Exploring Preservice Mathematics Teachers' Noticing of Students' Thinking on Probability

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Abstract. Noticing is one of the essential skills for preservice mathematics teachers. Therefore, it is vital to develop these skills so that students are ready to face their future work environment, especially in implementing mathematics learning that promotes mathematical thinking. This study aims to analyze preservice mathematics teachers' noticing of students' thinking in probability. The present study employed a qualitative method. The results show that analysis and discussions of classroom videos enabled the preservice mathematics teachers to analyze mathematics lessons and supported them to notice classroom instruction in more pedagogically relevant ways. However, the preservice mathematics teachers still require further work in interpreting and responding the students' mathematical thinking.

Keywords: Mathematical thinking · Probability · Teacher noticing

1 Introduction

A teacher asks his students to determine the probability of getting a sum of seven when rolling a pair of dice. A group of students answered that the probability is one per seven. The students also demonstrated that there were 21 possible outcomes, three of which had a sum of seven. The teacher said that it was incorrect and proceed to show them the correct answer and how to find it correctly. This vignette begs the question, does the teacher give appropriate response in ways that provide him the windows to the development of student thinking?

Effective mathematics teaching needs teacher's role to teach responsively in attending student's mathematical thinking [1, 2]. In order to carry out responsive teaching, it is very important for teachers to have a wide array of knowledge and skills [3, 4]. One of these skills is teacher noticing [5]. The teacher noticing is the teacher's skill in observing and reasoning about learning moments in the classroom.

Noticing is an important skill every teacher must have, and admittedly, it is not an easy task for teacher [6]. A teacher should have enough content knowledge and understand students' possible strategies in solving mathematical problems and why they use them. This knowledge would give teacher consideration in responding and developing students' thinking.

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The concept of professional noticing of students' mathematical thinking as defined by Jacobs et al. [7] comprises three interconnected skills namely *attending*, *interpreting*, and *deciding*. *Attending* refers to how well the teachers can observe and highlight the mathematical ideas that appear in their answers. *Interpreting* signifies the teachers' ability to understand the students' strategy by connecting it to current literature on mathematics learning and students' understanding. On the other hand, *deciding* focuses on how the teachers respond to and scaffold the students' idea.

Noticing students' mathematical thinking is not only important for teacher, but also for preservice mathematics teacher. Preservice mathematics teachers should be equipped with skills on interpreting students' mathematical thinking and use them to make an appropriate response. The present study aims to analyze preservice mathematics teachers noticing of students' thinking in the topic of probability.

2 Methods

The present study was part of a larger study in improving preservice mathematics teachers' noticing skills by analyzing students' thinking through classroom video clips. The present study employed qualitative method. The study involved 26 preservice mathematics teachers ($n = 26$,7 male, 19 female) in a private university in Yogyakarta, who are in their fourth semester and are enrolled in High School Mathematics Teaching and Learning course, as part of the study program offered by the university. In this paper, the term "preservice mathematics teachers" will be used interchangeably with "participants."

Data in this study were the written report of the preservice mathematics teachers in noticing students' mathematical thinking. In the report, they answered three prompt questions. First questions asked them to describe what they are thinking about students' answer on the given mathematical problem. Second question asked them to explain what they learn from the students' answer. Third question asked them to response students' thinking if they were teacher in the scenario. The written report should be done individually in after the class period.

Before the class period, the preservice mathematics teachers were asked to solve two problems, birthday problem [8, 9] and random variable problem [10]. The birthday problem is described as follow,

> *A football team consists of 23 players. What is the probability for two or more players to share the same birthday?*

- *Highly unlikely*
- *Unlikely*
- *Perhaps*
- *Very Likely*
- *Sure*

while the random variable problem is described as follow,

> *Ana will toss a die multiple times until she got 5. Predict how many times she needs to toss the die until she gets 5 and explain why.*

From this point onward, the birthday problem will be referred to as Problem 1 and the random variable problem will be referred to as Problem 2.

The student's mathematical thinking in solving the two problems will be the object of analysis of the preservice mathematics teachers. In the class, they were asked to watch two classroom video clips. The first video clip showed class discussion about birthday problem and the second showed the discussion about random variable problem. After that, they were asked to work in group to analyze the mathematical thinking of the students shown in those two video clips.

The data were analyzed by using Creswell's data analysis spiral [11], which states that the process of qualitative data analysis follows a spiral path instead of a fixed linear approach. Steps of data analysis are often interrelated and can happen simultaneously, and when a researcher has arrived at the end of the data analysis, they often has to step back to make sense of their interpretation in a larger context [11]. The data analysis spiral consists of four procedures, i.e. data managing; reading and memoing; describing, classifying, and interpreting; representing and visualizing.

The coding process was adapted from the study by Teuscher, Leatham, and Peterson [12], which is based on the construct of professional mathematics teacher noticing formulated by Jacob et al. [7] described earlier in this paper. The codes and the descriptions are described in Table 1.

