4, we state the significance and practice of the computer technology in advanced mathematics, including multimedia, mathematical software and online learning. Advantages and disadvantages of the hierarchical teaching based on majors are mentioned in Sect. 5 and conclusions are given in Sect. 6.

2 Exploration and Practice of Teaching by Major

2.1 The Reasons for Teaching by Major

Judging from the teaching status of higher mathematics, some contents cannot meet the actual needs. Students who majored in science and engineering need to study higher mathematics, which plays an important role in compulsory courses. However, some teaching contents are too theoretical and cannot be closely integrated with the majors, and it has become a serious phenomenon that the teaching contents are separated from the actual requirements. Students cannot apply high mathematics knowledge to their majors, which in turn affects their enthusiasm for learning higher mathematics.

2.2 Details of Teaching Based on Majors

First of all, preliminary research and preparation are needed. For the schools that give higher mathematics courses, we organized teachers and students to hold symposiums to understand their requirements for mathematics and grouped majors with similar requirements. Different majors have different requirements for students' mathematical foundation. We formulated the teaching plans and content of higher mathematics according to the requirements of goals for majors. Teaching plans and content were jointly developed by mathematics teachers with teachers of other subjects. Teachers communicated and discussed to determine which parts of mathematical content need to be emphasized in teaching and which parts can be appropriately deleted or weakened. We selected and optimized the teaching content of higher mathematics to meet the requirements of different majors. In the teaching process, teachers were encouraged to combine the related cases from their majors with mathematic and apply the learned knowledge to analyze and solve problems. In this way, students would know the importance of higher mathematics in their learning and know how to use what they have learned freely.

3 The Exploration and Practice of Hierarchical Teaching

3.1 Theoretical Basis of Hierarchical Teaching

Confucius is a famous Chinese thinker and educator. When educating his disciples, he always advocated the principle of teaching according to aptitude. The theoretical basis of hierarchical teaching comes from this principle, and it is imperative to teach hierarchically. The so-called hierarchical teaching refers to dividing students into several classes and establishing a new teaching mode according to the students' interests, knowledge base, learning, and understanding ability. Teachers carry out targeted hierarchical teaching according to the teaching objectives, content, and methods, so that students' interest in learning can be inspired, and a good learning atmosphere can be created. In this way, students at all levels can give full play to their own advantages, make progress and improve their quality, so as to achieve the standards and requirements of the established curriculum.

3.2 Details of Hierarchical Teaching Practice

At the beginning of the freshman year, we schedule an exam. Students are divided into two levels, designated A and B, according to the exam results. For A-level, in addition to the basic concepts, we expand the knowledge and improve the students' abilities as much as possible, and try to explain every method and conclusion thoroughly so that they can progress and have a solid mathematical foundation, a wide range of knowledge, and strong mathematical ability to solve basic problems in their majors. For B-level, we encourage students to think independently, explore various ways to solve problems, propose multiple solutions to one problem, infer other cases from one example, ask more questions, and dare to guess. Then, their ability to solve practical problems would improve. We set up several classes at A and B levels according to the number of students. Students can choose a class at their level by audition. In principle, once a class is chosen,

it will not be changed this semester. At the beginning of the second semester, students are free to choose A or B levels and classes according to their studies and final exam scores of the previous semester.

4 Analysis and Exploration of Computer Technology in Advanced Mathematics

4.1 The Significance of Introducing Computer Technology in Advanced Mathematics

Advanced mathematics learning needs more ability of reasoning and analysis, and a lot of the course is pretty abstract. With development of computer technology, multimedia teaching has been realized, and advanced mathematics can also be aided by mathematical software and online learning. On the one hand, with the aid of multi-media and mathematical software, we can present advanced mathematics for the students visually, such as figures plotted by Mathematica. Thus, students can grasp the knowledge well, and teachers will not impart knowledge tediously. On the other hand, mathematical software, such as Mathematica, can be used to solve practical problems through modeling to stimulate interest in students. In this way, we can cultivate capacity in solving problems using what they have learned.

In traditional advanced mathematics teaching, more attention is paid to the explanation of theoretical knowledge, while the cultivation of students' innovation consciousness and application consciousness is neglected. The introduction of the ideas of mathematical modeling stimulates students' interest in learning higher mathematics to a certain extent. Mathematical modeling ideas can make students understand the knowledge of higher mathematics more thoroughly and improve their ability to solve practical problems. Furthermore, the addition of mathematical modeling enriches the teaching process and makes students realize the significance of learning higher mathematics. It also expands the methods of learning mathematics and thus improves students' mathematical literacy.

