



The Indian Vedic Computational System: An Alternative Ways for Nurturing Students' Brain in Handling Mathematical Operations

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Abstract. In this age of calculators, the need for teaching computational standard procedures is often debated. However, instruction in four basic operations should provide students with understanding skill on computation. For students to perform computation successfully and meaningfully, their sensitivity to numbers and their flexibility in thinking about number are very important. However, it has become a common problem that students find it is difficult to perform some mathematical operations, and at the same time, students are not interested in calculation activities. Meanwhile, Indian Vedic mathematics, which is an ancient mathematical system, offer some different approached to computation methods. The term Vedic mathematics refers to the sixteen mathematical formulas or *sutras* and their corollaries (*sub-sutras*) that provide relationships or links through mathematics. This paper shall discuss some *sutras* derived from the Vedic mathematics system used in basic operations. The *sutras* with single-line phrases are easy to understand and remember. Some *sutras* discussed in this paper speaks for their coherence and simplicity which might develop aptitude and ability, and nurture students' brain in handling mathematical operations to develop their logical thinking and intelligence. Once students understand the mathematical concepts for particular operations, then they might use the related *sutras* to help them work faster and have more fun in completing mathematical operations.

Keywords: Mathematical operation · Sutra and sub-sutra · Vedic mathematics

1 Introduction

1.1 Background

The use of calculator is widely increasing in society. However, the use of a calculator that is too early for children who are still learning the concept of basic computation may not be recommended. Likewise, if the questions given to students are direct questions such as finding 45×72 , it is certainly not a wise thing to let students use a calculator to get the answer.

Many findings also found that prolonged use of calculators can cause significant brain atrophy. Sometimes, we can even see someone use a calculator to determine something

as simple as $15 + 27$. Moreover, it is also worth noting that some calculators are even prone to errors.

Moreover, in many cases, students are lazy and often feel bored with computations. They are interested in doing calculations, and rely heavily on calculators and computers. So that if the tools are not available, they give up because they are not able to perform calculation properly.

In order to address the problems mentioned above, various approaches have been taken to develop students' computational proficiency beyond traditional mathematics. One such approach is mental computation, which enables rapid calculation without the use of computing tools. Vedic mathematics is a set of techniques that utilizes this approach [1].

The existence and application of Vedic mathematics are expected to help students when doing mathematical calculation. Vedic (pronounced 'vaydik') mathematics is an ancient system formulated by Indian ancient sages and *rishis* many centuries ago. Singhal [2] and William [3] stated that around the year 1911 to 1918, these ancient texts on Vedic mathematics were rediscovered by Sri Bharati Khrisna Tistaji who lived from 1884–1960, from the ancient Sanskrit text called 'Veda'. The word *Veda* means knowledge and also refers to ancient Sanskrit writing contained in millions of manuscripts scattered in India. These texts contain incredible detail on various subjects including mathematics. The system of Vedic mathematics reconstructed by Tirtaji [3] provides much more unified, direct, and coherent mathematical calculations which are easier to learn or do. Vedic mathematics is a term used to describe a collection of sixteen mathematical formulas, known as *sutras*, and their fourteen accompanying corollaries, or sub-*sutras*, which have been derived from the Vedic system. This indicates its ability to efficiently and easily tackle mathematical problems through its consistency and simplicity [2].

Vedic system can be used to solve complex mathematical problems more easily. It has basic rules for particular computations. Each *sutra* has many implications in different computation methods. Two or three combined *sutras* or sub-*sutras* also yield more alternatives that can be used in calculations to help our mental computation.

1.2 Terminology in Vedic

The two most important terminology in Vedic mathematics are *vinculum* and *deficiency (complement)*. The integer system is made up of only nine digits consisting of the number 1 through 9 plus zero. Bases are numbers that have 1 as their initial digit followed by zeros [4]. These numbers repeat themselves in a specific order after the numbers 10, 100, 1000, and so on.

