



# Investment Portfolio Establishment Based on Mean, Variance and Sharpe Ratio

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**Abstract.** The objective of this article is to reveal the evolution of the investment portfolio establishment approach. The evolution from the diversification effect, whose primary goal is to minimize risk, to the use of the Markowitz model to find the portfolio with the highest return per risk, to finally taking capital constraints into account, so that investors can find the best portfolio under the constraints of limited capital. We select three assets with low correlation, named Putnminc, Fidel and Keystne, then collecting and analyzing their monthly return values to illustrate the evolution process. Simultaneously, the impact of risk-free assets is taken into consideration.

**Keywords:** Investment portfolio · I.I.D assumption · Markowitz Mean Variance Model · Sharpe Ratio

## 1 Introduction

Asset allocation is an important link in the investment process, and it indirectly influences the performance of an investment portfolio [1]. The importance and role of asset allocation have gradually become apparent, which can help investors reduce the non-systematic risks of a single asset. From the perspective of actual investment needs, asset allocation aims to minimize investment costs, maximize investment yields, and eliminate influences by determining the proportions of different asset classes in a portfolio based on their past performance and investors' risk preferences.

With the hope of establishing an optimal investment portfolio, this article briefly analyzing different results drawing from different asset allocation models. Based on the principle that small data correlation means small mutual influence between assets [2], three assets are selected: Putnminc, Fidel and Keystne. After testing, we confirm that their monthly returns follow the normal distribution and identical independent distribution,

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which means their monthly returns follow the probability density distribution function, so these data are predictable [3].

An important purpose of asset allocation is to diversify risks [4]. We randomly assign weight coefficients to targeted assets through random simulation experiments to obtain a certain number of investment portfolios and establish the optimal investment portfolio by comparing their risks. The same method can be used to find the portfolio with the highest expected value. To enrich the research methods, we introduced the Markowitz model. Harry Markowitz is the creator of Markowitz's mean-variance model. He was the first to use the risk represented by the mean and variance to study asset portfolio and selection issues [5] and Kulali I tested the model with 252 days of data in 2015 [6]. Steinbach M C used Markowitz's mean-variance model to analyse portfolios. The method in his paper inspired us [7]. Marling, Hannes and Sara Emanuelsson did a more specific study and description of the Markowitz model [8]. This approach was used in our analysis. Finally, we used the Sharpe ratio within our analysis as well. Riondato estimated the Sharpe ratio and computed confidence intervals around a point estimation of the Sharpe ratio [9]. Bailey D H, Lopez de Prado M. used Sharpe ratio efficient frontier to optimize portfolios and this concept is used in our paper [10]. We use this method to establish the optimal investment portfolio under the premise of capital constraints.

In this paper, we look at how to establish a portfolio investment based on different principles. Firstly, we use normal distribution and identical independent distribution to verify our hypothesis based on selected data. Then, Markowitz's Mean-Variance Model and Index model are both used in our paper to solve the related problem. With the help of data from 9 funds, from January 1 to 12 of each year from 1968 to 1982 we searched from the Yahoo database, built different models and illustrated three portfolios. The first method we capture the lowest risk figured by simulation experiment of random weight experiments. The second method is to calculate the highest expected return. The last method is to calculate the Sharp Ratio. Based on the above analysis we identify the applicable Sharpe ratio and weights of portfolios of these three assets, finding the optimal point in the plots. We have compared all the three methods. In the first model we focused on minimizing risks and to some degree, ignored profit. Through the second approach, we tried to capture the most profitable portfolio but in the meantime, we neglected the existence of risks. The last portfolio we captured has the highest Sharpe Ratio, which means that the additional amount of return an investor receives per unit of increase in risk is maximum.

The remainder of this article is organized as follows. In Sect. 2, we give a set of data that used in our paper. And next in Sect. 3, we use the data to test two hypothesis: Normal distribution and IID Assumption. In terms of sect. 4, we try to build portfolios from three different perspectives: the portfolio with the lowest variance, the portfolio with the highest expected return and the portfolio with the highest Sharpe Ratio. Finally, we conclude our results and explain some limitations.

## 2 Data Selection

We collect monthly return values of 10 assets during 1968 and 1982 and draw a descriptive statistical table for those data shown in Table 1. It is clear to see that, among the 10 assets, Eqmrkt has the biggest mean (0.011), and T-bills has the smallest standard

**Table 1.** Descriptive statistic

	Drefus	Fidel	Keystne	Putnminc	Scudinc	Windsor	Eqmrkt	Valmrkt	Mkt	Tbill
Mean	0.007	0.005	0.007	0.006	0.004	0.01	0.011	0.007	0.007	0.006
Std.dev	0.047	0.056	0.084	0.03	0.036	0.049	0.068	0.048	0.048	0.003
Min	-0.126	-0.159	-0.332	-0.079	-0.106	-0.145	-0.192	-0.118	-0.122	0.003
Max	0.171	0.21	0.22	0.115	0.122	0.187	0.333	0.164	0.166	0.014

**Table 2.** Correlation coefficient of assets

	Drefus	Fidel	Keystne	Putnminc	Scudinc	Windsor	Eqmrkt	Valmrkt	Mkt	Tbill
Drefus	1.000									
Fidel	0.942	1.000								
Keystne	0.865	0.876	1.000							
Putnminc	0.664	0.573	0.557	1.000						
Scudinc	0.798	0.760	0.697	0.833	1.000					
Windsor	0.912	0.876	0.810	0.638	0.817	1.000				
Eqmrkt	0.843	0.823	0.831	0.580	0.732	0.896	1.000			
Valmrkt	0.958	0.954	0.863	0.630	0.813	0.923	0.872	1.000		
Mkt	0.959	0.957	0.869	0.632	0.814	0.928	0.879	0.999	1.000	
Tbill	-0.025	-0.069	-0.061	0.036	-0.019	-0.032	-0.065	-0.077	-0.075	1.000

deviation (0.003), while Keystne has the minimum monthly return value (-0.332), and Eqmrkt has the maximum monthly return value (0.333).

