The Transversal Neighborhood Domination Number on Parachute Graph and Semi-Parachute Graph

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Abstract. Given V be a set of vertices on a graph *G.* A set $D \subseteq V$, is dominating set of $G = (V, E)$ if all the vertex that is not in the set *D* are neighbors to at least one vertex of *D*. The smallest number of elements in *D* is known as the domination number. A set $D \subseteq V(G)$ is called a transversal neighborhood-dominating set if *D* is the dominating set of *G* and intersects with every minimum neighborhood set. The transversal neighborhood domination number of G is the smallest number of elements in each transversal neighborhood-dominating set.. In this paper, we discuss transversal neighborhood domination number on a parachute graph and a semi-parachute graph. The transversal neighborhood domination number on parachute graph is (n + 3)/3 for every n = 3 k, (n + 5)/3 for every n = 3 k + 1, and (n + 4)/3 for every n = 3 k + 2. The neighborhood transversal domination number on the semi-parachute graph is (n + 1)/2 for every n = 2 k + 1 and (n + 2)/2 for every n = 2 k.

Keywords: Domination Set · Neighborhood Set · Neighborhood Transversal Dominating Set · Parachute Graph · Semi-parachute Graph

1 Introduction

One of the topics of graph theory is the dominating set on graphs. A graph *G* is described as a pair of sets (V, E), where V represent the set of vertices on G and E is the set of edges on G. The dominating set of a graph $G = (V, E)$ is a set $D \subseteq V$ which the vertices that are not in D are directly connected to at least one vertex on V. The smallest number of elements of all the dominating sets of graph G is called the domination number of G, denoted by $\gamma(G)$ [1]. There are several developments of dominating sets such as total domination [2], inversee domination [3], total inversee domination [4–6], transversal domination [7], transversal neighborhood-dominating set [8], etc.

In graph theory, a neighborhood of $v \in V(G)$ is closely related to the concepts of vertices adjacent to v. The open neighborhood of v is denoted by $N(v) = \{u \in V; (u, v) \in E\}$, while the closed neighborhood of a set is denoted by $N[v] = N(v) \cup \{v\}$. Moreover we define a subgraph $\langle N(v) \rangle$ that is induced by $N[v]$. A set

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$S \subseteq V$ is called a neighborhood set on $G = (V, E)$ if the set S forms $G = \bigcup_{v \in s} \langle N(v) \rangle$. A neighborhood set with a minimum number of elements is called a minimum neighborhood set [2]. Das [9], in 2022, investigated various structural similarities between G and $N(G)$ [3]. See [10–12] for more results related to neighborhood set.

If combined, the concept of the dominating set and the neighborhood set will yield a new concept, one of which is the transversal neighborhood-dominating set concept. A dominating set D is referred to as the transversal neighborhood-dominating set if it intersects with each minimum neighborhood set. A commanding set D is referred to as the transversal neighborhood-dominating set if it connects with each smallest neighborhood set. The transversal neighborhood domination number of G, denoted as $\gamma_{nt}(G)$, is the smallest number of elements of the transversal neighborhood-dominating set.

Related to transversal, some applications that possible have been introduced by Fellow [15] are fault-tolerant of data storage and complexity-related decision problems. Furthermore, Atakul [16] also discussed the possibility applications on communication networks, in particular, related to the vulnerability of the network to certain operational disruptions.

Research on the transversal neighborhood domination number has been carried out on several graphs, including complete bipartite (multipartite) graph and some basic graphs i.e. path graph, complete graph, cycle graph, wheel graph [8], and triangular snake graph [4]. In addition, there are various types and forms of other graphs. One example of such a graph is parachute graph, denoted by PC_n, and the semi-parachute graph, denoted by SP_{2n-1}. A parachute graph PC_n has the basic form of a fan graph and contains paths from v_1 to u_n, v_n to u_1, and u_i to u_{i+1} (see Fig. 4). Is contrast the semi-parachute graph SP_{2n-1} has the basic form of a fan graph and contains a path from v_i to v_{i+1} (see Fig. 5). Therefore, in this study, we discuss the transversal neighborhood domination number on parachute and semi-parachute graphs.

