



Analysis of Student Learning Outcomes of Derivative Materials

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Abstract. This study is to describe the learning outcomes and difficulties faced by students of the Mathematics Education Study Program in derivative materials. The method used in the study is descriptive quantitative. The subject's research is 48 students in the first semester of the Mathematics Education Study Program at PGRI Palembang University. Data was collected through essay tests and analyzed by average values and interpreted. The results showed that the learning outcomes of derivative material of 76.7 included in the good category and the difficulty lies in implicit derivative material, chain rules, and maximum and minimum problem solving. Therefore, lectures should making learning developments for derivative materials and applications that can overcome student difficulties in learning calculus.

Keywords: derivative material · learning outcomes · difficulties

1 Introduction

The differential calculus course is one of the basic courses before proceeding to the next course. The syllabus in the course studies, among others, the real number system, functions, limits, derivatives, and the use of derivatives. In the 2016 curriculum of the Mathematics Education Study Program, the differential calculus course is a prerequisite for taking courses in integral calculus, advanced calculus, differential equations, and initial problems and values. Therefore, it is hoped that students have mastered the material in differential calculus before proceeding to the next course.

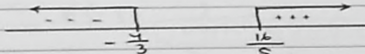
But in fact, some students still have difficulty solving problems related to derivative basic materials. Derivative material is given to first-semester students, where the material has been studied when in high school. The emphasis on learning concepts is due to students as prospective teachers of mathematics subjects.

In Figs. 1 and 2, students have difficulty determining the value of x on the absolute value material.

Figures 1 and 2 explain that students experience errors in using the concept of absolute value. Figure 1 explains error one, which is to decipher the absolute value using the concept of $|x| < a \Leftrightarrow -a < x < a$. Meanwhile, Fig. 2 explains error two, which is to decipher the absolute value by ignoring the absolute value sign. This shows students cannot distinguish between the use of absolute grades related to adversity. This derivative

$$\begin{aligned}
 a) \quad & 2|2x-3| < |x-10| \\
 & 14x-6 < |x-10| \\
 & 4x-8+8 < x-10+6 \\
 & 4x < x-4 \\
 & 4x-x < -4-x \\
 & 3x < -4 \\
 & x < -\frac{4}{3} \\
 \text{Hp: } & \{x \mid x < -\frac{4}{3}, x \in \mathbb{R}\}
 \end{aligned}$$

Fig. 1. Difficulties 1

$$\begin{aligned}
 \text{Jawab:} \\
 & 2|2x-3| < |x-10| \\
 & 14x-6 < |x-10| \\
 & -(x-10) < 4x-6 < x-10 \\
 & -x+10 < 4x-6 < x-10 \\
 & -x+10+6 < 4x-6+6 < x-10+6 \\
 & -x+16 < 4x < x-4 \\
 & -x+16 < 4x \qquad \qquad \qquad 4x < x-4 \\
 & -x-4x < -16 \qquad \qquad \qquad 4x-x < -4 \\
 & -5x < -16 \qquad \qquad \qquad 3x < -4 \\
 & x > \frac{16}{5} \qquad \qquad \qquad x < -\frac{4}{3}
 \end{aligned}$$


The diagram shows a horizontal number line with arrows at both ends. There are two points marked on the line: $-\frac{4}{3}$ and $\frac{16}{5}$. A vertical line segment with arrows at both ends is drawn above the number line between these two points, representing the interval $-\frac{4}{3} < x < \frac{16}{5}$. There are also dots on the number line to the left of $-\frac{4}{3}$ and to the right of $\frac{16}{5}$.

Fig. 2. Difficulties 2

basic material should be mastered by students because it will be related to the material for the use of derivatives. The results of the study [1] stated that students were still weak in presenting some concepts in some form of mathematical representation; students have difficulty using the first, and second derivative formulas, and have difficulty solving problem-solving problems [2], and most mistakes made by students were in process skills as much as 61% in solving trigonometric limit material questions [3].