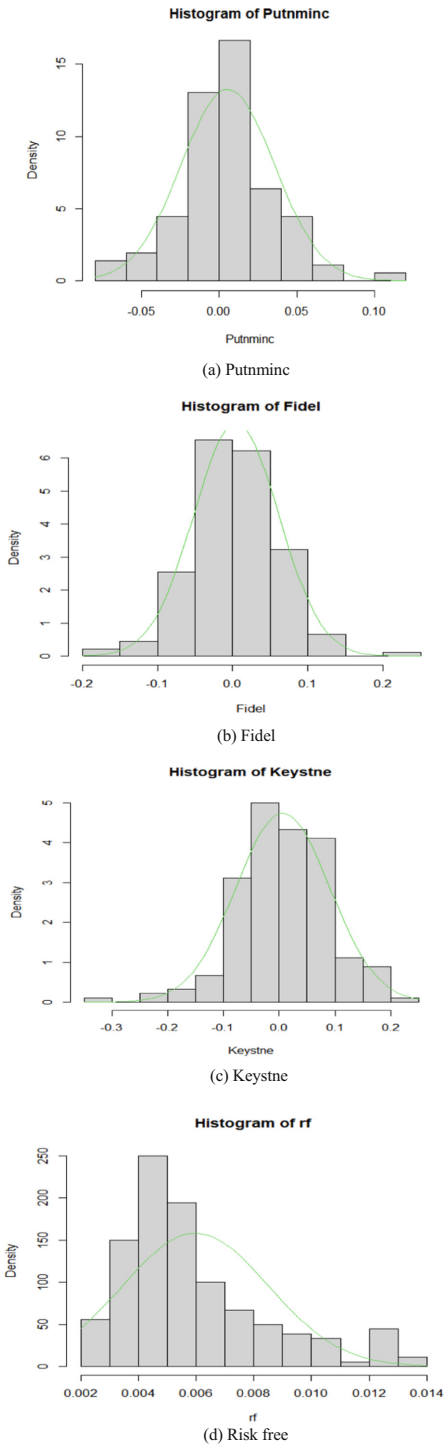
We calculate the correlation coefficient of the 10 assets. The result is presented in the Table 2. The correlation coefficient of Putnminc and Fidel (0.573), and Putnminc and Keystne (0.557) are the two minimum values. Since small correlation indicates that the mutual influence between assets is small, assets with small correlation coefficients are more worth investing in. Therefore, we choose Putnminc, Fidel and Keystne as our data sample for further analysis.

### 3 Hypothesis Test

In this section, we test the monthly return rate of the selected assets to verify whether they satisfy normal distribution and IID assumptions.

#### 3.1 Normal Distribution

We drew the frequency histogram of monthly return on the four assets and then drawing the normal distribution curve with the mean and variance of the monthly return of each asset. The results are shown in Fig. 1. It is clear that the decreasing and increasing trend of the frequency histogram is roughly the same as that of the normal distribution curve.



**Fig. 1.** Normal distribution test

**Table 3.** Parameters of normal distribution

	Putnminc	Fidel	Keystne
Mean	0.006	0.005	0.007
Std.dev	0.03	0.057	0.084

This result indicates that the return data of Putnminc, Fidel and Keystne follow the normal distribution. However, when it comes to the data of Tbill, the situation changes. The centre of the histogram and the curve are quite different. Therefore, the data of Tbill do not satisfy normal distribution.

We calculated the average and standard deviation of the monthly returns of the three assets which conform to the normal distribution and the result is shown in Table 3. It is clear that Keystne has the biggest mean (0.006), while Fidel has the smallest mean (0.004). Keystne also has the biggest standard deviation (0.084) while Putnminc has the smallest Standard Deviation (0.030).

### 3.2 IID Assumption

Through the independent identically distributed (IID) hypothesis test on the selected three assets, the test results are shown in Fig. 2, the correlation of the monthly return values is between  $-0.2$  and  $0.2$ . Those are small numbers, which means the monthly returns values of these 3 funds are independent. So, the data of these 3 assets satisfy the IID assumption. The monthly return values have strong correlations (above 0.5) with the data around 5 months, which means the data of T-bills do not satisfy the IID assumption.