2 Material and Methods

The graphs discussed in this study are graphs PC_n and graphs SP_{2n-1} with $n \geq 3$ and $n \in \mathbb{N}$. The transversal neighborhood domination number of PC_n can be determined by determining all the neighborhood sets S of the graph PC_n with minimum cardinality and determining the dominating set D starting for $n = 3$. If the set D intersects with each set S, then it is a transversal neighborhood-dominating set. If the set does not intersect with every S, then look for another set D that intersects with every S. Using the same method, we look for the transversal neighborhood domination number for several other and until we get the pattern of the transversal neighborhood for each $n \in \mathbb{N}$ with $n \geq 3$. From the pattern obtained, the exact value for the transversal neighborhood domination number of PC_n is obtained. The last step is to test the specified theorem. Using the same method, we determine the transversal neighborhood domination number of SP_{2n-1}.

Let a graph G with $V(G)$ being a non-empty set of vertices and $E(G)$ being a set of edges connecting a pair of vertices. A vertex $u, v \in V(G)$ is said to be adjacent if the two vertices are connected by an edge e in the graph or $e = (u, v) \in E(G)$[6]. A graph $P = (V(P), E(P))$ is a subgraph of $G = (V(G), E(G))$ if and only if $V(P) \subseteq V(G)$ and $E(P) \subseteq E(G)$. The induced subgraph of $G = (V(G), E(G))$ is $P\prime = (V(P\prime), E(P\prime))$ if for each $u, v \in V(P\prime)$ holds $(u, v) \in E(P\prime)$ if and only if $(u, v) \in E(G)$ [7].

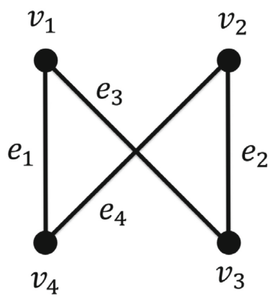

Fig. 1. Graph G with 4 vertices and 4 edges

The following is an example of a subgraph, and not a subgraph graph G in Fig. 1 and Fig. 2.

Definition 1 [11] **.** The set $S \subseteq V$ in the graph $G = (V(G), E(G))$ known as the neighborhood set if $G = \bigcup_{v \in s} \langle N(v) \rangle$, with $\langle N(v) \rangle$ is induced subgraph G by $N[v]$

Based on Fig. 1, the closed neighborhood set of each vertex on the graph G are $N[v_1] = N(v_1) \bigcup \{v_1\} = \{v_1, v_3, v_4\}$, $N[v_2] = N(v_2) \bigcup \{v_2\} = \{v_2, v_3, v_4\}$, $N[v_3] = N(v_3) \bigcup \{v_3\} = \{v_1, v_2, v_3\}$, and $N[v_4] = N(v_4) \bigcup \{v_4\} = \{v_1, v_2, v_4\}$

Observe that the induced subgraphs for each neighborhood of $v_i \in V(G)$ have the following sets of vertices and edges.

$V(\langle N(v_1) \rangle) = \{v_1, v_3, v_4\}$ and $E(\langle N(v_1) \rangle) = \{e_1, e_3\}$
$V(\langle N(v_2) \rangle) = \{v_2, v_3, v_4\}$ and $E(\langle N(v_2) \rangle) = \{e_2, e_4\}$
$V(\langle N(v_3) \rangle) = \{v_1, v_2, v_3\}$ and $E(\langle N(v_3) \rangle) = \{e_2, e_3\}$
$V(\langle N(v_4) \rangle) = \{v_1, v_2, v_4\}$ and $E(\langle N(v_4) \rangle) = \{e_1, e_4\}$

Subgraph is represented as follows.

Since there is no induced subgraph for each vertex in the graph G that forms graph G, then another subset of graph G is taken with cardinality greater than or equal to two. For example, take $S\prime = \{v_1, v_2\}$. Based on Fig. 3, it can be seen that $S\prime$ is a neighborhood set of G, because $\langle N(v_1) \rangle \bigcup \langle N(v_2) \rangle = G$. As a result, the neighborhood set of G has