In the Vedic system, the vinculum is defined as simplifying a number according to its base. For any given number, digits over 5 (that are 6, 7, 8, and 9) are substantially closer by 4, 3, 2, and 1 respectively to the next number with the base pattern. Using vinculum, we might change 8 into $\bar{2}$. An arithmetic operation can be greatly simplified by using vinculum. This also provides flexibility in choosing different ways of representing numbers. For example, since 48 is closed to 50, we may use $5\bar{2}$ or $(50 - 2)$ instead of 48. Similarly, we write 297 as $30\bar{3}$ or $(300 - 3)$, and more example, we notice 381 as $4\bar{2}1$ or $(401 - 20)$.

If the *vinculum* talks about simplifying the digit of a number, so that we only work with digits below 5, the *complement* serves to fit the numbers to the nearest base. From many arithmetic operations, it is easier to convert a big number into a smaller one by using its complement. In many books, the use of the term *complement* is interchanged by the term *deficiency* (what is the deficiency from the nearest base).

There are sixteen *sutras* (aphorism or formula) and their *sub-sutras* or corollary invented by the Swamiji [4, 5], and [6]. All *sutras* and their corollaries are as follows (Table 1):

Further exploration of the use and advantages of Vedic mathematics have been conducted by many researches. For example, Hazra [7] demonstrated the use of Vedic mathematics in the implementation of circular convolution circuit. Anjana et al. [8] showed the implementation of the Vedic multiplier using Kogge-stone adder, and validate their design is faster multiplier and adder than traditional way.

In this paper, we shall only discuss some of the *sutras*, which are related to the operation of multiplication that often used by students such as multiplication and squaring numbers.

2 Methods

This research is a literature study that discusses formulas (*sutras*) and corollaries (*sub-sutras*) in Vedic mathematics. From sixteen *sutras* of Vedic mathematics, the first three-*sutra* are selected, and their application for basic computations often performed by students is discussed. The discussion centers on the following considerations:

- i) The *sutras* used are focused on multiplication;
- ii) The *sutras* are applied to squaring numbers ended in 5;
- iii) The *sutras* can be applied to perform multiplication of two numbers that are closed to bases;
- iv) The *sutras* are used for the multiplication of algebraic expression, binomial and polynomial; and
- v) The application of *sutras* can be used to perform multiplications of specific number to apply them with an example of their uses.

Particularly for the multiplication of specific numbers, the thirteenth *sutra* and the ninth *sub-sutra* are also discussed. The relevant *sutras* discussed in this paper are as follows, with the meaning is described within bracket: a) the *Ekādhikena Pūrvēna sutra* (by one more than the one before); b) the *Nikhilam Navataścaramaṃ Daśataḥ sutra* (all from nine and the last from ten); c) the *Ūrdhva Tiryagabhyām sutra* (vertical and cross-wise); d) the *Sopāntyadvayamantyam sutra* (the ultimate and twice the penultimate); and e) the *Antyayoreva sub-sutra* (only the last two digits).

3 Sutras and Their Meaning in Vedic System

3.1 Ekādhikena Pūrvēna (by One More Than the One Before)

Ekādhikena means ‘the one more than the one before’ [9], which means we add 1 to the previous digit. For example, given the number 65, since 6 is the previous digit before 5, then we add 1 to this digit to get 7. So, 7 is called ‘one more than the one before’

Table 1. Vedic mathematics Sutra and sub-sutra according to [4]

Number	Sutra	Sub-sutra
1	Ekādhikena Pūrvana <i>By one more than the one before</i>	Anurupyena <i>Proportionately</i>
2	Nikhilam Navataścaramaṃ Daśataḥ <i>All from 9 and the last from 10</i>	Sisyate Sesasamjnah <i>The remainder remains constant</i>
3	Ūrdhva Tiryagabhyām <i>Vertically and Crosswise</i>	Adyamadyenantyamantyena <i>The first by the first and the last by the last</i>
4	Parāvartya Yojayet <i>Transpose and Apply</i>	Kevalaih Saptakam Gunyat <i>For 7 the multiplicand is 143</i>
5	Sūnyam Sāmyasamuccaye <i>If the samuccaya is the same, it is zero</i>	Vestanam <i>Osculation</i>
6	Anurūpye Sunyam anyat <i>If one is in ratio the other is zero</i>	Yavadunam Tavadunam <i>Lessen by the deficiency</i>
7	Sankalana Vyavakalanābhyām <i>By addition and by subtraction</i>	Yavadunam Tavadunikrtya Varganca Yojayet <i>Whatever the deficiency lessen by that amount and set up square of the deficiency</i>
8	Pūranāpūranābhyām <i>By the completion and non-completion</i>	Antyayordasakepi <i>Last totaling 10</i>
9	Calana Kalanābhyām <i>Differential Calculus</i>	Antyayoreva <i>Only the last term (last two digits)</i>
10	Yāvadūnam <i>By the deficiency</i>	Samuccayagunitah <i>The sum of the product</i>
11	Vyastisamastih <i>Specific and General</i>	Lopanasthapanabhyam <i>By alternate elimination and retention</i>
12	Sesānyankena Caramena <i>The remainder by the last digit</i>	Vilokanam <i>By mere observation</i>
13	Sopantyadvayamantyam <i>The ultimate and twice the penultimate</i>	Gunitasamuccayah Samuccayagunitah <i>The product of the sum – the sum of the product</i>
14	Ekānyūrnena Pūrvana <i>By one less than the one before</i>	Dhvajankah <i>On the flag</i>
15	Gunitasamuccayah <i>The product of the sum</i>	
16	Gunakasamuccayah <i>All the multipliers</i>	

