

Measuring the Contribution of Batik Education Treatment Due to the Optimal Control of the Dynamic Model of Drug Addicts

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Abstract. Central Sulawesi placed as the 4th of addicted drugs highest number in Indonesia. This paper proposes batik education treatment as an alternative for BNNK Palu in conducting its controlling program. There are three running programs; that are education (C_1), rehabilitation (C_2) and interaction monitoring (K). The contribution is measured by finding the optimal value of parameter control rates of the dynamic model of drug spread addicts population that divided into five sub-population namely the Vulnerable sub-population (\tilde{S}), the Drug addicted sub-population (\tilde{P}), Educated sub-population ($\tilde{S}a$), Rehabilitated patient sub-population ($\tilde{P}a$), and Post-rehabilitated patient sub-population ($\tilde{R}a$). Using Pontryagin Method, the programs that respectively treated to \tilde{S} and $\tilde{P}a$ give the optimal values $C_1 = 93.33\%$, $C_2 = 85,74\%$ and K = 10. The research result also shows that the contribution of batik education is 49,36%.

Keywords: Dynamic Model · Drug User · Controller · Pontryagin Method

1 Introduction

Batik has been stated by Unesco as Indonesian masterpiece of intangible cultural heritage of humanity. Indonesian government also declared 2nd of October as National Batik Day of Indonesia. Those two momentum drives Indonesian peoples to conserve and develop the batik motifs. BaDigit, the community of batik lovers in Palu, designed 9 certified batik motif namely The harmony nature, The waving cactus bloom, The Sigi orchidaceae fiesta, The tenun look of moringa chem, The kawung look of moringa chem, The eboni herb, Cesara [1], Maleomile and Kalem Bridge. The other four certified motif are DigiLog Tadulako, Black rock jasmine, Cute maleo, and SuTo Kaili.

Beside considering as a heritage cultural art product, actually batik also has several other function such as a supporting method of some treatment purposes. It's stated that batik therapy could increase the emotion regulation [2]. Related to controlling program

of BNNK Palu, national bureau of drugs controlling, has three programs to run that are education of the vulnerable people (C_1) , rehabilitation of drug addicted patient (C_2) and interaction monitoring between the rehabilitated people and the addicted patient (K). This paper proposes how batik education treatment to be considered as an alternative way for finding the optimal value of respected parameter control rates of the dynamic model of drug spread addicts population. The reason is batik education treatment could be considered as a compound process where the object of it (the motif), is design in such a philosophy value. For example the motif of cute maleo represents how restricted peoples would never bounds them for having incredible idea [3]. The question is how great the contribution of batik education treatment.

In this paper, the contribution of parameters is determined by investigate the optimal value parameters that represent by the program running by BNNK Palu. The contribution batik education treatment could be measured by the proportion of batik education parameter with respect to patient rehabilitation parameter and interaction monitoring parameter between the rehabilitated patient sub-population and the drug addicted sub-population. The proportion measures the rate of the success of batik education treatment in controlling the drugs addict spread. The optimal rate of the program is derived from the optimal value of such objective function that restricted by the dynamical system of drugs addict spread model. The method to be used in this paper is Pontryagin method. The principal of the method is to find the optimal solution of the state and co-state that satisfy the stationer condition [4]. Several problems using the method are shown in [5].

2 Material and Method

2.1 The Material

The preview paper [1] shows the seven heritage of Central Sulawesi batik motifs using fractal geometry concept that had been designed to develop the Wallacean batik motif of BaDigit in 2020 (see Fig. 1 a. - 1.g). As a result of community services activity, two other BaDigit motifs also had been deigned in 2021 (see Fig. 1.l. - 1.m) and 2022 (see Fig. 1.h. - 1.i). Two other motifs designed in 2022 are also shown in Fig. 1.j - 1.k. All motifs are designed based on such philosophy represented the diversity.

The philosophies of the motifs could be considered as a set of motivation words inside that potentially have positive impacts if it is used on mental therapy [6, 7]. This paper uses the BaDigit batik motifs as the material of the BNNK Palu education treatment program to educate the drug addict sub-population. The success of the treatment is measured from the proportion of the optimal rate of the education rate comparing to the other two programs of BNN Palu.