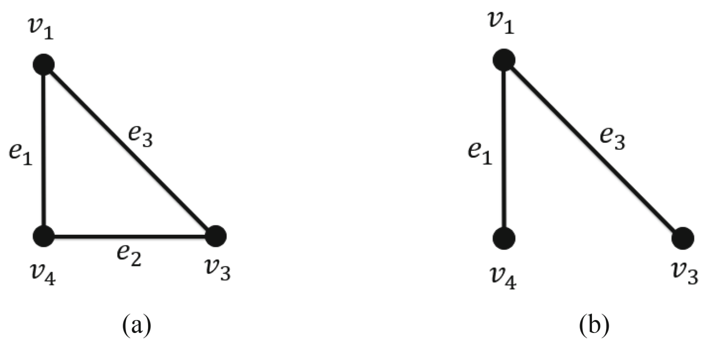

Fig. 2. (a) Not a subgraph, and (b) a subgraph of graph G

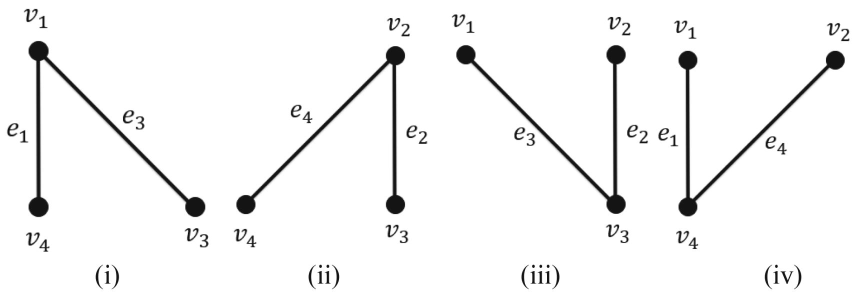

Fig. 3. (i) Subgraph $\langle N(v_1)\rangle$, (ii) Subgraph $\langle N(v_2)\rangle$, (iii) Subgraph $\langle N(v_3)\rangle$, and (iv) Subgraph $\langle N(v_4)\rangle$

the smallest cardinality, which is 2. Another neighborhood set in G with cardinality 2 is $S\prime\prime = \{v_3, v_4\}$.

Based on Fig. 1, suppose we take $D_1 = \{v_1, v_3\}$. We get $V - D_1 = \{v_2, v_4\}$ where v_2 is adjacent to v_3, and v_4 is adjacent to v_1. Since each vertex in $V - D_1$ is adjacent to at least one vertex of D_1, then D_1 is a dominating set of graph G.

Definition 2 [8] **.** The dominating set $D \subseteq V(G)$ of $G = (V(G), E(G))$ is known as the transversal neighborhood-dominating set if it intersects with each neighborhood set with least cardinality. The least cardinality of the set of transversal neighborhood-dominating set, denoted by $\gamma_{nt}(G)$ known as the transversal neighborhood domination number.

Based on Fig. 1, the smallest neighborhood set of G is $S\prime = \{v_1, v_2\}$ and $S\prime\prime = \{v_3, v_4\}$. We also know that $D_1 = \{v_1, v_3\} \subseteq V(G)$ is a dominanting set in G. Then,

$$D_1 \cap S\prime = \{v_1, v_3\} \cap \{v_1, v_2\} = \{v_1\}$$

$$D_1 \cap S\prime\prime = \{v_1, v_3\} \cap \{v_3, v_4\} = \{v_3\}$$

By Definition 2, D_1 is a transversal neighborhood-dominating set in G.

3 Result and Dicussion

The transversal neighborhood-domination number in the parachute graph PC_n and the semi-parachute graph SP_{2n-1} will be discussed in this part.

Definiton 3 [13]. A parachute graph PC_n with $n \geq 3$ is a graph with a set of vertices $V(PC_n) = \{v_i, u_i, a; 1 \leq i \leq n\}$ and a set of edges$E(PC_n) = \{(u_n, v_1), (u_1, v_n), (a, v_i)|i \in [1, n]\} \cup \{(v_i, v_{i+1}), (u_i, u_{i+1})|i \in [1, n-1]\}$.

