



# Research on Risk Measurement of Carbon Markets Between China and EU

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**Abstract.** This paper measures the risk of the carbon trading market, and studies the risk spillover effect between markets, which provides a theoretical reference for the policymaking of the carbon market. This paper mainly studies the risk spillover effect between Risk spillover effects between China's carbon trading markets in Shanghai, Beijing, Hubei, Guangdong, Shenzhen and the EU. The ARMA-GARCH (1,1) model is specifically constructed to simulate the carbon prices of the above six carbon markets under the t distribution. In particular, Vine Copula is used to obtain the dependency structure diagram between the six carbon trading markets, and CoVaR and % CoVaR are used to quantify the intensity of risk spillovers. This method is used to study the income fluctuation and risk spillover effect of the carbon trading market. The results show that there is a one-way asymmetric risk spillover effect between the EU and China's domestic carbon trading markets in Shanghai, Beijing and Hubei. China's unified carbon trading market was established in Hubei, and the EU carbon trading market is the most mature carbon market in the world, among which the EU has the strongest spillover effect on Hubei carbon market.

**Keywords:** Carbon market · GARCH · Copula · CoVaR · Risk spillover effect

## 1 Introduction

The carbon financial market will generate carbon dioxide emission rights through options, futures, spot trading and other forms, which will be affected by the carbon financial market mechanism [1]. Therefore, the common risk measurement methods in the financial market can also be applied to the carbon market, such as sensitivity analysis, volatility method and value at risk (VaR) method [2]. Among many analysis methods, the Basel Accord and EU Capital Adequacy Ratio Guidelines all use VaR as the supervision standard, which is also the method used by most national central banks to measure risk [3]. It is also widely used in the carbon market and other energy markets. This paper attempts to use CVaR to measure the risk of carbon market. Generally, the return on financial assets is characterized by peak and fat tail. In recent years, Copula method is also widely used in the energy market and carbon trading market. This function can be used to solve nonlinear and asymmetric dependence structures, and is a powerful tool

to explore the dependence structures between markets. Although the existing research has reference value for the risk measurement and spillover effect research of the carbon market, there are still some shortcomings: first, among the existing risk spillover measurement methods, the multivariate GARCH model and the copula GARCH model are typical models used to measure the dependence of the carbon market. Although GARCH model can measure the risk spillover between markets well, it can only be used to measure the linear risk spillover, it cannot be used to measure the nonlinear risk spillover effect. Secondly, in the existing copula GARCH model, binary copula or multivariate copula or special C-vine and D-vine copula are more used, while the more general and flexible R-vine copula is less used. Multi-copula uses the same copula function to construct the market dependency structure. In fact, the dependencies between financial assets are often inconsistent with expectations. Finally, most studies focus on the dependence between carbon markets and other markets and the risk spillover effect, while there are few studies on the risk spillover effect between carbon markets. Therefore, it is a valuable work to apply CVaR to measure the risk of the carbon market in China and the EU, and R-vine copula CoVaR to measure the risk spillover effect of the carbon market. This paper is arranged as follows: In the first part, GARCH-R-vine copula CoVaR model is built to measure the risk spillover effect of carbon market; In the second part, the model is used to measure the risk of carbon market in EU and China, and compared with VaR. The third part is conclusion and suggestion.

## 2 Theoretical Model

### 2.1 Edge Distribution Model

Due to the characteristics of “peak and thick tail” of carbon price time series and the existence of autocorrelation and heteroscedasticity, this paper constructs ARMA-GARCH (1,1) model with good fitting effect on the edge distribution, and its structure is as follows:

$$\begin{cases} r_t = \mu + \sum_{i=1}^p \varphi_i r_{t-i} + \varepsilon_t + \sum_{j=1}^q \theta_j \varepsilon_{t-j} \\ \varepsilon_t = h_t \sigma_t, \\ h_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}^2 \end{cases} \tag{1}$$

where,  $\varphi_i (i = 1, 2, \dots, p)$  and  $\theta_j (j = 1, 2, 3, \dots, q)$  represent the parameters of ARMA model.  $r_t$  It is the carbon price gain in  $t$  days.  $\varepsilon_t$  is a residual term and  $\sigma_t$  a white noise sequence. It is generally assumed that it follows the standard normal distribution, standardized  $t$  distribution, partial  $t$  distribution, generalized error distribution (*GED*), etc.