and we said the *Ekādhikena* for 65 is 7. Starting from digit 1, all digit can be generated by using this *sutra*. Some simple applications of this *sutra* are squaring numbers with specific digits.

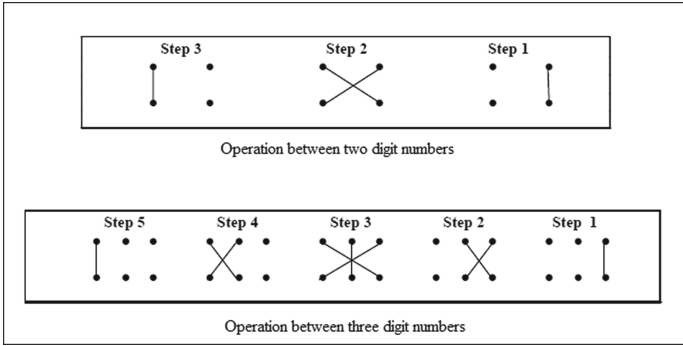


Fig. 1. Operation of vertical and crosswise

3.2 *Nikhilam Navataścaramaṃ Daśataḥ* (All from Nine and the Last from Ten)

This *sutra* means that the focus is on the number (digit 9). For example, what is ‘all from nine’ for the number 786? We take each of these digits from 9. So, taking away 7 from 9 is 2, and taking away 8 from 9 is 1. Since 6 is the last digit, take away 6 from 10 which is 4. So, all from nine and the last from ten for the number 786 is 234. This *sutra* is the most widely used and the easiest to understand. This *sutra* is also simple and very widely applied in subtraction, division, and multiplication.

3.3 *Ūrdhva Tiryagabhyām* (Vertical and Crosswise)

This *sutra* contains two direction in operating numbers, namely vertical and crosswise [10]. In this type, for operating on two-digit numbers and three-digits numbers, steps are as follows:

These steps can be expanded as the numbers have more digits. This *sutra* is a simple method for particularly for multiplication which is not only easy to remember, but also saves lot of time [10].

3.4 *Sopāntyadvayamantyaṃ* (The Ultimate and Twice the Penultimate)

This *sutra* refers to operation which focuses on the ultimate (last) digit and twice of the penultimate (the second last) digit. If it is given a number 234, then the last digit or the ultimate is 4, the penultimate is 3, so twice the penultimate digit is 6. This *sutra* is often used in multiplication.

3.5 *Antyayoreva* (Only the Last Two Digits)

This *sutra* means that the focus of the operation is just for the last two digits of the number given. The specific multiplications also apply for this formula.

4 Application of Sutras for Multiplications

In the Vedic Method, there are some easy ways of multiplying numbers by specific numbers [2]. There are many *sutras* that can be applied, either it is a single *sutra* or combined two or more *sutras*. The following will be discussed some multiplications using the *sutras* above.