Theorem 4. Given a parachute graph PC_n with $n \geq 3$, then.

$$\gamma_{nt}(PC_n) = \begin{cases} \frac{n+3}{3}, & forn \equiv 0(\text{mod}3) \\ \frac{n+5}{3}, & forn \equiv 1(\text{mod}3) \\ \frac{n+4}{3}, & forn \equiv 2(\text{mod}3) \end{cases}$$

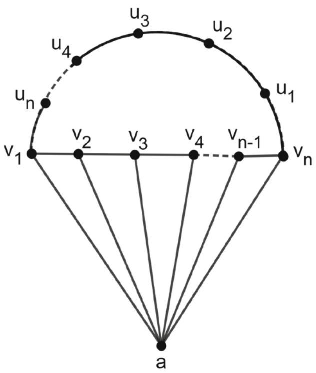

Fig. 4. Parachute graph PC_n

Proof: Given a parachute graph PC_n with $n \geq 3$ for each $n \in N$ with.

$$V(PC_n) = \{v_i, u_i, a;\ 1 \leq i \leq n\}$$

$$E(PC_n) = \{(u_n, v_1), (u_1, v_n), (a, v_i);\ 1 \leq i \leq n\} \cup \{(v_i, v_{i+1}), (u_i, u_{i+1});\ 1 \leq i \leq n-1\}$$

$V(\langle N(a)\rangle) = \{a, v_i;\ 1 \leq i \leq n\}$ and $E(\langle N(a)\rangle) = \{av_i, v_jv_{j+1};\ 1 \leq i \leq n;\ 1 \leq j \leq n-1\}$

$V(\langle N(v_1)\rangle) = \{a, v_1, v_2, u_n\}$ and $E(\langle N(v_1)\rangle) = \{(a, v_1), (v_1, v_2), (v_1, u_n)\}$

$V(\langle N(v_n)\rangle) = \{a, v_n, v_{n-1}, u_1\}$ and $E(\langle N(v_n)\rangle) = \{(v_n, a), (v_n, v_{n-1}), (v_n, u_1)\}$

$V(\langle N(u_1)\rangle) = \{v_n, u_2, u_1\}$ and $E(\langle N(u_1)\rangle) = \{(u_1, v_n), (u_1, u_2)\}$

$V(\langle N(u_n)\rangle) = \{v_1, u_n, u_{n-1}\}$ and $E(\langle N(u_n)\rangle) = \{(u_n, v_1), (u_n, u_{n-1})\}$

If $2 \leq i \leq n-1$ obtained.

$V(\langle N(v_i)\rangle) = \{a, v_i, v_{i-1}, v_{i+1}\}$ and $E(\langle \mathrm{N}(v_i)\rangle) = \{(a, v_i), (v_i, v_{i-1}), (v_i, v_{i+1})\}$

$V(\langle N(u_i)\rangle) = \{u_i, u_{i-1}, u_{i+1}\}$ and $E(\langle N(v_i)\rangle) = \{(u_i, u_{i-1}), (u_i, u_{i+1})\}$.

Let $\gamma(PC_n) = \frac{n+3}{3}$ for $n \equiv 0(\text{mod}3)$, $\gamma(PC_n) = \frac{n+5}{3}$ for $n \equiv 1(\text{mod}3)$, and $\gamma(PC_n) = \frac{n+4}{3}$ for $n \equiv 2(\text{mod}3)$.

1. For $n \geq 3$ and n odd numbers

Let $S \subseteq V$ on graph PC_n where $S = \{a, u_{2i-1} | i \in [1, \frac{n+1}{2}]\}$ then $\langle N(a)\rangle \cup \langle N(u_1)\rangle \cup \langle N(u_3)\rangle \cup \cdots \cup \langle N(u_{2i-1})\rangle = PC_n$. Thus S is a neighborhood set of the graph PC_n. Let $S_1 \subseteq V$ with $S_1 = \{a, u_{2i-1} | i \in [1, \frac{n+1}{2}]\}$ then $\langle N(a)\rangle \cup \langle N(u_1)\rangle \cup \langle N(u_3)\rangle \cup \cdots \cup \langle N(u_{\frac{n-1}{2}})\rangle \neq PC_n$ then S_1 is not a neighborhood set of the graph PC_n. Hence $S = \{a, u_{2i-1} | i \in [1, \frac{n+1}{2}]\}$ with $|S| = \frac{n+3}{2}$ is $\eta - set$ (neighborhood set with smallest cardinality) of graph PC_n. Let other S with $|S| = \frac{n+3}{2}$ and $S \neq \{a, u_{2i-1} | i \in [1, \frac{n+1}{2}]\}$ then $PC_n \neq \bigcup_{v \in S} \langle N(v)\rangle$ for $n \geq 5$. While for $n = 3$, there are $S\prime = \{v_1, v_3, u_2\}$ and $S\prime\prime = \{v_2, u_1, u_3\}$ which is another $\eta - set$ of PC_n.