### 2.2 Joint Distribution Model

Copula function is the connection function between marginal distribution function and joint distribution function. Sklar’s theorem [4]: Let the edge distribution function of

n-ary joint distribution function  $F(u_1, u_2, \dots, u_n)$  be  $F_1(u_1), F_2(u_2), \dots, F_n(u_n)$ , then there is a Copula function  $C : [0, 1]^n \rightarrow [0, 1]$ , so that:

$$F(u_1, u_2, \dots, u_n) = C(F_1(u_1), F_2(u_2), \dots, F_n(u_n)) \tag{2}$$

where  $\forall u_i \in [-\infty, +\infty], i = 1, 2, \dots, n$ .

For high-dimensional distribution, Joe put forward pair copula in 1996, aiming to use chain rule to decompose high-dimensional distribution into multiple binary distributions, and transform high-dimensional distribution into two-dimensional distribution for research. Because there are many pair copula structures corresponding to a high-dimensional distribution, Bedford and Cooke (2001) [5] proposed a graphical Vine model to intuitively describe the dependent structure between random variables. The number of variables in the model leads to different vine structures. When there are four variables in the model, they are simple C-vine and D-vine structures. When there are more than four variables in the model, a more general R-vine structure is added, including three vine structures. In this paper, Fuji Copula simulation can not only observe the cross dependency structure between multiple markets, but also analyze the volatility spillover effect of multiple carbon trading markets.

### 2.3 Risk Spillover Effect Measurement Based on CoVaR

CoVaR, as a risk measure, focuses on tail risk, satisfies the consistency axiom and subadditivity that VaR does not have, and can more accurately measure the risk of random variables. Adrain and Brunnermeier proposed a method to measure market risk spillover. When the maximum loss of the market is VaR, the maximum loss of the market is CoVaR, and the expression is as follows:

$$P(X^j \leq CoVaR_{\beta,t}^{j|i} \mid X^i \leq VaR_{\alpha,t}^i) = \beta \tag{3}$$

The relative risk spillover value of market i to market j is:

$$\%CoVaR_t^{j|i} = \frac{CoVaR_{\beta,t}^{j|i} - CoVaR_{\beta,t}^{j|i,\alpha=0.5}}{CoVaR_{\beta,t}^{j|i,\alpha=0.5}} 100\% \tag{4}$$

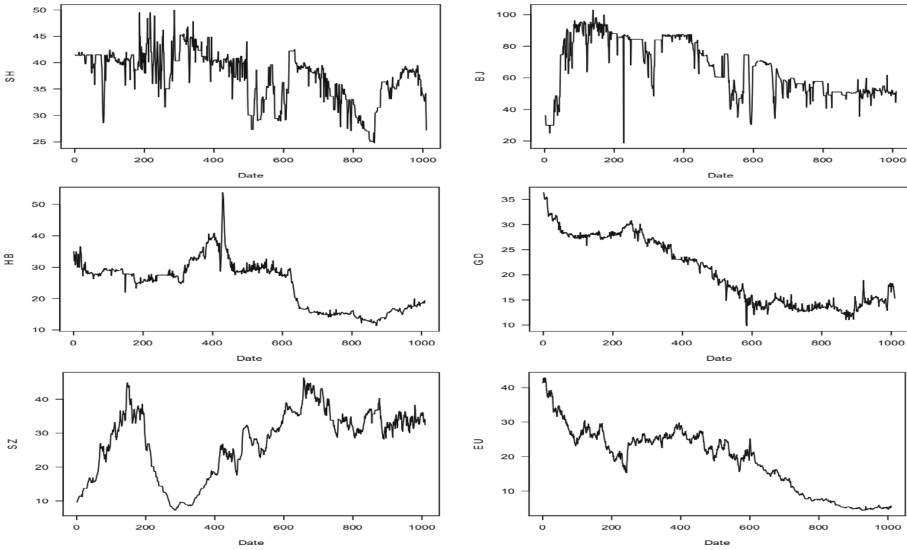
If the bivariate joint distribution copula function  $C$  of  $(X_i, X_j)$  is Archimedes copula, its generator is  $\phi$ ,  $\phi$  is strict and its first derivative  $\phi'$  is reversible, then the expression of  $CoVaR_{\beta}^{i|j}$  is [6]:

$$CoVaR_{\beta}^{i|j} = F_i^{-1}(\phi^{-1}(\phi(\phi'^{-1}(\frac{\phi'(F_i(VaR_{\alpha}^i))}{\beta})) - \phi(F_i(VaR_{\alpha}^i)))) \tag{5}$$

## 3 The Empirical Analysis

### 3.1 Data Selection and Processing

Since the two carbon markets in Fujian and Sichuan in China started late and their development lags behind other markets, this paper selects six carbon trading markets in Shanghai, Beijing, Hubei, Guangdong, Shenzhen and the European Union as research



**Fig. 1.** Time sequence of carbon market closing price

objects. The sample data is the daily closing price from January 3, 2017 to March 23, 2021 (1010 trading days), and the empirical research is mainly implemented by R software. The data in this paper are from the carbon K line and wind database.

In Fig. 1, the increase and decrease trend of carbon price is unstable. In order to reduce the error, the carbon price is logarithmically differentiated to obtain the carbon price yield:

$$R_{i,t} = \ln P_{i,t} - \ln P_{i,t-1} \tag{6}$$

### 3.2 Descriptive Statistics

According to the descriptive statistics in Table 1, the average carbon market in Beijing and Shenzhen is positive, while the other four markets are negative. The minimum values of the five markets are negative, and Beijing market has the largest price fluctuation range and degree. The most stable is the EU carbon market, which is consistent with the fact that the EU carbon market is the most mature carbon market in the world. The kurtosis coefficients are greater than 3, and the skewness values are greater than zero, indicating that all sequences show the characteristics of “peak and thick tail”. And the P values of J-B statistics are all zero, which indicates that all yield series refuse to obey the original assumption of normal distribution.

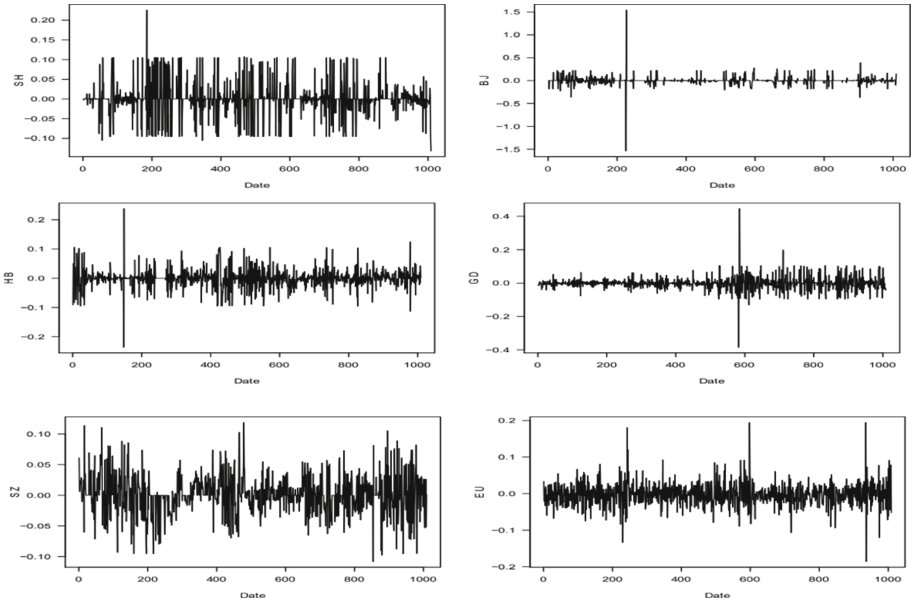
### 3.3 Establishment and Estimation of Edge Distribution Model Based on GARCH