## 4 Portfolio Establishment

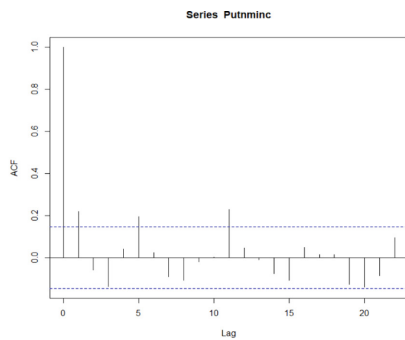
Portfolio establishment can be divided into 3 sections. Firstly, assigning weights to target assets through random simulation experiments to get a certain amount of portfolio, and only consider the risk dimension to find the optimal investment portfolio. Secondly, introducing the Markowitz model considers the two dimensions - risk and expectation, to arrive at the optimal investment portfolio. Thirdly, based on the principle of the Sharp ratio, we incorporate the capital factor into our analysis and draw the efficient frontier, which situates the tangent point and yields the optimal investment portfolio.

### 4.1 Diversification Effect

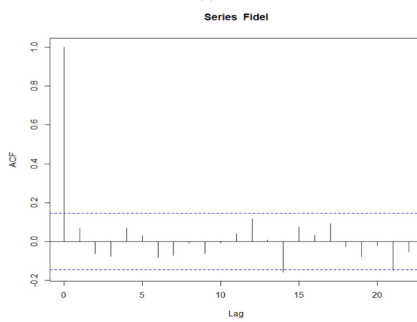
In this section, we only start from the perspective of risk through randomly allocating weight coefficient and comparing risk per unit to explore the optimal weight of the portfolio.

#### 1) Constraint Condition

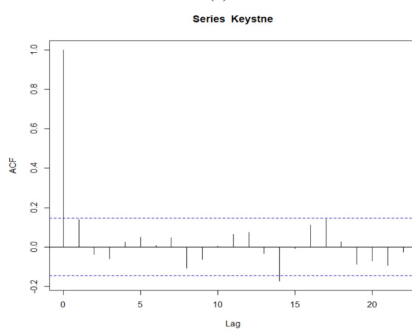
*a) A Portfolio Without Risk-Free Asset*



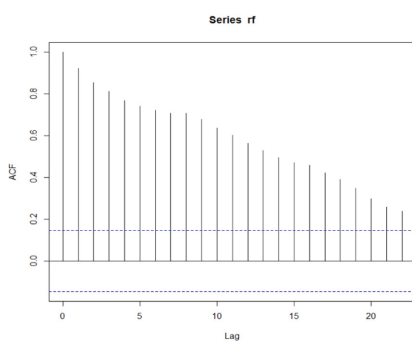
(a) Putnminc



(b) Fidel



(c) Keystne



(d) Risk free

**Fig. 2.** IID assumption test

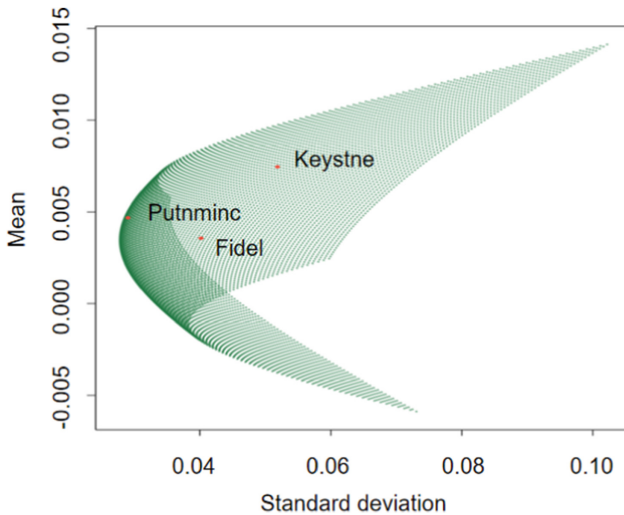
A portfolio without risk-free assets contains Puntminc, Fidel and Keystne. Generally speaking, the sum of the weight coefficient of the portfolio is 1. Under the premise that the sum of random arrays is 1, we select random numbers from  $-1$  to  $2$  and assign them to the selected three assets. Thus, we got a certain amount of portfolio. Furthermore, we use the variance of the portfolio as the abscissa and the mean as the ordinate to draw the scatter diagram as shown in the Fig. 3. As we can see that the red point refers to the portfolio with the least risk because it has the smallest standard deviation.

When we print out the weight coefficient of the redpoint, we get the weight coefficient table as shown in Table 4.

*b) Portfolio With a Risk-Free Asset*

A portfolio with risk-free assets contains Puntminc, Fidel, Keystne and Tbill. We select random arrays with a sum 1 from  $-3$  to  $4$  and assign them to these four assets, so we get a certain amount of portfolio. Similarly, according to the principle mentioned above, standard deviation as abscissa and the mean as ordinate, we draw the scatter diagram as shown in Fig. 4. The leftmost point in the scatter diagram represents.

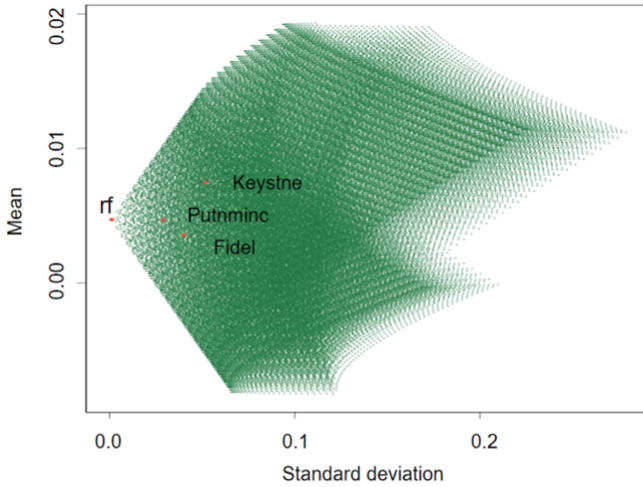
The leftmost point (redpoint) in the scatter diagram represents the portfolio with the least risk. Interestingly, it almost coincides with the portfolio that putting 100% of the capital into T-bills. If printing the weight coefficient of the redpoint out, we get the weight coefficient table as shown in Table 5.