For $n \geq 5$ and n odd numbers, let the neighborhood set of graph PC_n is $S = \{a, u_{2i-1} | i \in [1, \frac{n+1}{2}]\}$.

For n $\equiv 0(\mathbf{mod}\,3)$, let $D = \{a, u_{3i-1} | i \in [1, \frac{n}{3}]\}$, then $D \cap S = \{a, u_{3i-1} | i \in [1, \frac{n}{3}]\} \cap \{a, u_{2i-1} | i \in [1, \frac{n+1}{2}]\} = \{a, u_{6i-1} | i \in [1, \frac{n-3}{6}]\}$.

For n $\equiv 1(\mathbf{mod}\,3)$**,** let $D = \{a, u_{n-1}, u_{3i-1} | i \in [1, \frac{n-1}{3}]\}$, then $D \cap S = \{a, u_{n-1}, u_{3i-1} | i \in [1, \frac{n-1}{3}]\} \cap \{a, u_{2i-1} | i \in [1, \frac{n+1}{2}]\} = \{a, u_{6i-1} | i \in [1, \frac{n-1}{6}]\}$

For $n \equiv 2(\mathbf{mod}\,3)$, let $D = \left\{a, u_{n-1}, u_{3i-1} | i \in \left[1, \frac{n-2}{3}\right]\right\}$, then $D \cap S = \left\{a, u_{n-1}, u_{3i-1} | i \in \left[1, \frac{n-2}{3}\right]\right\} \cap \left\{a, u_{2i-1} i \in \left[1, \frac{n+1}{2}\right]\right\} = \left\{a, u_{6i-1} | i \in \left[1, \frac{n-2}{6}\right]\right\}$

Thus $\gamma(PC_n) = \gamma_{nt}(PC_n)$ for $n \geq 5$ and odd numbers.

Let $D \subseteq V$ with $|D| = \gamma(PC_3)$. there is $D = \{v_3, u_3\}$ which intersects with $S, S\prime$, and $S\prime\prime$. Thus $\gamma(PC_3) = \gamma_{nt}(PC_3)$.

2. For $n \geq 4$ and n even numbers

Let $S \subseteq V$ of graph PC_n with $|S| = \frac{n}{2} + 2$. Then there is $\bigcup_{v \in s} \langle N(v) \rangle = PC_n$ with

$$S = \left\{a, u_1, u_{2i}; 1 \leq i \leq \frac{n}{2}\right\}$$

$$S_m = \left\{a, u_{2i-1}, u_{2j}; 1 \leq i \leq m \leq j \leq \frac{n}{2}\right\}$$

$$S_{\frac{n}{2}+1} = \left\{a, v_1, u_{2i-1}; 1 \leq i \leq \frac{n}{2}\right\}$$

$$S_{\frac{n}{2}+2} = \left\{a, v_n, u_{2i}; 1 \leq i \leq \frac{n}{2}\right\}$$

We get $S, S_m, S_{\frac{n}{2}+1}$ and $S_{\frac{n}{2}+2}$ is the neighborhood set of graph PC_n. Let $S_1 \subseteq V$ with $|S_1| = \frac{n}{2} + 1$. Obtained $PC_n \neq \bigcup_{v \in s_1} \langle N(v) \rangle$. Hence S_1 is not a neighborhood set of the graph PC_n. Thus $S, S_m, S_{\frac{n}{2}+1}$, and $S_{\frac{n}{2}+2}$ are $\eta - set$ of graph PC_n. Let other $S \subseteq V$ with $|S| = \frac{n}{2} + 2$ and $S \neq S_m \neq S_{\frac{n}{2}+1} \neq S_{\frac{n}{2}+2}$ then $PC_n \neq \bigcup_{v \in s} \langle N(v) \rangle$ for $n \geq 6$ and there is $S = \{v_1, v_3, u_1, u_3\}$ and $S = \{v_2, v_4, u_2, u_4\}$ for $n = 4$.