4.2 Detailed Practices of Integrating Computer Technology into Advanced Mathematics

In our teaching, with the multimedia technology and mathematical software aided, we provide more accurate and interesting details for students. For examples, creation background of some concepts or stories of domestic and international mathematicians are given by PPT or videos, trend of functions or regions of integration are plotted accurately by mathematical software, to name a few. The commonly used mathematical software today includes Mathematica, MATLAB and Maple. In the process of teaching, we choose mathematical software according to the requirements of the major. Then, students can master mathematical software and improve their ability to solve practical problems. We encourage students to practice independently, at the same time we can give open problems. Students use mathematical software to model and solve problems in groups to enhance their understanding of theoretical knowledge. Furthermore, Rain Classroom, an excellent online learning way, serves our teaching. We share teaching

materials, upload exercises and do tests through Rain Classroom, and we achieve good teaching effects.

Below, an example of a practical problem is given to illustrate the important role of mathematical software in advanced mathematics:

In order to shorten the sliding distance on the runway when an aircraft lands at the airport, the deceleration parachute is released from the rear of the aircraft when touching the ground. In this way, resistance increases and makes the aircraft slow down and stop. Suppose that the weight of the aircraft is 9,000 kg and the velocity is 700 km/h. The resistance of the aircraft experiences when landing is proportional to its velocity, and the coefficient of proportionality is $k = 6.0 \times 10^6$. The problem is to determine the total sliding distance from the landing point.

Solution: According to the known facts, the weight of the aircraft and the horizontal velocity of the aircraft during landing are denoted as $m = 9000kg$ and $v_0 = 700km/h$ respectively. We set the sliding distance and the velocity at time t as $s(t)$ and $v(t)$, respectively. Then, it can be obtained that $v(0)=v_0$, $s(0) = 0$, and the resistance of the aircraft is $-kv(t)$. According to Newton's second law, the following differential equation and the initial conditions are obtained

$$m\frac{\mathrm{d}v}{\mathrm{dt}} = -kv \tag{1}$$

$$s|_{t=0} = 0,\ v|_{t=0} = v_0 \tag{2}$$

In addition, it holds that

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} \tag{3}$$

Then, the following differential equation can be derived

$$m\frac{\mathrm{d}v}{\mathrm{d}s} = -k \tag{4}$$

By using the law of separation of variables, we have

$$\mathrm{d}s = -\frac{m}{k}\mathrm{d}v \tag{5}$$

By integrating both sides, the general solution can be expressed as

$$s(t) = -\left(m\big/k\right)v(t) + C \tag{6}$$

where C is an arbitrary constant. After substituting the initial condition into the above general solution, we can get $C = \left(m\big/k\right)v_0$ and $s(t) = \left(m\big/k\right)[v_0 - v(t)]$. It is known that the aircraft stops when $v(t)=0$. Finally, we obtain the sliding distance of the aircraft

$$s(t) = \frac{mv_0}{k} = \frac{9000 \times 700}{6.0 \times 10^6} = 1.05km \tag{7}$$

Alternatively, $v(t)$ can be directly solved as $v(t)=700e^{-2000t/3}$ from the definite solutions problem (1) and (2), and we know that the aircraft stops when $v(t)=0$. From

the exact result, we can get $v(t)=0$ as $t \to \infty$. Based on the definite integral, the sliding distance of the aircraft can be expressed as

$$s = \int_0^{\infty} v(t)dt = \int_0^{\infty} 700e^{-2000t/3}dt = 1.05\,\text{km} \tag{8}$$

To make students master the basic mathematical software and use it proficiently, we use Mathematica software to recalculate, as shown in Fig. 1. The detailed procedure of Expressions (1)–(8) is given as follows:

In Fig. 2, the sliding distance of the aircraft is just the gray area, which can be calculated via the definite integral.

By comparison, we can find that using Mathematica software can get results faster and can be displayed visually. In this example, mathematical software shows a positive effect on problem-solving. We should bring more applicable examples in teaching to introduce the use of mathematical software. In this way, higher mathematics would attract students' great interest and improve their abilities.