According to the yield sequence diagram in Fig. 2, it can be seen that it has a volatility agglomeration effect. First, the stability test is carried out, and the results are shown in

**Table 1.** Descriptive Statistics of Yield Series

statistics	SH	BJ	HB	GD	SZ	EU
mean value	-0.00042	0.00035	-0.00059	-0.00086	0.00121	-0.00201
Max	0.22587	1.54343	0.23807	0.44612	0.11874	0.19467
Min	-0.13201	-1.53883	-0.23627	-0.38423	-0.10826	-0.18594
standard deviation	0.04237	0.09462	0.03232	0.03858	0.03202	0.03149
skewness	0.25167	0.22202	0.07653	0.81139	0.06221	0.40737
kurtosis	5.26995	142.06884	10.37920	31.28056	3.99861	8.48166
P-value of J-B statistic	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 2. The results of ADF single check root test and PP test show that the original hypothesis of non-stationary sequence is rejected, and the results of KPSS (level) test show that there is no sufficient evidence to reject the original hypothesis of stationary sequence. Secondly, the L-B autocorrelation test of the yield series with a lag of 5 orders is carried out. All the series have autocorrelation, so the order of the ARMA model is determined by the AIC, BIC and other criteria. After the L-B test of the yield series with a lag of 15 orders, the P values are greater than 0.05, and the autocorrelation of the series has been eliminated.



**Fig. 2.** Time Series of Carbon Market Yield

**Table 2.** Stability Test of Carbon Market Yield Sequence

statistics	SH	BJ	HB	GD	SZ	EU
ADF	-11.065	-12.586	-11.625	-12.646	-9.299	-10.249
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
PP	-799.26	-1184.434	-1143.952	-969.452	-1000.08	-1036.946
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
KPSS (level)	0.024	0.069	0.101	0.088	0.192	0.099
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)

Note: The corresponding P value is in brackets

Then the heteroscedasticity test is conducted on the sequence, and the results show that the sequence has heteroscedasticity, so GARCH model is used to eliminate the conditional heteroscedasticity of the sequence. Normal, t, partial t and GED distributions are used to fit the distribution of the normalized residuals. The optimal marginal distribution obtained by fitting is t distribution. The parameter estimation results of the marginal distribution model of each market are shown in Table 3.

Table 3 shows that most parameters of edge distribution are significant. It shows that ARMA-GARCH (1,1) model can better describe the volatility characteristics of carbon income series. The values of the six markets are close to 1 and less than 1, indicating that the current market shock has no lasting impact on the future conditional variance. Finally, the standardized residuals are transformed by probability integral to make them obey the distribution.

