Analysis of the Validity of the Capital Asset Pricing Model: A Comparison Based on the FTSE ALL-Share and the UK Three-Month Treasury Bills

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Abstract. CAPM (Capital Asset Pricing Model) is the most important model in modern finance that can be used to: calculate the systematic risk of assets, estimate the expected rate of return on assets, and evaluate the performance of a portfolio. In this study, FTSEALL-Share and UK three-month Treasury bills are used as the market portfolio and risk-free rate to compare and determine whether there is correlation between beta coefficient and excess return. The purpose of this study is to conduct two regression analyses and empirical tests of the capital asset pricing model using R-studio coding and finally conclude the validity of the capital asset pricing model.

Keywords: systematic risk of assets · expected rate of return on assets · two regression analysis · validity · empirical test of capital asset pricing model

1 Introduction

First for capital asset pricing theory, which is based on modern investment theory proposed by Markowitz, who advocates certain assumptions about how investors should construct portfolios (or rather, how investors should select investments in portfolios) with respect to investor behaviour and the nature of capital markets. Markowitz demonstrates that rational investors (i.e., those who are risk averse and seek to maximize utility) investors) should evaluate potential portfolio allocations based on the mean and variance associated with the return distribution [3]. Given two investments with equal theoretical returns (measured by the mean of returns), a risk averse investor will choose the lowest risk investment (measured by the variance), but his theory is premised on assumptions such as that capital markets are perfect, which implies that there are no taxes or transaction costs; all traders have free access to all available information; perfect competition exists among all market participants.

Returns are normally distributed. Then came the Capital Asset Pricing Model by William Sharpe and went on to propose that the CAPM proves that by combining assets into portfolios, the unique risk of each asset can be eliminated [6]. This makes market risk the only risk exposure of the portfolio. Because unique risks can be eliminated in

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a diversified portfolio, unique risks are also referred to as diversifiable risks. Because systematic risk cannot be eliminated even in a well-diversified portfolio, systematic risk is also referred to as no diversifiable risk. The derivation of the CAPM includes several key assumptions, some of which are the same as those used by Markowitz in deriving the MPT:

All market participants have access to information, which means that all information is freely available and immediately absorbed.

That all market participants have the same expectations.

that all market participants make investment decisions based on the mean and variance of returns.

the absence of transaction costs, taxes, or other frictions

allocations can be made in any fraction of the amount invested (i.e., full divisibility)

the ability of all participants to borrow and lend at a common risk-free rate.

no allocation decision by any individual investor can change the market price. The assumptions made by the two above are rather idealistic, are markets really perfectly efficient, so I think there are some shortcomings for example the assumptions are too simple: the CAPM model assumes that markets are perfectly efficient and that investors are rational and that all investors have the same investment preferences and risk attitudes. These assumptions can sometimes be too simple and do not match the real market situation, so the CAPM model's prediction results may be inaccurate.

Ignoring unsystematic risks in the market: CAPM models focus only on the systematic risks of the market as a whole and ignore the unsystematic risks unique to individual assets [5]. These unsystematic risks may have an impact on the expected return of individual assets, and therefore need to be considered when making investment decisions as well [4]. Therefore, this study deliberately selects data such as the market portfolio and the risk-free rate are represented by the FTSE ALL-Share index and the UK three-month Treasury roll. Through a two-step regression Hypothesis testing compares whether there are more explanatory variables capturing the non-systematic risk, which is useful for the model to be valid.

Heading into the report we need to first establish what the CAPM actually is. The CAPM is widely used financial modelling tool for in any area of the securities market but typically used in Stocks. This tool gives individuals a gauge of the market risk in relation the expected returns for a specific asset [2]. The CAPM is commonly used for pricing securities which could be considered volatile or risky. The CAPM formula can be shown as the following:

$$\mathrm{E}(r_i) - r_f = \beta_i[E(r_m) - r_f]$$

where:

$\mathrm{E}(r_i) =$ **Expected Return**

This notation simply shows, given all additional variables in the equation, the expected return of this specific asset over its lifetime.

$r_f =$ **Risk-free rate**

When investing, individuals demand to have a form of compensation for the risk of their asset and the time value of money (TVM), put simply is that a sum of money is worth more now than in a date in the future. The risk-free rate makes up for this TVM.

$\beta_i =$ **beta of the asset**

The beta of the asset indicates the systematic risk, for example if the asset was a stock, it would give the investor an indication of how volatile the stock is compared to the whole market, therefore how risky it is. With a Greater Beta, the more risk is involved [7]. The positive of a greater beta is, a greater margin for profits and the negative being increased risk of capital loss [1]. In the CAPM formula it is assumed the beta will reduce the risk of your portfolio.

$[E(r_m) - r_f] =$ **Risk Premium**

When an individual holds an asset seen as risky, the risk premium is an additional return the investor receives or is expected to receive.