Most students are unable to make intellectual connections in higher mathematics learning. They believe that mathematical knowledge is isolated, and it is impossible to solve practical problems with higher mathematical knowledge. In this regard, after taking higher mathematics courses in the freshman year, we offer mathematical modeling courses in the sophomore year. The mathematical modeling courses facilitate students to learn and apply advanced mathematical knowledge to solve practical problems. For

```
In[•]:= m = 9000;
        k = 6 * 10^6;
        DSolve[{m D[v[t], t] + k v[t] == 0, v[0] == 700},
         v[t], t]
Out[•]= {{v[t] → 700 e^(-2000 t/3)}}

        v[t] = 700 e^(-2000 t/3);
        Plot[Callout[700 e^(-2000 t/3), "v(t)", 0.001],
         {t, 10^-17, 0.009}, PlotStyle → {Red},
         AxesLabel → {t, v[t]}, Filling → Bottom,
         FillingStyle → {Gray}]
In[•]:= v[t] = 700 e^(-2000 t/3);
        s = ∫_0^∞ v[t] dt
Out[•]= 21/20
```

Fig. 1. Mathematica program

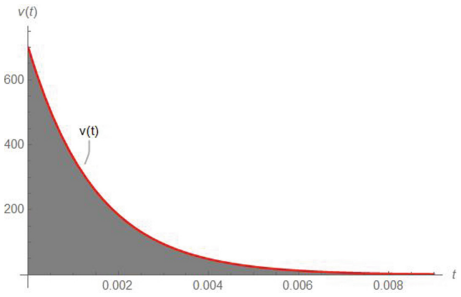

Fig. 2. Graphic of *v(t)*, units of vertical and horizontal axes are km/h and hour, respectively.

mathematical modeling courses, there should be tests at the mid-term and end of the course to focus students' attention on mathematical modeling and test their ability to model.

5 Advantages and Disadvantages of Hierarchical Teaching Based on Majors

In traditional teaching, students of different majors use the same teaching materials, syllabus, and lesson plans, and it is difficult for students to apply the learned higher mathematics knowledge to professional fields. Higher mathematics teaching based on majors adopts different teaching contents according to the requirements of majors. We closely integrate the knowledge of higher mathematics with majors and familiar fields. It can stimulate the students' interest and enthusiasm in learning higher mathematics. In addition, in traditional teaching, the knowledge and abilities of students in the same class vary greatly. For the same content, students with a good foundation and strong learning ability may find it too easy to meet their needs for knowledge, while students with relatively poor foundation and weak learning ability may find it difficult to keep up with the teaching progress. Hierarchical teaching can solve this problem well by arranging students with similar learning foundations in the same class. This will facilitate teachers to adjust the course design to meet the needs of students and improve teaching effectiveness. In addition, students can freely choose classes according to individual circumstances. This practice reflects that our teaching is student-centered.

The number of students is 42. In every unit test, the problems remained similar difficulty. From the data in Table 1, we can find that scores increased. Table 2 shows that the enthusiasm of the students gradually increased which can be reflected by the numbers of interaction and click of Rain Classroom. Since the introduction of the mathematical modeling course, the number of participants in the Mathematical Model Competition has doubled. Furthermore, the number of awards increases significantly.

Table 1. SCORES OF UNIT TESTS

Chapters	**1**	**2**	**3**	**4**	**5**
Average	73.4	79.2	79.4	81.2	83.5

Table 2. NUMBER OF INTERACTIONS AND CLICKS

Chapters	**1**	**2**	**3**	**4**	**5**
Interaction	12	23	27	34	38
Click	46	59	84	95	157

Note: The number of students is 42

Hierarchical teaching based on major benefits student development, but everything has two sides. Although hierarchical teaching is carried out according to academic performance and students' wishes, the method of dividing classes is not conducive to protecting self-esteem and is prone to lead to psychological gaps.

6 Conclusions

This paper mainly introduces the teaching reform of advanced mathematics. First of all, we arrange corresponding teaching according to the requirements of different majors for students, which is good for students to apply advanced mathematics to their majors. Secondly, we carry out hierarchical teaching in the case of similar requirements of majors. Students with similar knowledge foundations and abilities are grouped into the same class, and teachers use suitable teaching content and methods. In this way, we can ensure almost every student learns mathematics well, facilitate teaching students in accordance with their aptitude, and improve students' interest in advanced mathematics. In our teaching practice, we introduce related computer technology including multimedia technology, Mathematica and Rain Classroom, to serve our teaching and improve students' interest of learning and ability to analyze and solve real-world problems. We have achieved good teaching effects in this practice.

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