The other material of this paper is the following governed mathematical model [8].

$$\frac{d\tilde{S}}{dt} = A - \mu \tilde{S} - \varepsilon C_1 \tilde{S} - \frac{\sigma_1 \tilde{S} \tilde{P}}{N} + (1 - \alpha_1) \tilde{P}$$
(1)

$$\frac{d\tilde{P}}{dt} = \frac{\sigma_1 \tilde{S}\tilde{P}}{N} + \frac{\omega K\tilde{P}\tilde{P}a}{N} - (\mu + \gamma)\tilde{P} - \beta C_2\tilde{P} - (1 - \alpha_1)\tilde{P}$$
(2)

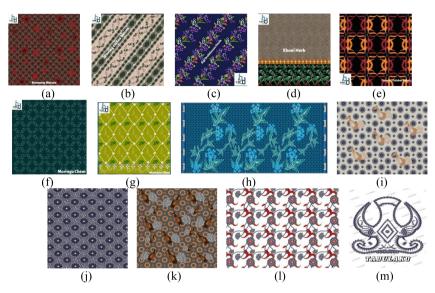


Fig. 1. The BaDigit Certified [11–19] a. The harmony nature b. The waving cactus bloom c. The Sigi orchidaceae fiesta d. The eboni herb e. Cesara f. The kawung look of moringa chem g. Kalem Bridge h. Maleomile i. The tenun look of moringa chem j. Black rock jasmine k. Cute maleo l. SuTo Kaili m. DigiLog Tadulako

$$\frac{d\tilde{S}a}{dt} = \varepsilon C_1 \tilde{S} - \mu \tilde{S}a - \frac{\sigma_2 \tilde{S}a\tilde{P}}{N}$$
(3)

$$\frac{dPa}{dt} = \beta C_2 \tilde{P} - \frac{\omega KPPa}{N} - \mu \tilde{P}a - \alpha_1 \tilde{P}a + \frac{\sigma_2 SaP}{N}$$
(4)

$$\frac{dRa}{dt} = \tilde{P}a\alpha_1 - \tilde{R}a\mu \tag{5}$$

The model is governed by consider the changing of five sub-populations, namely the vulnerable (\tilde{S}), the drug users (\tilde{P}), the educated people ($\tilde{S}a$), the rehabilitated patient ($\tilde{P}a$), and the post-rehabilitated patient ($\tilde{R}a$). The individual movement of such subpopulation is caused by the transition and interaction activity such that each differential equation of the system represents the growth. The parameters are the recruitment of 15-years old-individual into $\tilde{S}(A)$, the total population (N), the natural death rate (μ), the drugs over dosage death rate (γ), the education rate (ε), the rehabilitation rate (β), the recovered opportunity rate of $\tilde{P}a$ (α_1), the recovered opportunity rate of \tilde{P} ($1 - \alpha_1$), the individual interaction rate between of $\tilde{P}a$ and $\tilde{P}(\omega)$, the individual interaction rate between of \tilde{S} and $\tilde{P}(\sigma_1)$, and the interaction rate between individual of $\tilde{S}a$ and $\tilde{P}(\sigma_2)$.

2.2 The Method

The Stability. The stability of the dynamical system (1-5) is identified from the eigen value of the Jacobian matrix of the approximation linear system of (1-5) due to the stability criteria stated in [8].

The Optimum Control. Solution of the optimal control problem is determining the signal that will be processed in such constrained plant. At the same time the signal is the maximum/ minimum value that satisfy the performance index, as a quality measurement criteria, that formulated as follow.

$$J = \theta(x(t_f), t_f) + \int_{t_0}^{t_f} f(x(t), u(t), t) dt$$
(6)

The constrain of the performance index is.

$$\dot{x} = g(x(t), u(t), t), \text{ where } x(t_0) = x_0, \ a < u < b$$
 (7)

u * (t) is an optimal control if the substitution of it to (2.23) gives an optimal state x * (t) and optimizes the performance index J(u * (t)). Optimal state x * (t) is constrained by the boundary condition a and b.

For such Halmitonian function

$$H = f(x, u, t) + \lambda g(x, u, t)$$
(8)

the minimum/ maximum Pontryagin principal stated that the solution have to satisfy the (\dot{x})

state equation
$$\dot{x}(t) = \begin{pmatrix} I \\ \dot{P} \\ \dot{X} \end{pmatrix}$$
, co-state $\dot{\lambda} = \begin{pmatrix} \lambda_1(t) \\ \dot{\lambda}_2(t) \\ \dot{\lambda}_3(t) \end{pmatrix}$ and stationary condition where $\dot{T} = \frac{\partial H}{\partial \lambda_1}, \dot{P} = \frac{\partial H}{\partial \lambda_2}, \dot{X} = \frac{\partial H}{\partial \lambda_3}, \lambda_1 = -\frac{\partial H}{\partial T}, \lambda_2 = -\frac{\partial H}{\partial P}, \lambda_3 = -\frac{\partial H}{\partial X}$ and $\frac{\partial H}{\partial u} = 0$.