For $n \geq 6$ and n even numbers, let the minimum neighborhood set of graph PC_n is $S_m = \left\{a, u_{2i-1}, u_{2j}; 1 \leq m \leq \frac{n}{2}, 1 \leq i \leq m \leq j \leq \frac{n}{2}\right\}$,$S_{\frac{n}{2}+1} = \left\{a, v_1, u_{2i-1}; 1 \leq i \leq \frac{n}{2}\right\}$, and$S_{\frac{n}{2}+2} = \left\{a, v_n, u_{2i}; 1 \leq i \leq \frac{n}{2}\right\}$.

For $n \equiv 0(\mathbf{mod}\,3)$ let $D = \left\{a, u_{3i-1}; 1 \leq i \leq \frac{n}{3}\right\}$, then

$$D \cap S_m = \left\{a, u_{3i-1}; 1 \leq i \leq \frac{n}{3}\right\} \cap \left\{a, u_{2i-1}, u_{2j}; 1 \leq i \leq m \leq j \leq \frac{n}{2}\right\} = \{a\}$$

$$D \cap S_{\frac{n}{2}+1} = \left\{a, u_{3i-1}; 1 \leq i \leq \frac{n}{3}\right\} \cap \left\{a, v_1, u_{2i-1}; 1 \leq i \leq \frac{n}{2}\right\}$$

$$= \{a, u_{6i-1}; 1 \leq i \leq \frac{n}{6}$$

$$D \cap S_{\frac{n}{2}+2} = \left\{a, u_{3i-1}; 1 \leq i \leq \frac{n}{3}\right\} \cap \left\{a, v_n, u_{2i}; 1 \leq i \leq \frac{n}{2}\right\}$$

$$= \left\{a, u_{6i-4}; 1 \leq i \leq \frac{n}{6}\right\}$$

For $n \equiv 1(\mathbf{mod}\,3)$ let $D = \left\{a, u_{n-1}, u_{3i-1}; 1 \leq i \leq \frac{n-1}{3}\right\}$, then

$$D \cap S_m = \left\{a, u_{n-1}, u_{3i-1}; 1 \leq i \leq \frac{n-1}{3}\right\} \cap \left\{a, u_{2i-1}, u_{2j}; 1 \leq i \leq m \leq j \leq \frac{n}{2}\right\} = \{a\}$$

$$D \cap S_{\frac{n}{2}+1} = \left\{a, u_{n-1}, u_{3i-1}; 1 \leq i \leq \frac{n-1}{3}\right\} \cap \left\{a, v_1, u_{2i-1}; 1 \leq i \leq \frac{n}{2}\right\}$$

$$= \{a, u_{n-1}, u_{6i-1}; 1 \le i \le \frac{n-4}{6}$$

$$D \cap S_{\frac{n}{2}+2} = \left\{a, u_{n-1}, u_{3i-1}; 1 \le i \le \frac{n-1}{3}\right\} \cap \left\{a, v_n, u_{2i}; 1 \le i \le \frac{n}{2}\right\}$$

$$= \left\{a, u_{6i-4}; 1 \le i \le \frac{n+5}{6}\right\}$$

For $\boldsymbol{n} \equiv 2(\mathbf{mod}\,3)$ let $D = \left\{a, u_{n-1}, u_{3i-1}; 1 \le i \le \frac{n-2}{3}\right\}$, then

$$D \cap S_m = \left\{a, u_{n-1}, u_{3i-1}; 1 \le i \le \frac{n-2}{3}\right\} \cap \left\{a, u_{2i-1}, u_{2j}; 1 \le i \le m \le j \le \frac{n}{2}\right\} = \{a\}$$

$$D \cap S_{\frac{n}{2}+2} = \left\{a, u_{n-1}, u_{3i-1}; 1 \le i \le \frac{n-2}{3}\right\} \cap \left\{a, v_1, u_{2i-1}; 1 \le i \le \frac{n}{2}\right\}$$

$$= \left\{a, u_{n-1}, u_{6i-1}; 1 \le i \le \frac{n-4}{4}\right\}$$

$$D \cap S_{\frac{n}{2}+2} = \left\{a, u_{n-1}, u_{3i-1}; 1 \le i \le \frac{n-2}{3}\right\} \cap \left\{a, v_n, u_{2i}; 1 \le i \le \frac{n}{2}\right\}$$

$$= \left\{a, u_{6i-4}; 1 \le i \le \frac{n-2}{6}\right\}$$

Hence $\gamma(PC_n) = \gamma_{nt}(PC_n)$ for $n \ge 6$ and even numbers.