2 Data Selection and Processing

2.1 Research Pathway and Research Methodology

This study approaches the study through a capital asset pricing model and then implements a two-step regression and empirical test, first using R-studio coding to obtain the stock market line to see if the relationship between beta and risk premium is consistent with the CAPM hypothesis, and then using the graph to derive the value of R-square to determine the change in the average excess return and beta coefficient in the sample stocks, and then through a two-step regression Hypothesis testing compares whether there are more explanatory variables capturing the non-systematic risk, which is useful for the model to be valid.

2.2 Data Selection

For the processing of data, this study uses the G.csv file in canvas as a unique sample of monthly time series data of stock returns spanning the period January 2009 to December 2019, which serves as a large sample of stocks. In addition, the market portfolio and the risk-free rate are represented by the FTSE ALL-Share index and the UK three-month Treasury roll.

3 CAPM Two-Step Regression Analysis and Empirical Test

3.1 Introduction of Stock Market Line

For the two-step regression analysis, we need to first obtain some information through the equity market line, which captures the relationship between security payoffs and the beta coefficient of systematic risk, as well as the relationship between equilibrium expected returns and risk for all risky assets in the market.

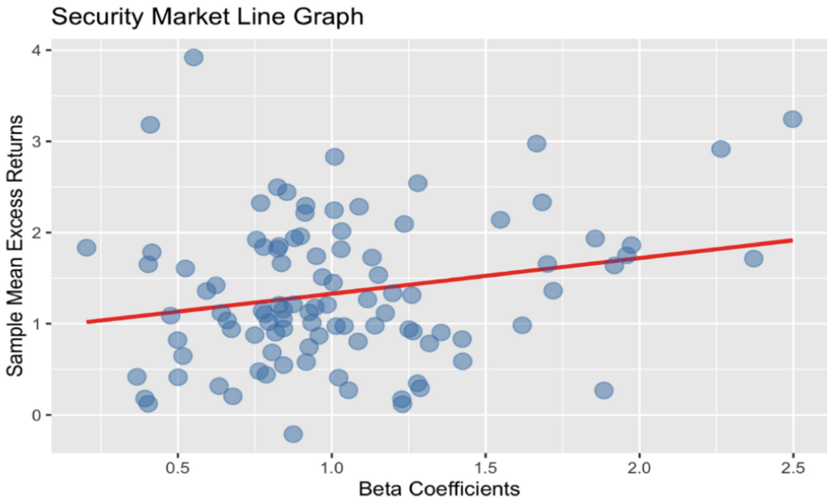

Fig. 1. Stock market lines prepared by R-studio with selected data

3.2 CAPM Two-Step Regression Analysis

Step (i)

This study derives the security market lines by R-studio coding. As shown in Fig. 1, the relationship in the above graph appears to be broadly linear and there is a positive relationship between beta coefficients and the risk premiums for each stock. The graph depicts that stocks with higher beta coefficients will have higher risk premiums and those with smaller beta coefficients will have lower risk premiums. This corresponds with the CAPM assumption that, "investors are rewarded for bearing systematic risk".

Step (ii)

As shown in Table 1, this study has obtained an R^2 value of 0.049 which is rather quite low. The adjusted R^2 is even lower at 0.039. This suggests that only 4.9% of the variation in the sample mean excess returns of the stocks is explained by the variation of the beta coefficients for each stock. These results don't necessarily go against the CAPM assumption that higher beta corresponds to higher returns, however the relationship between the two variables is only a very small explanation for the high variation in the risk premiums for each stock. Due to the lack of explanatory power, This study continues with the CAPM hypothesis test

3.3 Hypothesis Testing Analysis

Second-pass regression model:

$$\overline{r_l} - \overline{r_f} = \gamma_0 + \gamma_1 \widehat{\beta}_i + \varepsilon_{i,t}$$

As shown in Table 1 the model should be:

$$\overline{r_l} - \overline{r_f} = 0.938 + 0.391 \widehat{\beta}_i + \varepsilon_{i,t}$$

Table 1. Test results of two-step regression analysis of the asset capital pricing model

Statistic	N	Mean	St. Dev.	Min	Max
betas	100	1.028	0.448	0.203	2.498
Error.var	100	59.823	62.780	2.590	441.856
sample.meanxs	100	1.340	0.793	−0.212	3.920

	Dependent variable:
	sample.meanxs
betas	0.391^{**}
	(0.174)
Constant	0.938^{***}
	(0.195)
Observations	100
R^2	0.049
Adjusted R^2	0.039
Residual Std. Error	0.777 (df = 98)
F Statistic	5.025^{**} (df = 1; 98)
Note:	$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

The CAPM implies that $H_0 : \gamma_0 = 0$ should not be rejected. Here This study introduces the alternative hypothesis $H_A : \gamma_0 \neq 0$ to empirically test the CAPM. In the t-test, t value = 4.805, which is greater than the critical value $t^*_{98} = 2.627$. Thus, this study rejects the null hypothesis by a t-test at the 1% level of significance, which contradicts the CAPM implication, meaning that γ_0 cannot equal to zero. Therefore, this study can interpret that when investors invest in the UK 3-month Treasury bills, the mean excess return for each sample stocks cannot be zero.