### 3.4 Establishment and Estimation of Joint Distribution Model Based on R-vine Copula

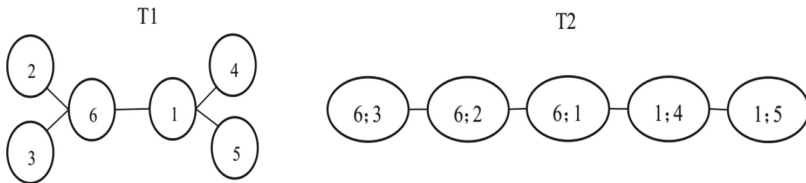
The obtained distribution is taken as the input variable of R-vine copula. According to Kendall rank correlation coefficient, the maximum spanning tree MST algorithm is used to establish the R-vine copula model. As shown in Fig. 3, it shows the dependency structure of the carbon market in the first and second trees of the rattan structure. The first tree is a general R-vine structure, and the second tree is a special D-vine structure. The Arabic numerals 1 to 6 represent the carbon markets of Shanghai, Beijing, Hubei, Guangdong, Shenzhen and the EU respectively. The structure related binary copula and parameter results are shown in Table 4. From the structure of the first tree, it can be found that in the four carbon trading markets of Shanghai, Beijing, Hubei and the European Union, the structure is centered on the European Union, while the carbon markets of Guangdong and Shenzhen are far away. It shows that the EU carbon market has a relatively direct impact on the carbon markets in Shanghai, Beijing and Hubei. Since the launch of China's regional carbon market, the development of Hubei's carbon market has been relatively mature. As the most mature carbon market in the world, the EU carbon trading market has played a certain role for China's carbon market in terms of trading mechanism and policy formulation, The most affected area is Hubei carbon

**Table 3.** Parameter Estimation Results of ARMA-GARCH (1,1) Model

parameter	SH	BJ	HB	GD	SZ	EU
$\mu$	-0.0002*** (0.0000)	0.0000 (0.0000)	0.0001 (0.0004)	-0.0006*** (0.0003)	0.0005 (0.0011)	-0.0023*** (0.0008)
$Ar1$	1.0000*** (0.0025)		-0.2203*** (0.0339)	0.3954*** (0.1297)	0.7033*** (0.0889)	-0.0551 (0.0328)
$Ar2$				0.0989 (0.0529)		
$Ar3$				-0.0011 (0.0379)		
$Ma1$	-1.0395*** (0.0000)	-0.0180 (0.0243)		-0.7214*** (0.1243)	-0.5497*** (0.1061)	
$Ma2$	0.0171 (0.0368)					
$Ma3$	-0.0204 (0.0372)					
$\omega$	0.0001*** (0.0000)	0.0000 (0.0000)	0.0001*** (0.0000)	0.0001*** (0.0000)	0.0001*** (0.0000)	0.0001*** (0.0000)
$\alpha$	0.6861*** (0.0058)	0.8787*** (0.0698)	0.6715*** (0.0923)	0.4152*** (0.0615)	0.4139*** (0.1131)	0.1276*** (0.0503)
$\beta$	0.3129*** (0.0447)	0.0970** (0.0203)	0.3275*** (0.0604)	0.5838*** (0.0565)	0.5804*** (0.0964)	0.7399*** (0.1357)
Shape	2.3925*** (0.0567)	2.2825*** (0.0231)	3.1798*** (0.2188)	3.3558*** (0.2552)	5.0355*** (0.0936)	6.6385*** (0.2499)
Log (L)	2194.037	2934.073	2358.203	2302.785	2142.145	2151.181

Note: The values in brackets represent the standard deviation of the corresponding values, shape represents the shape parameter, and \* \* \* represents significant at the 1% significant level

market, where carbon trading activity is relatively high and carbon price is relatively stable.



**Fig. 3.** Vine structure of six carbon markets

**Table 4.** Vine Structure Results

T	edge	Cop	par	tau
1	1,5	F	-0.26(0.19)	-0.03
	1,4	F	0.33(0.19)	0.04
	6,1	F	-0.30(0.19)	-0.03
	6,2	F	-0.43(0.19)	-0.05
	6,3	C270	-0.06(0.04)	-0.03
2	4,511	F	0.07(0.19)	0.01
	6,411	C90	-0.06(0.05)	-0.03
	2,116	F	0.15(0.19)	0.02
	3,216	F	-0.25(0.19)	-0.03

**3.5 Analysis of Risk Spillover Effect Under R-vine Copula CoVaR**

The VaR value and CVaR value are calculated by Monte Carlo simulation. In Table 5, the CVaR values of the six carbon markets are greater than the corresponding VaR values, indicating that the VaR value will underestimate the market risk. The CVaR value of Guangdong is the highest among the six markets, reaching 0.8120 at the 99% significance level, followed by Shenzhen. The CVaR value of the two carbon markets in Hubei and the European Union is the lowest. Although China’s carbon market was established relatively late, Hubei has established a national unified carbon financial market. The local government attaches great importance to its development. The management and development mechanisms are relatively perfect and mature, and the carbon price fluctuates less. As the most mature carbon market in the world, the EU is also very mature in market risk management, with small market risk and stable carbon price. The relative risk spillover value between EU, Shanghai, Beijing and Hubei carbon markets is calculated on the basis of the obtained rattan structure, as shown in Table 6.