3 Results and Discussion

The set of motivation words inside to be considered in this paper that captured from philosophies sight of the motifs is tabulated in Table 1.

Table 1. The motivation words captured from philosophies sight of the motifs

Motif	Motivation inside		
The Harmony Nature	Harmony, power, unity, togetherness, spirit		
The waving Cactus	Struggle, dynamic, toughness, optimize		
The Sigi Orchidacea Fiesta	Diversity, happiness, beneficial, solidarity		
The Moringa Chem Tenunlook	Beneficial, populist, useful, accepted		
The Moringa Chem Kawunglook	Beneficial, populist, useful, hand in hand		
The Eboni Herb	Tough, rigid, diverse, helpful		
Cessara	Careful, prevent, hopeful, not giving up		
Maleomile	Strong, not inferior, useful		
Kalem Bridge	Teamwork, open minded, solutive, team work		

3.1 Scaling Model and Steady State Condition

The dynamical system is normalized to observe the spread of drug abusers such that the new variables $S = \frac{\tilde{S}}{N}$, $P = \frac{\tilde{P}}{N}$, $S_a = \frac{\tilde{S}_a}{N}$, $P_a = \frac{\tilde{P}_a}{N}$, and $R_a = \frac{\tilde{R}_a}{N}$ are in the interval of (0, 1). In case of no death drug overdose cases in Palu, it is assumed that $\gamma = 0$ so that the growth of the human population is just influenced by natural birth and death such that the steady condition for the population growth human population gives $N = \frac{A}{\mu}$ and by neglecting the tilde, could be derived by keeping the expression of Eqs. (2)–(5), while Eq. (1) comes to the following expression.

$$\frac{\partial S}{\partial t} = \mu - \mu S - \varepsilon C_1 S - \sigma_1 S P + (1 - \alpha_1) P \tag{9}$$

The system has two critical points, namely disease free equilibrium and endemic equilibrium states, that are respectively $E_{DFE_{-1}} = (S, P, Sa, Pa, Ra) = \left(\frac{\mu}{\mu+C_{1}\varepsilon}, 0, \frac{C_{1}\varepsilon}{\mu+C_{1}\varepsilon}, 0, 0\right)$ and a parameter dependently $E_{END_{-1}} = (S, P, Sa, Pa, Ra) = \left(\frac{\frac{\mu}{\mu+C_{1}\varepsilon}, 0, \frac{C_{1}\varepsilon}{\mu+C_{1}\varepsilon}, 0, 0\right)$ and a parameter dependently $E_{END_{-1}} = \left(\frac{\frac{-Pa_{1}+P+\mu}{P\sigma_{1}+cC_{1}+\mu}, -\frac{((a_{1}-1)P-\mu)\varepsilon C_{1}}{(P\sigma_{2}+\mu)(P\sigma_{1}+scC_{1}+\mu)}, \frac{P\left(\frac{P^{2}\beta C_{1}\sigma_{1}\sigma_{2}+\left(\frac{\beta C_{2}(\sigma_{1}+\sigma_{2})\mu}{+\sigma_{2}cC_{1}(\beta C_{2}-\alpha_{1}+1)}\right)^{P+\mu}\left(\frac{\beta \mu C_{2}}{\mu+\sigma_{2}cC_{1}(\beta C_{2}-\alpha_{1}+1)}\right)}{(P\sigma_{2}+\mu)(P\sigma_{1}+scC_{1}+\mu)},$

where *P* is a polynomial root that needed the rule of Descrates to give the requirements $\sigma_1 > \frac{(\epsilon C_1 + \mu)(\beta C_2 + \mu + \alpha_1)}{\mu}$ and $\alpha_1 < \frac{\mu + P}{P}$.