**Fig. 3.** Portfolio without risk-free asset (SD-Risk)

**Table 4.** Weight coefficient of Portfolio without risk-free asset

	Putnminc	Fidel	Keystne
Portfolio 1	-0.254	0.271	0.983



**Fig. 4.** Portfolio with risk-free asset (SD-Risk)

**Table 5.** Weight coefficient of the portfolio with a risk-free asset

	Putnminc	Fidel	Keystne	Tbill
Portfolio 2	-0.085	0.085	-0.034	1.034

## 2) Results

We obtain two optimal portfolios through the above discussion, which are considering and not considering risk-free assets, and the results are shown in Table 6. It is clear to see that the optimal portfolio without risk-free assets (Portfolio1) includes Putnminc, Fidel, Keystne and T-bills, and their weight are  $-0.254$ ,  $0.271$  and  $0.983$  respectively. Furthermore, the optimal portfolio which considers risk-free assets (Portfolio2) contains Putnminc, Fidel, Keystne and Tiblls with the weight of  $-0.085$ ,  $0.085$ ,  $-0.034$  and  $1.034$  respectively.

As we can see from Table 7, although the mean of portfolio 1 ( $0.028$ ) is almost 7 times larger than that of portfolio 2 ( $0.006$ ), the return per risk of portfolio 2 ( $1.617$ ) is almost 9 times more than that of portfolio1 ( $0.177$ ). In addition, the standard deviation, which can also be regarded as a risk, of portfolio 2 ( $0.004$ ) is slightly smaller than that of portfolio 1 ( $0.005$ ). Therefore, portfolio 2 is a better choice.

**Table 6.** Optimal weight coefficient

	Putnminc	Fidel	Keystne	Tbill
Portfolio 1	$-0.254$	$0.271$	$0.983$	
Portfolio 2	$-0.085$	$0.085$	$-0.034$	$1.034$



**Table 7.** Descriptive statistical table of the 2 obtained portfolio

	Portfolio 1	Portfolio 2
Mean	0.028	0.006
Std.dev	0.005	0.004
Return per risk	0.177	1.617

## 4.2 Leveraging

In the second section, we will first introduce the Markowitz Mean Variance Model, and briefly explain this model's application methods and usage scenarios. The Markowitz model takes both risk and return into consideration to analyze investment portfolios. We use assets selected to conduct practical applications and discuss the limitations of the model.

### 1) Introduction

Asset Allocation refers to the allocation of investment funds among different asset classes according to investment needs. Usually, assets are allocated between low-risk, low-yield securities and high-risk, high-yield securities [11].

Asset allocation is one of the most important links in the investment process, and it is also the main factor that determines the relative performance of the investment portfolio. According to the research on Asset allocation and portfolio performance, the contribution rate of asset allocation to portfolio performance is constantly increasing [1]. The importance and role of asset allocation have gradually become apparent, which can help investors reduce the non-systematic risks of a single asset. From the perspective of actual investment needs, asset allocation aims to determine the proportions of different asset classes in the portfolio based on the historical performance of asset classes and investors' risk preferences, to reduce investment risks and, increase investment returns.

### 2) Introduction to Markowitz Model

There are two core issues that investment needs to solve first: the expected return and risk. So how to determine the risk and return of portfolio investment and balance these two indicators for asset allocation are issues that market investors urgently need to solve. The solution to making a trade-off between "expected return" and "variance of return" is Markowitz Mean-Variance Model.

Markowitz pointed out that investment portfolio risk is not simply adding up the risks of various assets, but considering the correlation between them [5]. The risk of determining the investment portfolio is not the risk of the assets themselves but the mutual influence.

The mean value refers to the expected rate of return of the portfolio. Variance measures the deviation between the actual rate of return and the mean and characterises the risk of the investment portfolio [12]. Assuming that investors are risk averse, they will choose less risky assets under the same level of return. Similarly, when the level of risk is the same, investors will tend to choose high-yield assets.

The most efficient point of the Markowitz Model is the efficient frontier. Following the logic mentioned above, the efficient frontier represents all the most effective risk

asset portfolios. In the mean-variance coordinate system, the frontier of the portfolio is a parabola, and the return rate is higher than the return rate corresponding to the minimum variance portfolio.

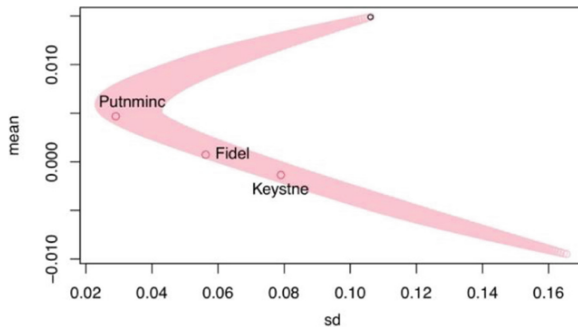
**3) Practical Application of Markowitz Model**

Using the same drawing method mentioned above, we draw the graphs shown in Figs. 5 and 6. The highest point in each graph represents the portfolio with the highest return. It can also be seen from Fig. 5 that only investing in one certain asset is not able to obtain the maximum benefit, because their points are not on the efficient frontier of the Markowitz Mean-Variance Model.