4.1 Squaring of Two-Digit Numbers Ending in 5

When squaring two-digit number ending in 5, the Ekādhikena Pūrvena *sutra* [by one more than the one before] provides a simple way to solve it [2] and [9]. For example, multiplying 85 by itself, or squaring 85. In this case, we consider the digit before 5 to be 8. The one more than 8 is 9. The first part of the answer is by multiplying the one digit before 5 (that is 8) by the one more than itself (that is 9) to get 72. The second part of the answer is by multiplying 5 itself which is 25. So, finally the square of 85 is 7225. It can be described as follow:

$$85^2 = 7225 \text{ which comes from } 72 / 25 \text{ where } 72 = 8 \times 9, \text{ and } 25 = 5^2.$$

4.2 Multiplying the Two Numbers Closed to Unity-Based Numbers

When multiplying two-digit numbers closed to unity bases, it can be solved by applying two simple *sutras*, namely *all from nine and the last from ten* and *vertical and crosswise* [2] and [6]. If we want to multiply 97×86 , the multiplication involving large digits such as 7, 8, and 9 are sometimes considered quite difficult by students. However, using ‘all from nine and the last from ten’ to the numbers 97 and 86 results in 3 and 14 respectively. This implies that 97 is 3 under 100 as well as 86 is 14 under 100 (recall that we do the same for deficiency).

The answer of the multiplication above will be acquired by dividing it into two parts and then applying ‘vertical and crosswise’. We cross-subtract (crosswise) the deficiencies from other number (in this case, either $97 - 14$ or $86 - 3$, whichever we like) to get the left-hand part of the answer and multiply vertically these two deficiencies to get the right-hand part of the answer. So, the answer of 97×86 is 8342 which comes from $83 / 42$. Simply, we can write as in Fig. 2 below:

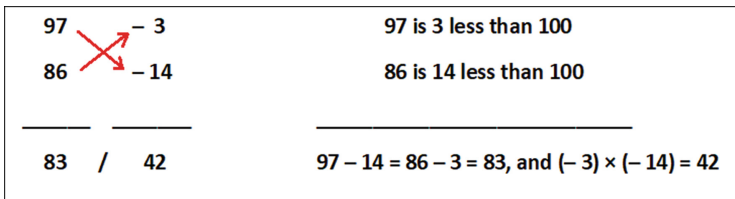


Fig. 2. Applying sutras, all from nine and the last from ten and vertical and crosswise.

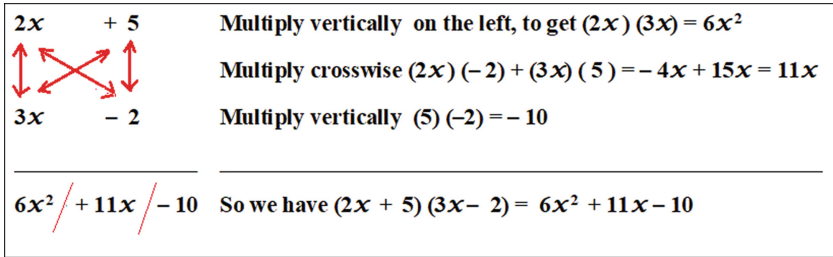


Fig. 3. Applying sutra, *Vertical and crosswise* for multiplying the binomials.

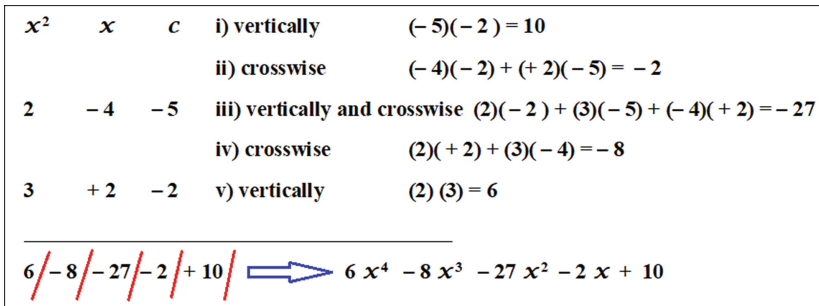


Fig. 4. Applying sutra, *Vertical and crosswise* for multiplying the polynomials.