Based on the description, it can be seen that $\gamma_{nt}(PC_n) = \gamma(PC_n)$ for $n \ge 3$ and $n \in N$. ■

Before discussing the domination number of the transversal neighborhood on a semi-parachute graph, the definition of a semi-parachute graph is given.

Definition 5 [14]. A semi-parachute graph SP_{2n-1} is a graph with $V(SP_{2n-1}) = \{v_i, u_{i-1}, a | i \in [1, n]\}$ and $E(SP_{2n-1}) = \{(a, v_i) | i \in [1, n]\} \cup \{(v_1, v_n)\} \cup \{(v_i, u_i) | i \in [1, n-1]\} \cup \{(u_i, v_{i+1}) | i \in [1, n-1]\}$.

Theorem 6. Given a semi-parachute graph SP_{2n-1} with $n \ge 3$, then.

$$\gamma_{nt}(SP_{2n-1}) = \begin{cases} \frac{n+1}{2}, & for\, n \equiv 1(mod 2) \\ \frac{n+2}{2}, & for\, n \equiv 0(mod 2) \end{cases}$$

Proof. Given a semi-parachute graph SP_{2n-1} with $n \ge 3$ for $\forall n \in N$

$$V(SP_{2n-1}) = \{v_i, u_{i-1}, a; 1 \le i \le n\}$$

$$E\big(SP_{2n-1}\big) = \{(a, v_i); 1 \le i \le n\} \cup \{(v_1, v_n)\} \cup \{(v_i, u_i), 1 \le i \le n-1\} \cup \{(u_i, v_{i+1}); 1 \le i \le n-1\}$$

$$V(N(a)) = \{a, v_i; 1 \le i \le n\},\ E(N(a)) = \{(a, v_i), (v_1, v_n); 1 \le i \le n\}$$

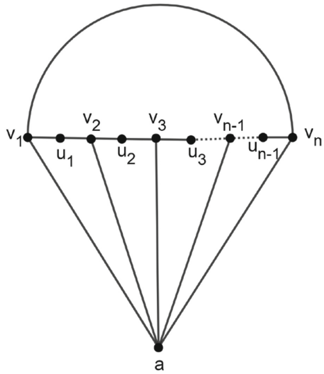

Fig. 5. Semi-parachute graph SP_{2n-1}

$$V(N(v_1)) = \{a, v_1, u_1, v_n\},\ E(N(v_1)) = \{(a, v_1), (a, v_n), (v_1, u_1), (v_1, v_n)\}$$

$$V(N(v_n)) = \{a, v_n, u_{n-1}, v_1\},\ E(N(v_n)) = \{(a, v_n), (a, v_1), (v_n, u_{n-1}), (v_n, v_1)\}$$

For $1 \le i \le n-1$

$V(\langle N(u_i)\rangle) = \{u_i, v_i, v_{i+1}\}$ and $E(\langle N(u_i)\rangle) = \{(u_i, v_i), (u_i, v_{i+1})\}$

For $2 \le i \le n-1$

$V(\langle N(v_i)\rangle) = \{a, v_i, u_i, u_{i-1}\}$ dan $E(\langle N(u_i)\rangle) = \{(a, v_i), (v_i, u_i), (v_i, u_{i-1})\}$.

Let $S \subseteq V$ with $|S| = n$. Then $\bigcup_{v\in s}\langle N(v)\rangle = SP_{2n-1}$ with

$$S_1 = \{v_i;\ 1 \le i \le n\}$$

$$S_2 = \{a, u_i;\ 1 \le i \le n-1\}$$

$$S_3 = \{u_1, v_i;\ 2 \le i \le n\}$$

$$S_4 = \{u_{n-1}, v_i;\ 1 \le i \le n-1\}$$

Is neighborhood set of graph SP_{2n-1}.