The effective frontier is a parabola, each point in this parabola represents an efficient portfolio [8]. Since the largest profit is required, only the highest point located at the upper right of the parabola satisfy. Therefore, we can get the result in Table 8. by analyzing the target point on the upper right, which is a reasonable investment portfolio and can obtain considerable profits.

Markowitz’s effective frontier only considers risky assets, and no risk-free assets appear here. When risk-free assets are added to the market, investors can improve portfolio performance by investing in risk-free assets, so the original Markowitz efficient frontier is no longer effective.

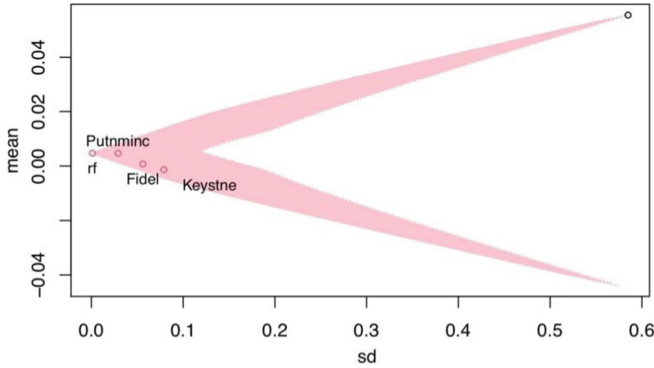
After adding the risk-free assets, re-draw and get Fig. 6. After introducing risk-free assets, capital can be freely allocated between risk-free assets and risky asset portfolios. When risk-free assets are introduced, the return on investment is significantly higher. At this time, the investment portfolio corresponding to the target point in the upper right corner is the data in Table 9.



**Fig. 5.** Portfolios without risk-free asset (Mean-Expectation)

**Table 8.** The weight of each asset when the return is maximum (without risk-free asset)

	Putnminc	Fidel	Keystne
Weight	2.000	2.000	-3.000



**Fig. 6.** Portfolios with risk-free asset (Mean-Expectation)

**Table 9.** The weight of each asset when the return is maximum (with risk-free asset)

	Putnminc	Fidel	Keystne	Tbill
Weight	4.000	4.000	-11.000	4.000

#### 4) Result

Investors want to choose the maximum return, so choosing the highest point in the effective front must be suitable, just like the investment portfolio that appears in Tables 8 and 9. In fact, through the Markowitz model, an optimal combination can be obtained for each target rate of return. Investors can find a balance between risk and return. By comparing Figs. 5 and 6, we can also see that after the introduction of risk-free assets, the efficient frontier of the investment portfolio composed of risk-free assets and multiple risk assets is different, which significantly improves the return of the same risk. Income provides investors with more considerable investment options.

Although the Markowitz model is a reliable analytical tool, it still has shortcomings in practice. The Markowitz model measures the return with the expected rate of return and measures the risk with the variance of the rate of return. But in practice, these return rates may be different. We will go into detail about this shortcoming in the conclusion part.

#### 4.3 Sharpe Ratio

After considering about diversification effect and leveraging, the Sharpe Ratio is the next compared factor in this essay. The definition and formula of the Sharpe Ratio are introduced at first. Subsequently, we explained the function of the Sharpe Ratio and introduced what is a capital market line. Finally, according to whether with the risk-free asset or without risk-free asset, judging which one is the optimal Sharpe Ratio in these assets we choose.

### 1) Definition

After considering about diversification effect and leveraging, the Sharpe Ratio is the next compared factor in this essay. The definition and formula of the Sharpe Ratio are introduced at first. Subsequently, we explained the function of the Sharpe Ratio and introduced what is capital market line. Finally, according to whether with risk-free asset or without the risk-free asset, judging which one is the optimal Sharpe Ratio in these assets we choose.

### 2) Function

The Sharpe Ratio was created by William F. Sharpe and is named by his name. It is utilised to assist investors clearly understand the return of an investment compared to its risk. Volatility is a measure of the price movement of an asset or portfolio [9]. Here is the formula below:

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad (1)$$

$R_p$  = return of the portfolio.

$\Sigma_p$  = standard deviation of the excess portfolio return.

$R_f$  = risk-free rate.

The Sharpe Ratio is simply calculated by taking the average of the fund's NAV (Net Asset Value) growth rate minus the  $R_f$  (risk-free rate) and dividing it by the standard deviation of the fund's NAV growth rate to obtain the fund's Sharpe Ratio. It reflects the extent to which the growth rate of the fund's NAV per unit of the risk exceeds the risk-free rate of return. A positive Sharpe Ratio shows that the average NAV growth rate of the fund exceeded the risk-free rate during the measurement period. That makes it better to invest in the fund than in bank deposits if the bank deposit rate is used as the risk-free rate over the same period. A higher Sharpe ratio indicates that the fund is earning a higher risk-reward per unit of risk. When the Sharpe ratio is negative, ranking by size is meaningless. The Sharpe Ratio uses the capital market line (Fig. 7) as a benchmark for evaluating investment performance.