Furthermore, understanding the concept of derivative material is very necessary, because derivative material is material related to the next course. Derived material is part of the material in the differential calculus course. While the differential calculus course is a prerequisite for taking the advanced calculus course. Therefore, it is necessary to analyze student learning outcomes regarding derivative material so that lecturers can develop learning that makes students understand and overcome student difficulties in solving problems related to derivative material.

Some previous studies on the difficulty of learning calculus include [4] calculus is seen by students as a difficult mathematics lesson, and students misunderstand the idea of function. Another study stated that the difficulty of learning calculus 1 in Informatics Engineering students is a low interest in learning and basic ability of calculus, assuming that Calculus 1 is not related to the Informatics Engineering Study Program[5]; student learning difficulties in integral application materials include difficulty drawing graphs,

determining integral boundaries, areas, and understanding integrals [6]; 55% of students have difficulty relating calculus knowledge to physics problems, both derivative and integral materials [7] and difficulty learning integral material due to weak students' understanding of the basic theorem concepts of calculus [8]. The results of the study obstacles in learning two-variable functions with the greatest difficulty in sketching graphs of two-variable functions in 3 dimensions [9]. The results of the study on difficulties in learning calculus state that the difficulty of students in general on drawing function graphs and performing trigonometric manipulations and specifically determining domains and ranges, lack of mastery of rules in determining the value of limit functions, determining maximum and minimum values in stories, errors using integral rules that are often used derivative rules and unable to distinguish the use of integral substitution techniques and partial as well as in completing the volume of the rotating object [10].

Based on the results of the research above, it is necessary to further examine the difficulties of learning calculus, especially in derived material, by paying attention to which part of the difficulty occurs in the material. The difficulties of learning calculus include difficulties in the description of limit restrictions on functions; difficulty translating real-world problems into calculus formulations; the use of Leibniz notation; difficulty in choosing the appropriate representation between numerical, symbolic, and visual; algebraic manipulations; difficulty absorbing complex new ideas in a limited time; difficulty dealing with summations in multiple definitions of magnitudes; difficulties for students who only use procedural without thinking or using flexible knowledge [11].

The purpose of this study is to describe the learning outcomes and difficulties faced by students of the Mathematics Education Study Program in derivative materials.

2 Method

The research subjects of students in semester 1 (one) of the Mathematics Education Study Program in the odd semester of the 2018/2019 Academic Year totaled 48 people. This research method is quantitative research. Data is collected through tests. The test in the form of essay questions totals 5 (five) questions, consisting of derivative material, tangent equations, chain rules, implicit derivatives, maximum and minimum problem solving, ascending and descending function intervals, upward and downward concave intervals, turning points, and sketching graphs using derivatives. Furthermore, the study was carried out on January 7, 2019. The data were analyzed using descriptive qualitative, calculated the percentage of learning outcomes based on the material on the test questions and then calculated the average and categories of learning outcomes. Then described a picture of the difficulties that occurred in some students who experienced errors in working on the test through interviews to find the cause of the difficulties.

3 Result and Discussion

Researchers conduct research at the end of the semester. Research data is the result of the final semester exam. Based on the results of the tests that have been processed, the data in Table 1 are obtained.

Table 1 explains that the average learning outcomes were 76.7 with good categories. This shows that there are still mistakes made by students in solving problems or problems. Although the concept of derivatives and tangent equations can be understood by students, namely 96% and 85%. However, on implicit derivative material of 58%, it means that students have difficulty in solving problems related to implicit derivatives. Then the 70% chain rule and the maximum and minimum problem solving also needs to be reviewed because it is only 76%.

Findings from student research results have many difficulties in implicit derivative materials, chain rules, and solving maximum and minimum problems. Figure 3 shows the error that occurred in solving problem number 3, determining the implicit derivative, namely, the derivative of and the tangent equation at the point (2,2). (Modified [12]).