It can be observed from Table 6 that the confidence levels of the risk spillover effect at 95% and 99% are 1.62%, 2.08%, 3.52% and 2.75%, 3.67% and 8.38% respectively.

**Table 5.** VaR Value and CVaR Value of Carbon Market

Carbon Market	VaR <sub>0.95</sub>	VaR <sub>0.99</sub>	CVaR <sub>0.95</sub>	CVaR <sub>0.99</sub>
SH	0.2175	0.3314	0.3293	0.5775
BJ	0.2016	0.4331	0.3690	0.7391
HB	0.0029	0.0175	0.0164	0.0535
GD	<b>0.2880</b>	0.5627	0.4709	0.8120
SZ	0.2747	0.5521	0.4531	0.8103
EU	0.1619	0.2719	0.2327	0.3648



**Table 6.** Relative risk spillover effect value between carbon markets

Carbon Market	EU → SH	EU → BJ	EU → HB	SH → EU	BJ → EU	HB → EU
% CoVaR <sub>0.05</sub> <sup>ij</sup> (%)	1.62	2.08	3.52	0.00	−.09	−0.14
% CoVaR <sub>0.01</sub> <sup>ij</sup> (%)	2.75	3.67	8.38	0.00	−3.89	−0.15

Note: “→” indicates the direction of risk spillover

Among them, the EU carbon trading market has the strongest risk spillover effect on Hubei carbon market, which means that for the losses faced by the EU carbon trading market, the risk spillover effect of Hubei carbon market accounts for 8.38% of its (Hubei) market losses (the confidence level is 99%). The other relative risk spillover values are not positive, indicating that the risk of the European Union carbon trading market on the carbon markets in Shanghai, Beijing and Hubei is net acceptance. Therefore, when managing the carbon market, Shanghai, Beijing and Hubei should also pay attention to the risk spillover effect of the development of the EU carbon market when considering their own carbon market development status, so as to improve various mechanisms and risk management strategies to better improve the carbon market.

## 4 Conclusions

This paper uses CVaR to measure the risk of China’s carbon market and the EU’s carbon trading market, and proposes to use R-vine copula CoVaR to study the risk spillover effect between Shanghai, Beijing, Hubei, Guangdong, Shenzhen and the EU’s carbon markets. The following conclusions are drawn. First of all, the carbon price in Guangdong and Shenzhen fluctuates greatly, and there are significant risks in the operation of the carbon market, while the risks in Hubei and EU markets are the least. Secondly, there is only one-way risk spillover effect from EU to Shanghai, Beijing and Hubei carbon markets. Among them, the EU carbon trading market has the strongest risk spillover effect on Hubei carbon market, which means that for the losses faced by the EU carbon trading market, the risk spillover effect of Hubei carbon market accounts for 8.38% of its (Hubei) market losses (the confidence level is 99%). Finally, China’s unified carbon market has just been built. On the way to maturity of China’s carbon market, we need to learn from the European Union’s carbon trading market experience in market management, further optimize the risk management system, timely adjust and improve the development and management mechanisms of China’s carbon market, so as to promote the development of China’s carbon market.

**Acknowledgment.** Young Scientists Fund of the National Natural Science Foundation of China (71903018); Youth Fund for Humanities and Social Sciences Research of the Ministry of Education (18YJC790148); Hunan provincial key project of educational information technology research (No. HNETR22042).

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