3.2 The Stability

The local stability of E_{DFE_1} is identified from the Jacobian matrix that evaluated at the zero critical point that gives the following characteristic polynomial equation.

$$(\varepsilon C_1 + \mu + \lambda) \left(\frac{\sigma_1 \mu}{\varepsilon C_1 + \mu} - \beta C_2 - \mu + \alpha_1 - \gamma_1 - 1 - \lambda \right) (\mu + \lambda)^2 (\mu + \alpha_1 + \lambda) = 0$$
(10)

Four eigen values derived from the polynomial are negative; $\lambda_1 = -\mu - \alpha_1$, $\lambda_2 = -\varepsilon C_1 - \mu$, $\lambda_3 = -\mu$, and $\lambda_4 = -\mu$; while the other one $\lambda_5 = \frac{-\mu^2 + (-\beta C_2 - \varepsilon C_1 - \alpha_1 + \sigma_1)\mu - \varepsilon C_1(\beta C_2 + \alpha_1)}{\varepsilon C_1 + \mu}$ gives the requirement of $\sigma_1 < \frac{(\varepsilon C_1 + \mu)(\beta C_2 + \mu + \alpha_1)}{\mu}$. This result appears the need of a control treatment to keep the long-term behavior stable solution.

3.3 The Optimum Control

The preview result relied to the treatment program of BNNK Palu to maximize the level of drug addicted rehabilitation patient treatment and the interaction rate of rehabilitated patients and drug addicts. The treatments are education (C_1) , rehabilitation (C_2) and interaction monitoring (K). The contribution is measured by finding the optimal value of parameter control rates of the dynamic model of drug spread addicted population. A

control ability test shows that the system is control able. The performance index for C_1 , C_2 and K are constructed by $J[C_1, C_2, K] = \frac{1}{2} \int_0^T (P^2 - C_1^2 - C_2^2 - K^2) dt$.

Using the Minimum Pontryagin method, the performance index related to Eq. (8) gives the following Hamiltonian function.

$$H = \left(\frac{1}{2}(P^2 - C_1^2 - C - +K^2) + (\mu - mS - +C_1S - s_1SP + a_1P)l_1 + (s_1SP + \&KPPa - \mu P - bC_2P - a_1P)l_2 + (+C_1S - mSa - s_2SaP)l_3 + (Paa_2 - Ram)l_5 + (bC_2P - \&KPPa - \mu Pa - a_2Pa + s_2SaP)l_4)$$
(11)

An optimal Hamiltonian function is derived by consider the steady condition of the controlled system $\frac{\partial H}{\partial C_1} = 0$, $\frac{\partial H}{\partial C_2} = 0$, and $\frac{\partial H}{\partial K} = 0$. The control value that restricted under $0 \le C_1^* \le 1, 0 \le C_2^* \le 1, 0 \le K^* \le 1$ give $C_1^* =$ min{max(0, $(-S(-\lambda_1 + \lambda_3)\varepsilon)), 1$ }, $C_2^* = min{max(0, <math>(-P(-\lambda_2 + \lambda_5)\beta)), 1$ }, and $K^* = min{max(0, (PPa\omega\lambda_2 - PPa\omega\lambda_4)), 1}$. Furthermore, the optimal Hamiltonian function is obtained by substitute the optimal control parameters value to Eq. (11).

The state and co-state equation of an optimal tomor parameters value to Eq. (17). The state and co-state equation of an optimal Hamiltonian function are respectively in form of $S^*(t) = \left(\frac{\partial H^*}{\partial \lambda_1}\right), P^*(t) = \left(\frac{\partial H^*}{\partial \lambda_2}\right), Sa^*(t) = \left(\frac{\partial H^*}{\partial \lambda_3}\right), Pa^*(t) = \left(\frac{\partial H^*}{\partial \lambda_4}\right), Ra^*(t) = \left(\frac{\partial H^*}{\partial \lambda_4}\right); \text{ and } \dot{\lambda}_1 = -\frac{\partial H(x, C_1^*, C_2^*, K^*, t)}{\partial S}, \dot{\lambda}_2 = -\frac{\partial H(x, C_1^*, C_2^*, K^*, t)}{\partial P}, \dot{\lambda}_3 = -\frac{\partial H(x, C_1^*, C_2^*, K^*, t)}{\partial Sa}, \dot{\lambda}_4 = -\frac{\partial H(x, C_1^*, C_2^*, K^*, t)}{\partial Pa}, \dot{\lambda}_5 = -\frac{\partial H(x, C_1^*, C_2^*, K^*, t)}{\partial Ra}.$

3.4 The Numerical Simulation

In this section will be shown the dynamic solution with respect to time before and after applying the control of (C_1, C_2, K) . Simulation is figured by provide the parameter values that satisfy the existence and stability conditions of the critical point. The figure is showed for such initial values of variables. For a total population of Palu, N = 367.403. to the population value shown in the simulation results. The initial values of the variables and the parameter values are given in Table 2.