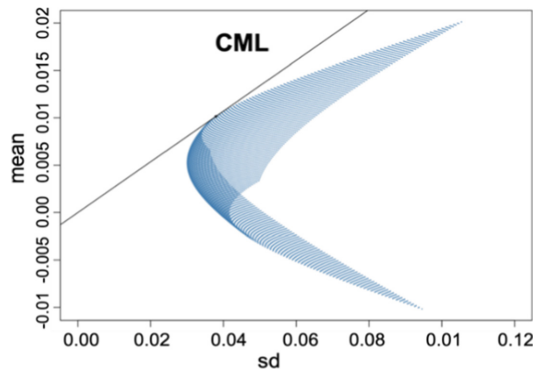
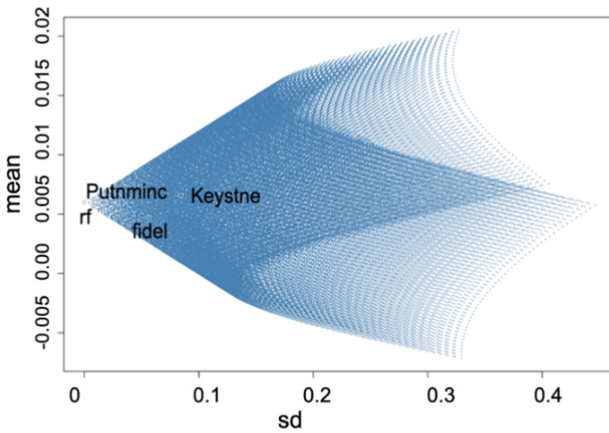


Fig. 7. Capital market line (black line)

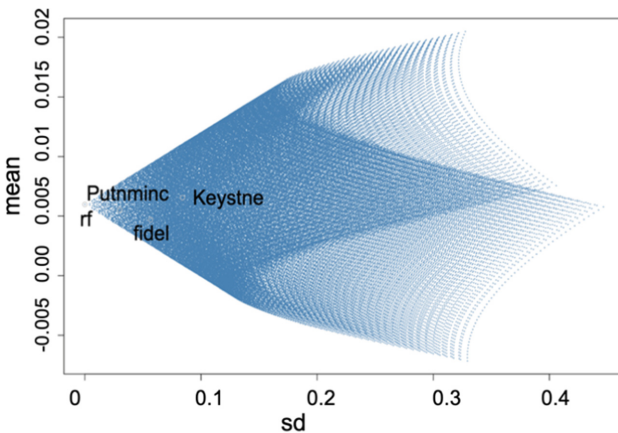
**3) Constraint Condition**

*a) With a Risk-Free Asset*

The reason why we choose these three assets is introduced above. We take into consideration of the risk-free asset at first. In terms of the risk-free asset, there are four assets that we need to calculate. Therefore, creating the code of four weights of each asset and derived the value of weights by using circulation in R. From this progress, we obtain the mean and standard deviation of each asset, including the risk-free asset (it got zero standard deviation). And then, the mean-standard deviation plot with the risk-free asset is demonstrated below (Fig. 8). It can be seen that the highest risk and return in this mean-standard deviation plot with risk-free asset plot. Meanwhile, these two plots (Figs. 8 and 9) also present each fund's applicable weight, which ensures the best portfolio of these three risk assets and one risk-free asset.



**Fig. 8.** Mean-std plot with a risk-free asset



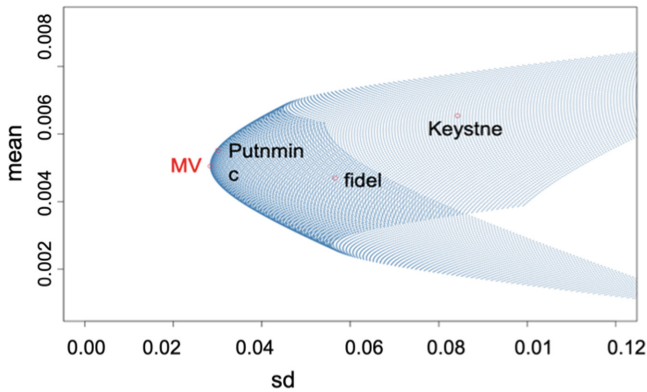
**Fig. 9.** Mean-std plot with risk-free asset adding three risky assets

*b) Without Risk-Free Asset*

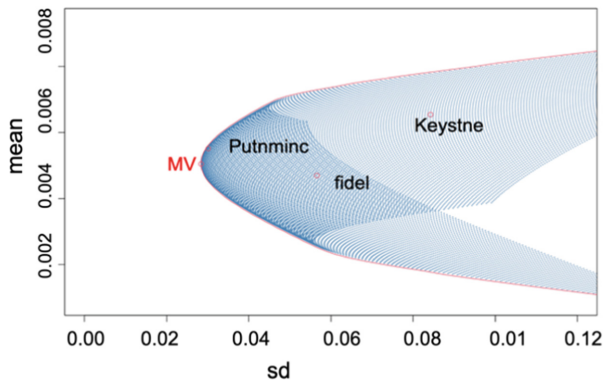
As for without the risk-free asset, we only need to consider three assets, not including T-bills. Similarly, creating three weights of each asset and find each optimal weight and the Sharpe Ratio of the portfolio. We found the distribution of assets in the plot below (Fig. 10).