Let $S \subseteq V$ with $|S| = n-1$. We get $SP_{2n-1} \ne \bigcup_{v\in s_2}\langle N(v)\rangle$. Thus S with $|S| = n-1$ is not a neighborhood set of graph SP_{2n-1}. Hence S_1, S_2, S_3, and S_4 is $n-set$ of graph SP_{2n-1}.

Let $\gamma(SP_{2n-1}) = \frac{n+1}{2}$ for $n \equiv 1(\text{mod}2)$ and $\gamma(SP_{2n-1}) = \frac{n+2}{2}$ for $n \equiv 0(\text{mod}2)$

1. For $n \equiv 1(\text{mod}2)$

Let $D = \{a, v_{2i};\ 1 \le i \le \frac{n-1}{2}\}$

$D \cap S_1 = \{a, v_{2i};\ 1 \le i \le \frac{n-1}{2}\} \cap \{v_i;\ 1 \le i \le n\} = \{v_{2i};\ 1 \le i \le \frac{n-1}{2}\}$.

$D \cap S_2 = \{a, v_{2i};\ 1 \le i \le \frac{n-1}{2}\} \cap \{a, u_i;\ 1 \le i \le n-1\} = \{a\}$.

$D \cap S_3 = \{a, v_{2i};\ 1 \le i \le \frac{n-1}{2}\} \cap \{u_1, v_i;\ 2 \le i \le n\} = \{v_{2i};\ 1 \le i \le \frac{n-1}{2}\}$.

$D \cap S_4 = \{a, v_{2i};\ 1 \le i \le \frac{n-1}{2}\} \cap \{u_{n-1}, v_i;\ 1 \le i \le n-1\} = \{v_{2i};\ 1 \le i \le \frac{n-1}{2}\}$.

Hence $\gamma(SP_{2n-1}) = \gamma_{nt}(SP_{2n-1})$ for $n \equiv 1(\text{mod}2)$

2. For $n \equiv 0(\text{mod}2)$

Let $D = \{a, v_{2i}; 1 \leq i \leq \frac{n}{2}\}$

$D \cap S_1 = \{a, v_{2i}; 1 \leq i \leq \frac{n}{2}\} \cap \{v_i; 1 \leq i \leq n\} = \{v_{2i}; 1 \leq i \leq \frac{n+6}{4}\}$.

$D \cap S_2 = \{a, v_{2i}; 1 \leq i \leq \frac{n}{2}\} \cap \{a, u_i; 1 \leq i \leq n-1\} = \{a\}$.

$D \cap S_3 = \{a, v_{2i}; 1 \leq i \leq \frac{n}{2}\} \cap \{u_1, v_i; 2 \leq i \leq n\} = \{v_{2i}; 1 \leq i \leq \frac{n+6}{4}\}\}$.

$D \cap S_4 = \{a, v_{2i}; 1 \leq i \leq \frac{n}{2}\} \cap \{u_{n-1}, v_i; 1 \leq i \leq n-1\} = \{v_{2i}; 1 \leq i \leq \frac{n+6}{4}\}$.

Hence $\gamma(SP_{2n-1}) = \gamma_{nt}(SP_{2n-1})$ for $n \equiv 0(\text{mod}2)$

Based on the description, it can be seen that $\gamma_{nt}(SP_{2n-1}) = \gamma(SP_{2n-1})$ for $n \geq 3$ and $n \in N$. ■

4 Conclusion

Let $G = (V, E)$ and $D \subseteq V$. Set D is transversal neighborhood-dominating set in G if set D included in dominating set with the condition that it must intersect with neighborhood set that has the samllest cardinality. The transversal domination number of graph PC_n and SP_{2n-1}:

$$\gamma_{nt}(PC_n) = \begin{cases} \frac{n+3}{3}, \text{ untuk n} \equiv 0(\text{mod3}) \\ \frac{n+5}{3}, \text{ untuk n} \equiv 1(\text{mod3}) \\ \frac{n+4}{3}, \text{ untuk n} \equiv 2(\text{mod3}) \end{cases}$$

$$\gamma_{nt}(SP_{2n-1}) = \begin{cases} \frac{n+1}{2}, \text{ untuk n} \equiv 1(\text{mod2}) \\ \frac{n+2}{2}, \text{ untuk n} \equiv 0(\text{mod2}) \end{cases}$$

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