4.3 Multiplying Binomials and Polynomials

When we become proficient in using the ‘vertical and crosswise’ technique for multiplying numbers, it makes multiplying algebraic expressions easier. This method can be applied to the multiplication of binomials and polynomials as well. The ‘vertical and crosswise’ sutra serves as the basis for this approach, and the process remains the same. For example, to multiply $(2x + 5)$ by $(3x - 2)$. By applying the steps of this *sutra*, we might lead to the following process (Fig. 3):

For polynomial cases, the way is the same. For example, to multiply polynomial $(2x^2 - 4x - 5)$ by $(3x^2 + 2x - 2)$. The steps are carried out with the second line rule in Fig. 1 as follow (Fig. 4):

4.4 Multiplying Two Numbers Whose First Figures Are the Same, and Whose Last Figures Add Up to the Unity Numbers (Ends by Zero)

The *Ekādhikena Pūrvvena sutra* [by one more than the one before] also provides a simple way to solve multiplication whose first numbers are the same, while the last digits add up to zero of the unity number. For example, to multiply 43×47 . In this case, the same digit is 4, and the last digits add up to 10 (from $3 + 7$). For this case, we do the following steps: the one more than 4 is 5, so $4 \times 5 = 20$, and $3 \times 7 = 21$. The first part of the answer is 20, and the last part of the answer is 21. It guides us to the answer of $43 \times 47 = 2021$.

Another example is when we multiply 302×398 . The first digit is 3, and the last digit add up to 100 which comes from $(02 + 98)$. By the same steps as before, the first part

of the answer is $3 \times 4 = 12$. The last part of the answer is to multiply $02 \times 98 = 0196$. It can be noted that we need four digits for the last part because we multiply two digits by two digits. So, it comes to the answer that 120196.

4.5 Multiplying Some Specific Numbers

The Vedic approach involves several simple techniques for multiplying numbers by certain values, making the process of multiplication more interesting [2]. Since in Vedic mathematics systems provides us with large number of options at each stage of working, then it will depend upon the pattern of the problem we are working with [11]. In this paper we shall deal with some of the multiplications of numbers.

Multiplication by 11. In multiplication by 11, the sub-*sutra* *Antyayoreva* [only the last two digits] makes it very easy since we only need to consider the last two digits. Using this *sutra* only requires addition instead of multiplications, and interestingly enough, the answer is the same as multiplying the number by 11.

For example, to find the result of the multiplication of 3425×11 . By applying the *sutra*, we do the following steps (Fig. 5):

Multiplication by 12. In multiplication by 12, the *sutra* *Sopāntyadvayamantyam* [the ultimate and twice the penultimate] simplify the process of multiplication. Using this *sutra*, similar to the previous multiplication by 11, we also make a zero and which for both ends, then add the ultimate (last) digit and the twice of the penultimate (second last) digit. For example, to find the multiplication of 32154×12 . By applying the *sutra*, we do the following steps (Fig. 6):

3425 x 11 = ...	
Step 1.	0 3 4 2 5 0
	place zeros on both ends Convert it to the zeros and which such that it does not give different value, since zeros are not part of original numbers
Step 2.	
	keep adding the last two digits of the original Number, $0 + 5 = 5$ $5 + 2 = 7$ $2 + 4 = 6$ $4 + 3 = 7$ $3 + 0 = 3$
The answer of $3425 \times 11 = 37675$	

Fig. 5. Applying *sutra*, only the last two digits for multiplying a number by eleven

$32154 \times 12 = \dots$	
Step 1.	0 3 2 1 5 4 0
	place zeros on both ends Convert it to the zeros and which such that it does not give different value, since zeros are not part of original numbers
Step 2.	0 3 2 1 5 4 0
	keep adding the last digits and twice of the second last digit of the original number, $0 + (2 \times 4) = 8$ $4 + (2 \times 5) = 14$, so we put 4 and carry 1 $5 + (2 \times 1) = 7 + 1 = 8$, add 1 as carry over to get 8 $1 + (2 \times 2) = 5$ $2 + (2 \times 3) = 8$ $3 + (2 \times 0) = 3$
The answer of $32154 \times 12 = 385848$	

Fig. 6. Applying sutra *the ultimate and twice the penultimate* for multiplying a number by twelve

5 Discussion

Hundreds of methods of computations can be generated from the *sutras* of Vedic mathematics, not only for multiplication but also for addition, subtraction, division, exponentiation and roots. Vedic mathematics also not only applies to integers, but also fractions, decimals and others. Savadi and Yamanshetti [12] after reviewing many papers on design of digital signal processor, concluded that the multiplier designed using *Ūrdhva Tiryagabhyām* sutra and binary dividers implemented using *Nikhilam Navataścaramaṇ Daśataḥ* sutra and *Parāvartya Yojayet sutra* are superior of speed, are, complexity as compared to regular multiplication and division method.