And then find out where is the efficient frontier in this plot. It can be seen that there is a red parabola line in this plot which is called frontier. (Fig. 11). Here we can see that a black line at the top of this plot from minimum variance to the peak, which is called the efficient frontier. The portfolio with the highest Sharpe ratio would have been the tangent point with the black line passing through the efficient frontier. So here we can see the black point, which is the highest Sharpe ratio (Fig. 12).

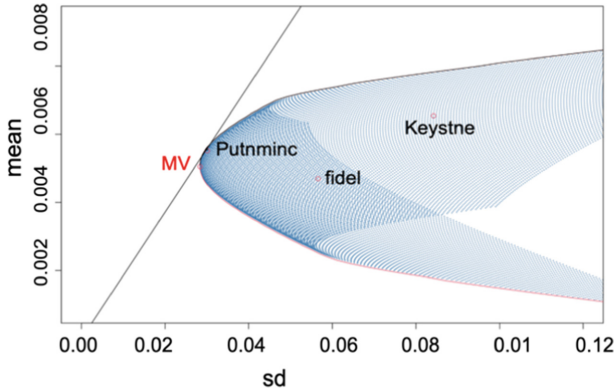
Finally, demonstrating the weight of each asset in an optimal portfolio and using the Sharpe Ratio plot illustrate that portfolio that maximises the value of Sharpe Ratio with the increasing index. However, we could see that the index from 8 to the end



**Fig. 10.** Mean-std plot without risk-free asset



**Fig. 11.** Mean-std plot without risk-free asset with frontier



**Fig. 12.** Mean-std plot without risk-free asset with the efficient frontier

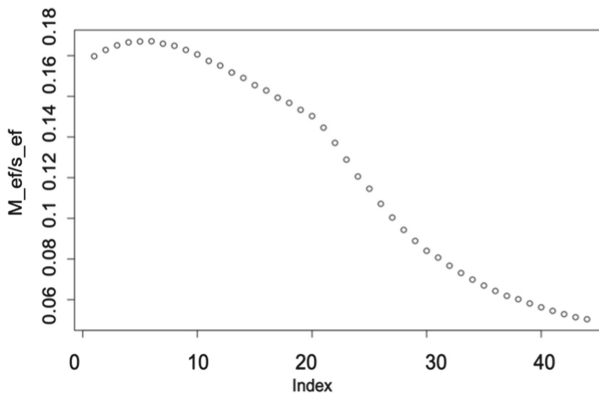
is continuous descending. This results from the efficient frontier, which is a parabola, reaching the peak point then declining (Fig. 13).

**4) Result**

*a) Portfolio with a Risk-Free Asset*

At first, to calculate the Sharpe Ratio of each asset, we use the mean and standard deviation of these four assets below (Table 10). To speak of T-bills, the standard deviation of risk-free asset (Tbills) is 0. Therefore, the Sharpe Ratio of Tbills is definitely zero. Keystne has the highest mean (0.65%) as well as standard deviation (8.42%). This is regular because generally speaking the higher risk gains a higher return. As for Putnminc and Fidel, their means are lower than that in risk-free asset and standard deviation is not zero.

And weights of each asset are added with two lines (Fig. 14). Also, the Sharpe ratio is calculated by the mean of each asset divided by the standard deviation of each asset (Table 11 below). Apparently, because of the higher mean and lower standard deviation, the Sharpe ratio of Putnminc is the highest. In terms of weights, the optimal choice



**Fig. 13.** Sharpe ratio plot without risk-free asset

**Table 10.** Mean and standard deviation of four assets

	Mean	Standard Deviation
Putnminc	0.006	0.030
Fidel	0.005	0.057
Keystne	0.007	0.084
T-bills	0.006	0

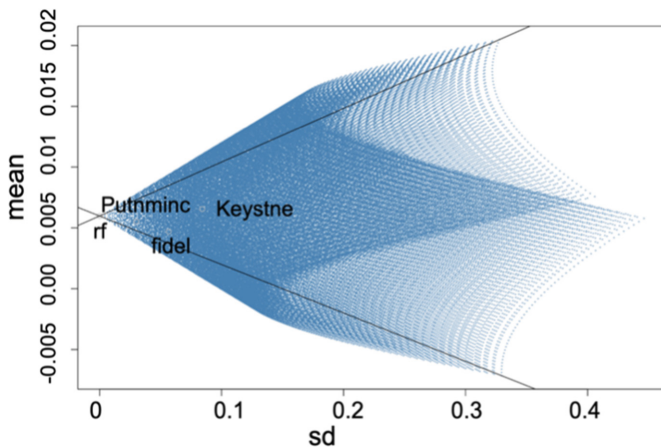
is absolutely allocating more to the risk-free asset. Therefore, T-bills is allocated the greatest weight.

*b) Portfolio Without Risk-Free Asset*

We calculate the weights and Sharpe ratio of each risky asset (Table 12). We talked about Putnminc above. This asset is better than others. Therefore, we put more weight on it. Meanwhile, the Sharpe Ratio of minimum variance is also calculated, at 0.1779 which is the minimum risk of all variety of assets.