Table 1. Percentage and Category of Learning Outcomes of Derived Material

| No | Material | % | Category |
|---------|---|------|-----------|
| 1 | Derivative | 96 | Excellent |
| | Tangent equation | 85 | Excellent |
| 2 | Chain Rules | 70 | Good |
| 3 | Implicit Derivatives | 58 | Enough |
| 4 | Maximum and minimum problem solving | 76 | Good |
| 5 | Maximum and minimum values | 86 | Excellent |
| | Interval of ascending and descending function | 96 | Excellent |
| | Interval of concave upward and downward | 84.4 | Good |
| | Turning point | 93.8 | Excellent |
| | Graphic sketch | 82 | Good |
| Average | | 76.7 | Good |

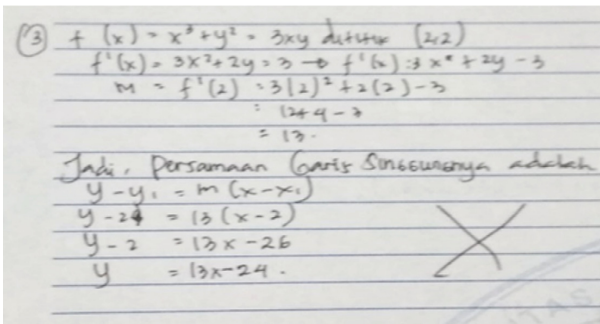


Fig. 3. Difficulties question number 3

The results of interviews between researchers and students (M01) include the following:

Researcher : “When you solve problem number 3, what strategy do you do “

M01 : “I will look for derivatives and tangent equations.”

Researcher : “How do you work on the derivative.”

M01 : “Initially I was able to work on the derivative but when in $3xy$ I was confused to work on it.”

Researcher : “Can you distinguish between derivative work on explicit and implicit functions”

M01 : “ I’m confused and don’t remember how to work on the derivation of the implicit function.”

In the results of the interview and the M01 job there are errors in the writing of the y^2 derivative against x and the $3xy$ derivative against x . M01 was unable to write correctly the notation of the derivative result and was confused when it was about to lower $3xy$. M01 writes the derived result notation as an explicit function. So it can be concluded that students are still unable to distinguish between explicit and implicit derivatives. The results of this study support research that the difficulty of students in understanding the derivative of implicit functions is 56% with sufficient categories and also research states that students’ ability to understand implicit derivative material is still low [12].

Furthermore, Fig. 4 shows the error that occurred in solving problem number 2.

Figure 4 shows an error that occurred in solving problem number 2, determining the derivative using the chain rule from $f(x) = \sin^4(x^3 + 3x)$ (modification [13]).

The results of interviews between researchers and students (M02) include the following:

Researcher : “Try to pay attention to question number 2, what you think of in order to be able to solve the problem”.

M02 : “I will use the derivative of the repeating chain rule.”

Researcher : “Derivative of the repeating chain rule, how is the procedure?”

M02 : “First I made a forgery. Then I took it down.”

$$\begin{aligned}
 2) \quad & f(x) = \sin^4(x^3 + 3x) \\
 & y = \sin^4 \rightarrow D_x y = 4 \cos^3 u \\
 & v = x^3 + 3x \rightarrow D_x v = 3x^2 + 3 \\
 & D_x y = D_x y \cdot D_x v \\
 & = 4 \cos^3(x^3 + 3x)(3x^2 + 3) \\
 & = (12x^2 + 12) \cos^3(x^3 + 3x)
 \end{aligned}$$

Fig. 4. Mistake question number 2

Researcher : “Good, but the forging you did is not yet correct, so the derivatives resulting from the forging are not correct. Take a look at the forging you made, $y = \sin^4$, what does this mean? Then the derivative result becomes $D_{x,y} = 4\cos^3 u$ ”

M02 : “Sorry ma’am, I was wrong, my forging should be $y = \sin^4 u$ ”

Researcher : “not yet right son, it should be $y = u^4$ so that the derivative results $D_{x,y} = 4u^3$. So $y = \sin^4(x^3 + 3x)$ is changed to $y = u^4$.”

M02 : “yes ma’am, sorry I’m not careful enough.”