Also, the CAPM implies that $H_0 : \gamma_1 = \overline{r}_m - \overline{r}_f$ should not be rejected. Similarly, this study introduces the alternative hypothesis $H_A : \gamma_1 \neq \overline{r_m} - \overline{r}_f$ to empirically test the CAPM. As a result, t value = - 0.684, which is greater than the critical value. Thus, $H_0 : \gamma_1 = \overline{r}_m - \overline{r}_f$ is not rejected by a t-test at the 1% level of significance. Moreover, this means that investors earn a market risk premium when taking the systematic risk, which is still supportive of the CAPM.

Furthermore, there is also an extended version of this approach that can be used to test the non-systematic risk is not priced as the CAPM predicts.

The extended version:

$\overline{r_l} - \overline{r}_f = \gamma_0 + \gamma_1\widehat{\beta}_i + \gamma_2\widehat{\sigma}^2_{\varepsilon_i} + \mu_i$. This extended version involves the estimated variance $\widehat{\sigma}^2_{\varepsilon_i}$ as a regressor in the second pass-regression to capture non-systematic risk.

The regression table:

As shown in Fig. 2 the model should be:

$$\overline{r_l} - \overline{r}_f = 0.995 - 0.063\widehat{\beta}_i + 0.007\widehat{\sigma}^2_{\varepsilon_i} + \mu_i$$

This study can test $H_0 : \gamma_2 = 0$ using t-test to see if the non-systematic risk is not priced as the CAPM predicts. And the alternative test is $H_A : \gamma_2 > 0$. Because if $\gamma_2 > 0$, it suggests that investors earn a risk premium for bearing non-systematic risk. In the right -tailed t-test, this study can see that the t value = 5.540, and the critical value $t^*_{97} = 2.365$, which means that I can reject the null hypothesis, and the alternative hypothesis is not rejected by a t-test at the 1% level of significance. And this result is not supportive of the CAPM which predicts that only market risk matters, measured by β (Sollis, R, 2012).

Additionally, the R-squared of the extended version model is now 0.2775, which means that the explanatory variable $\widehat{\beta}_i$ and $\widehat{\sigma}^2_{\varepsilon_i}$, explain around 27.75% of the variation in the sample mean excess returns, meaning there is 72.25% of residual variation in sample mean excess returns left to explain. This explanatory power increased from about 4.9% (the R-squared of the second-pass regression model) to 27.75%, meaning that this model is an improvement over the second-pass regression model, because it includes more explanatory variable to capture the non-systematic risk.

	Dependent variable:	
	Sample Mean Excess Returns	
	(1)	(2)
Beta hat	0.391**	-0.063
	(0.174)	(0.173)
Estimated Error Variance		0.007***
		(0.001)
Constant	0.938***	0.995***
	(0.195)	(0.171)
Observations	100	100
R^2	0.049	0.277
Adjusted R^2	0.039	0.263
Residual Std. Error	0.777 (df = 98)	0.681 (df = 97)
F Statistic	5.025** (df = 1; 98)	18.626*** (df = 2; 97)
Note:		*p<0.1; **p<0.05; ***p<0.01

Fig. 2. Test results of an extended version of the regression analysis of the capital asset pricing model on capturing unsystematic risk

4 Conclusion

4.1 Results of the Empirical Analysis

The above empirical analysis indicates that in both regression experiments the estimated value of the stock beta coefficient is greater than 0 and passes the significance test at the 0.01 level, By the SML we know that individual stock returns are positively correlated with systematic risk and investors are rewarded for taking risk, which is consistent with the assumptions of the model. However, in the results of the hypothesis test, the t-value is greater than the critical value, and I reject the null hypothesis by t-test at 1% significance level, so it does not support CAPM. Also, in this extended model experiment, the t-value is 5.540 and the critical value is 2.365, the alternative hypothesis is not rejected by t-test at 1% significance level, and this result does not support CAPM. In addition, the value of R-square in the second regression is only 0.049, indicating that the explanatory power of the regression equation is not sufficient compared to the later model, however, the explanatory power of R-square through the extended model test has significantly increased to 0.277, and there are more variables to explain the non-systematic risk in the model.

4.2 Shortcomings of the Capital Asset Pricing Model

In summary, the CAPM hypothesis test is not fully valid and cannot determine the expected return through systematic risk alone, because there are limitations to the assumptions that are fully consistent with the model, and pricing is not only related to systematic risk but also to unsystematic risk. In addition, the assumptions of the capital asset pricing model are not fully valid because it requires that investors are recipients of security prices and cannot influence equilibrium prices, and that markets are perfectly competitive. All investors are risk averse, seeking to minimize portfolio variance under a certain return, or maximize return under a certain variance, and investors seek the highest Sharpe ratio, etc. In general, the CAPM is still widely used in both academia and industry at present. The model is not that perfect, but it is acceptable.

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