**Table 11.** Weights and Sharpe Ratio of four assets

	Weights	Sharpe Ratio
Putnminc	-3	0.183
Fidel	-3	0.083
Keystne	-3	0.078
T-bills	10	0



**Fig. 14.** Mean-std plot with risk-free asset adding two lines



**Table 12.** Weights and Sharpe Ratio of three assets.

	Weights	Sharpe Ratio
Putnminc	1.094	0.183
Fidel	-0.074	0.083
Keystne	-0.020	0.078

**Table 13.** Two optimal portfolios

	Sharpe Ratio
Portfolio 1	0.098
Portfolio 2	-0.007

**5) Summary**

Finally, we summarise the above analysis in this paragraph. Here are two optimal portfolios in Table 13. Portfolio 1 is without the risk-free asset, and portfolio 2 is including the risk-free asset. Obviously, the Sharpe Ratio of the portfolio with the risk-free asset is lower than that in a portfolio without the risk-free asset because the Sharpe ratio of the risk-free asset is closed to 0. (standard deviation is 0 in T-bills). Hence, portfolio 1 is superior to portfolio 2.

**4.4 Comparison**

The first two portfolios we captured by Diversification Effect have the lowest risk. It is the steadiest one that meets the needs of conservative investors. We got two portfolio through this method. One portfolio without risk-free asset consists of -0.254 Putnminc, 0.271 Fidel and 0.983 Keystne. The weight of portfolio with risk-free asset is -0.085 Putnminc, 0.085 Fidel, -0.034 Keystne and 1.034 tbill. Whereas low risks mean low return. Earning of this portfolio may not achieve some investors' expectations. The second two portfolios are calculated by Markowitz Mean-Variance Model (M.M.V model), having the highest expected return. The portfolio without risk-free asset is made up of 2.000 Putnminc, 2.000 Fidel and -3.000 Keystne. The portfolio with risk-free asset is made up of 4.000 Putnminc, 4.000 Fidel, -11.000 Keystne, and 4.000 Tbill. It is suitable for those investors who are ambitious to maximize profit. However, to some extent, this portfolio neglects the existence of risks. The Sharp Ratio calculates the last portfolio that pays the most per unit of risk. During the establishment of this portfolio, we took returns as well as risks into consideration. The portfolio without risk-free asset is comprised of 1.094 Putnminc, -0.074 Fidel and -0.020 Keystne. The portfolio with risk-free asset is comprised of -3 Putnminc, -3 Fidel, -3 Keystne and 10 tbills. The portfolios we provided have the highest Sharpe Ratio which means this one has the highest investment efficiency. It can satisfy most investors' demands.

## 5 Conclusion

All in all, we analyse the data from the Yahoo database which is called 'mfunds' contains 9 risky assets and a risk-free asset. We wish to find three suitable assets, and then analyse them with and without the risk-free asset. Subsequently, calculating each asset's mean and standard deviation, choose the most profitable portfolio in the three assets.

Then, we analyse the correlation between a range of assets to choose the three most applicable assets that we used and created the distribution histogram to identify the three assets whether or not it is a normal distribution. Meanwhile, using time series plots to check if the I.I.D assumption is reasonable in these three assets. Next, we analyse the chosen assets in three aspects: diversification effect, leveraging, and Sharpe ratio. Based on Markowitz Mean-Variance Model (M.M.V model), we identify the optimal Sharpe ratio and weights of portfolios of these three assets, finding the optimal point in the plot.

According to the M.M.V model, we figured out each asset's optimal weights and calculated the optimal Sharpe ratio of each asset. We figured out the optimal Sharpe ratio of the portfolio with the risk-free asset and without the risk-free asset. The result in the highest Sharpe ratio of each asset is Putnminc, at 0.18 roughly. The lowest one is Keystne, at 0.08 roughly. In terms of portfolios, portfolio with the risk-free asset is lower than that without risk-free asset (0.098;  $-0.006$  respectively). Therefore, because of adding T-bills which is a low return and low-risk asset, the Sharpe ratio of the portfolio with the risk-free asset is surely shorter than that without the risk-free asset. We recommend this one that captured by Sharpe ratio over the two methods mentioned in the previous article because this portfolio has a higher investment efficiency.

The above three methods for establishing investment portfolios have their own set of limitations. The consideration of the Diversification effect is too singular, only taking the risk as a constraint for establishing an portfolio and, to some extent, ignoring the importance of return. In terms of the Markowitz model, the variance of the historical return rate of assets is used to measure asset risk and correlation. There may be a certain deviation from the actual variance. Due to the expected rate of return, which is highly operable, it is not enough to simply calculate the average rate of return of the investment portfolio. The rate of return must be adjusted according to the level of risk, namely the risk-adjusted rate of return is calculated. Pursuing high profits blindly is very dangerous because it neglects latent risks. Mishra S K, Panda G and Majhi B proposed a modified model for choosing constrained portfolio assets selection based on the Markowitz mean-variance model [13]. As for the Sharpe ratio takes into account total risk and is high risk dependent, which is assessed by measuring annualised volatility over a historical or holding period. We did not use other measures like the Treynor Measure used by Scholz and Wilkens in their paper, which takes market risk into account [14]. The higher the value of the Treynor Measure, the higher the excess return per unit of systematic risk taken.

Subsequently, when it comes to data selection, we should consider selecting all the assets and giving a conclusion of them. In future research, we would calculate the data of each asset, calculating the weights and Sharpe ratio of portfolios. Meanwhile, we should choose a range of comprehensive and more applicable methods to analyse these assets.

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