The results of the interview showed that students still did not understand making forging and derivative procedures for repeat chain rules. The results of this study support research solving problems with a symbolic, numerical approach without visualization will make it difficult for students to learn calculus [11]. The same thing is conveyed that learning equipped with graphic visualization can make it easier for students of all knowledge levels [14, 15]; research about students have difficulty in applying derivative rules [16]; students difficulties in the skill of deriving functions, accuracy in using basic concepts of derivatives and flow of solving algorithms [17].

Furthermore, Fig. 5 shows the error that occurred in solving problem number 4.

Figure 5 shows the error that occurred in solving problem number 4, the maximum and minimum problem solving, namely: The garden cage is made of wire whose circumference is 150 m. Determine the sizes x and y in Fig. 6 that make the maximum garden area (modification [18]).

The results of interviews between researchers and students (M03) include the following:

Researcher : “When you do question number 4, what strategy do you do to solve the problem?”

M03 : “I made a mathematical model first of the problem. Then determine the maximum area using the first child.”

Researcher : “Yes, good. Try explaining how you got the mathematical model circumference = $2p + 4l$ ”

M03 : “ $l = p$ and $x = l$ so that the circumference of the cage is $2p + 2l$ ”

1. $K = 150$
dik rel = $2P + 2L$
 $150 = 2P + 2L$
* Dit = Luas maks
 $P = 150 - 2L$
 $P = 75 - L \dots (1)$
* $A = P - L$
 $A = (75 - L) \dots (2)$
* $f(x) = 75 - 2L$
 $75 - 2L = 0$
 $2L = 75$
 $L = 32,5 \dots (3)$
* (3) ke (1) $\rightarrow P = 75 - 32,5$
 $P = 42,5 \dots (4)$
Luas = $P \times L$
 $= 42,5 \times 32,5$
 $= 1381,25$
* Luas kebun maks = 1381,25 m

Fig. 5. Difficulties question number 4

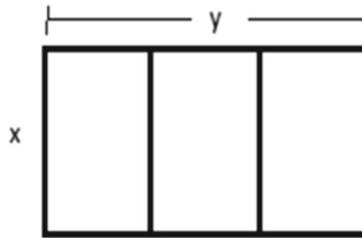


Fig. 6. Garden Cage

Researcher : “Let you pay attention to the y in the picture of the question, how many y lines are there?”

M03 : “There are two”

Researcher : “Good, you are right. Now consider the x in the image of the question, how many x lines are there?”

M03 : “There are four”

Researcher : “Good, you are right. So then how much is the circumference of the cage.”

M03 : “circumference = $2p + 4l$ ”

Researcher : “Yes, that’s right, so you have realized the mistake you made.”

M03 : “Yes, ma’am, I won’t repeat it again.”

The results of the interview showed that students made mistakes in making mathematical models. It should be Circumference = $2y + 4x$ or circumference = $2p + 4l$, if p is Length = x and l is width = y . So that the calculation determines the maximum area of the garden has an error. The results of this study support research [10] that students have difficulty in determining the maximum and minimum scores on story questions; difficulties in translating real-world problems into calculus formations [11, 19] and most students consider calculus subjects difficult or very difficult [20].

Based on the analysis of student work and the aforementioned interviews, students still have difficulty in terms of lowering the implicit function, unable to distinguish which function is explicit and which is implicit. Furthermore, students are still not right in making reasoning on derivatives using chain rules and repeated chain rule procedures. Students are still confused about the trigonometric function that is ranked, so the rank and trigonometric function are both directly lowered. Then on the problem-solving problem, students have difficulty translating real-world problems into calculus formations, and students are less observant in observing the problems presented.

4 Conclusions and Suggestions

Based on the results of the study, the results of learning derivative material for students of the Mathematics Education Study Program amounted to 76.7, including the good category. Students’ difficulty in learning derivative material is dominant in implicit derivative material, chain rules, and problem-solving is maximum and minimum. The error that occurs, students still cannot distinguish between explicit and implicit derivatives. Students still do not understand the derivative procedures of repeated chain rules, especially

in trigonometric functions. Students make mistakes in making mathematical models for problem-solving problems.

The findings of this study can be followed up by lecturers by making learning developments for derivative materials and applications that can overcome student difficulties in learning calculus.

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