The dynamics are figured with respect to the horizontal axis time (*t*) in year unit time, and the vertical axis states the number of \tilde{S} , \tilde{P} , $\tilde{S}a$, $\tilde{P}a$ or $\tilde{R}a$ population. The figure represents the change of such population by developing the time in free disease condition. The green curve describes the uncontrolled condition, while the other one describes the controlled condition. It could be seen from Fig. 2 that the first condition the number or population increased for some time interval and become stagnant somewhere after reached stable condition. Meanwhile, the increasing will not happen because of the successes of the controller to reduce the number of the uneducated peoples.

The sub-population of uneducated communities experienced a significant increase and stagnated on day 22,212 with the number of uneducated people of 408,259. This can happen due to the non-improvement of the parameters ε which is where the parameter ε role in educating the sub-population of the uneducated society (\tilde{S}) and the presence of parameter A as the recruitment rate in the uneducated subpopulation led to a continuous addition to stagnation on day 22,212. This shows that drug dealing requires a long time and continuous effort. Then in the system that has been controlled, the subpopulation

Variables/ Parameters	Value	References	Variables/ Parameters	Value	References
Κ	0 < K < 1	PI	β	0.161	GOP
$\tilde{S}(0)$	0.80005	CBS	α1	0.152	GOP
$\tilde{P}(0)$	0.00330	Assumption	ω	0.032	GOP
$\tilde{S}a(0)$	0.19631	CBS	σ_2	0.052	GOP
$\tilde{P}a(0)$	0.00023	GOP	α1	< 442.677	ToE E _{END_1}
$\tilde{R}a(0)$	0.00016	GOP	σ1	< 3.715	SC E _{DFE_1}
A	11.312	CBS	$1 - \alpha_1$	0.852	PtH
μ	1.457	CBS	<i>C</i> ₁	$0 < C_1 < 1$	PI
ε	0.706	GOP	<i>C</i> ₂	$0 < C_2 < 1$	PI

Table 2. The initial value of variables and parameters

Annotation: PI: Performance IndexCBS: Central Bureau of Statistics. GOP: Government Office of PaluToE: Terms of Existence. SC: Stability ConditionPtH: Probability to Heal.

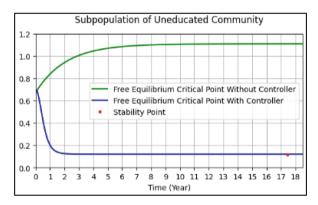


Fig. 2. Simulation of free disease condition

of uneducated communities experienced a significant decrease then stagnated on day 6,374 with the number of uneducated people of 44,591. It is clear that C_1 controller can minimize the subpopulation of uneducated society.

The endemic condition is determined from the existence of the drug addicted subpopulation. Figure 3.a shows. The uncontrolled number of this sub-population is well controlled. It is happened because of the educated society sub-population is driven to become greater such it shown in Fig. 3.b. As a consequence, the number of rehabilitation and post rehabilitation patients in Figs. 3.c and 3.d become reduces. The proportion of optimal controlled parameter values C1, C2, and K are shown in Fig. 3.e.



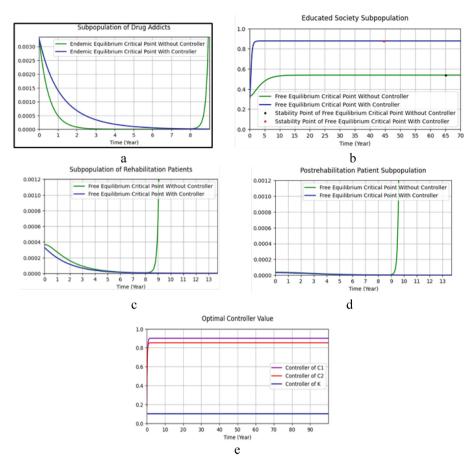


Fig. 3. Simulation of the endemic condition

Applying the Pontryagin method, the treated programs to \tilde{S} and $\tilde{P}a$ gives the optimal values of $C_1 = 93.33\%$, $C_2 = 85.74\%$ and K = 10% and the proportion among the parameters are 49.36%:45.35%:5.29% and the contribution of batik education is 49,36%.

4 Conclusion

The governed dynamic model of drug addicts by involving preventive treatment in the form of education, rehabilitation, and interaction monitoring has two critical points. E_{DFE_1} represents the free disease condition while E_{END_1} represents the existing of all populations. The model has been controlled for the optimal parameters C_1 , C_2 , and K.

Acknowledgements. Thank you to the Dean of Mathematics and Natural Sciences Faculty of Tadulako University for the funding and BaDigit for many valuable discussions and works.

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