From the *sutras* discussed in this paper, it can be seen that the use of *sutras* can facilitate faster calculations with interesting tricks so that students are not easily tempted to rely on pressing the calculator or computer buttons in an effort to get answers. In the previous examples it is also seen that one *sutra* may be applied to several different calculation methods, whereas more than one *sutra* is sometimes combined to solve a computational problem.

One of the benefits of these *sutras* is that students can play more with mathematical or arithmetic operations where they can pose problems with handpicked numbers and then calculate the answer by themselves. For example, once students know how to find the square of 85 using the *sutra* ‘by one more than the one before’, they can play around and try for themselves how this *sutra* works when squaring the numbers 65, 95, 115, and other numbers ended in 5. Although in this paper only small examples are discussed such as squaring numbers ending in 5s, but in Vedic mathematics, there are many methods to calculate square numbers that are simpler when compared to regular ways, and them make students easily use and enjoy [13].

Likewise, when students calculate the multiplication of binomials or polynomials, they may try calculations by choosing as many binomials or polynomials as they want, and have fun working with the appropriate *sutra*.

What may also interested for students are to figure out how a computational operation may be partially separated to produce a final answer as in the case of the sub-*sutra*

'only the last two digits' and the *sutra* 'the ultimate and twice the penultimate'. Long calculations can be done partially so that the operation of numbers is very simple and only requires a one-line process, compare to the traditional ways that takes a long time and process to arrive at the desired answers. This is also in accordance with the fact stated by Kandasamy [5] where many mathematics teachers claimed that Vedic is a concise method.

Some critics and experts also argue that carrying out the arithmetic calculation process by using programs in computer software or calculators continuously will only lower the students' numeracy skills. The brain, like a muscle, also need exercise, when doing computation mentally without using calculator, we basically train ability to concentrate.

Singhal in [2] also stated that Vedic mathematics can nurture our brain and direct full concentration. These qualities make mathematics easy, enjoyable, and flexible. Their qualitative approaches also make use of both part of brain. Even in more general perspective, in the Vedic world view, the process in the sky, on earth, and within the mind are taken to be connected [14]. The Vedic system has been incorporated in the educational syllabi so widely, even the NASA (National Aeronautics and Space Administration) scientists apply its principle in the area of artificial intelligent (AI).

Vedic mathematics can be easily used and enjoyed by anyone. As learning it can be done with minimal time and effort, these methods can be an attractive alternative for calculation in numerous competitive exams [13]. It means that Vedic mathematics is also worthful in preparing students for competitive examination, since it provides some extra skills in computing. However, the real beauty and effectiveness of Vedic mathematics cannot be fully appreciated without actually putting it into practice. Without practice, the Vedic mathematics will be soon forgotten. In this modern period, summer camps are often organized predicated on Vedic fine calculations for easy and interesting activity for students [14]. So, it would be beneficial if these methods are also taught in regular schools and teachers encourage students to apply it in schools and daily life.

6 Conclusion

The Vedic mathematics is an ancient system of mathematics. Its *sutras* and sub-*sutras* can turn operations into single line or phase that are easy to understand and remember. The qualities of sutras make mathematics easy, enjoyable, and flexible to students to solve problems. Once we understand the concept of mathematical operations, some sutras can be applied to simplify them and make problems interesting and logic.

In Vedic mathematics, problems and calculations that seem difficult or calculation involving large numbers can often be completed in simpler ways, often mentally. It can make someone, particularly students become more creative, interested and skilled in counting technique. This is because the Vedic system itself is a mental computational system.

This Vedic science has been widely integrated into the syllabus of many educations worldwide. Even many scientists applied it to artificial intelligent and named it the *Cosmic Calculator*.

Although many people may think that in this modern era calculations can be done faster because of the availability of calculators and computers with various calculation

software to obtain the answer in seconds, but the use of sutras still helps students to keep doing mental calculations to nurture their brains in handling mathematical operations.

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