Table 1. The codes on teachers' noticing

Code	Description
Attending	
General observations	No descriptions on the students' mathematical thinking.
The students' mathematical thinking	There are some levels of descriptions that makes it possible to infer on the students' mathematical thinking.
Interpreting	
General interpretation	No interpretation on the students' mathematical thinking.
Root interpretation	Sufficient evidence on inferring the students' understanding on the problem and identifying the possible mathematical reason behind the students' response.
Responding	
No clear connection	No connection on the interpretation of the students' thinking.
Elaborated	Descriptions on how the teachers would address the students' understanding (or misunderstanding) by discussing the mathematics concepts.
Facilitated	Descriptions that is not only focused on how the teachers would explain the mathematics related to the students' answers, but also on how they plan to scaffold or build on the students' understanding.

3 Result and Discussion

Table 2 shows the percentages of the preservice mathematics teachers' written reports coded in each category. Based on the table, even though the participants can describe students' mathematical thinking, they still have difficulties in understanding it and giving proper response. This section will further describe how the preservice mathematics teachers attend, interpret, and respond to the students' thinking in solving the given problems.

How the preservice mathematics teachers *attend* the students' mathematical thinking can be categorized into two types, namely *general observations* and *student' mathematical thinking*. For the first one, the participants simply make general observation about the students' performance in solving the problem. The first type, as exemplified by Fig. 1, does not give enough information, if any, to infer about the students' thinking in solving a particular mathematics problem. Instead, the participants just provide general observation which are mostly based on whether the answers is correct, in which the explanation does not reflect the mathematical thinking needed to solve a specific topic and can easily be applied to other problems.

In the second subcategory, as exemplified in Fig. 2, the participants provide more detailed explanation on the student' answer in solving the problem. Such explanation provides more sufficient evidence of what the students are likely thinking in answering the problem. The preservice mathematics teachers give enough detail so that the student's thinking is possible to infer. For example, the student maybe said that, "I think the probability is so small because the proportion of 22 and 365 is very small."

In *interpreting* student's mathematical thinking, there are also two subcategories, namely *general interpretation* and *root interpretation*. The response of preservice mathematics teachers that fall in the first subcategory, as exemplified in Fig. 3, give broad generalizations about the student's thinking. The preservice mathematics teachers do

Table 2. The percentage of the participants' written report corresponding to each of the code

Code	**Percentage (%)**
Attending	
General observations	41
The student' mathematical thinking	59
Interpreting	
General interpretation	75
Root interpretation	25
Responding	
No clear connection	41
Elaborated	36
Facilitated	23

> Peserta didik masing-masing memiliki jawaban yang berbeda dan pemahaman yang berbeda dalam mengerjakan persoalan. Secara umum, jawaban peserta didik masih kurang tepat dan juga dalam penyelesaian permasalahan ini belum sesuai dengan konsep peluang. Dalam artian, proses perhitungan masih belum sesuai.

[each of the student has different answer and understanding in solving the problem. Generally, the students' answers are still incorrect and the solution of the problem is still not in accordance to the concept of probability. Meaning, the calculation is still not suitable]

Fig. 1. Example of general observations of student's thinking in general

> Sebagian besar siswa menjawab sangat kecil. Hanya beberapa siswa yang menjawab kecil. Alasan beberapa siswa yang menjawab sangat kecil adalah peserta didik 1 menjawab peluangnya sangat kecil karena mendekati nol atau hamper tidak mungkin ulang tahunnya ada yang sama. Peserta didik 2 menjawab peluangnya sangat kecil jika peluangnya antara 0 sampai 0,2. Peserta didik 3 menjawab peluangnya sangat kecil namun tidak mengetahui alasan menjawabnya. Peserta didik 4 menjawab peluangnya sangat kecil karena missal ada ulang tahun yang sama yaitu pemain 1 dan pemain 2, maka hanya ada 22 tanggal berbeda yang menjadi ulang tahun ke-23 pemain tersebut. 22 ini sangat kecil jika dibandingan dengan 365 hari dalam 1 tahun. 22 : 365 hasilnya mendekati 0,06.

[the majority of the students answered 'really small'. Only some students that answer 'small'. The reasoning of some students who answer 'really small' is Student 1 answer the probability is really small because its close to zero or almost impossible for the birthdays to happen on the same date. Student 2 answer the probability is really small if the value is between 0 to 2. Student 3 answer the probability is really small but cannot explain the reasoning behind the answer. Student 4 answer the probability is really small because for example there are coincidental birthdays of player 1 and player 2, there will only be 22 different dates who become the birthdays of the 23 players. These 22 dates are really small if compared to 365 days in a year. The result of 22 : 36 is close to 0.06.]

Fig. 2. Example of sufficient evidence about student's mathematical thinking

not focus interpreting the student's thinking, but on the difference between one student' thinking to the others.

The second category is exemplified by Fig. 4, in which the preservice mathematics teachers interpret student's mathematical thinking by giving possible reasons behind the students' answer. The participants interpret that the error of the student lie on their assumption that the probability can be determined by dividing the number of persons in football group by the number of days in a year.

Setiap siswa mengutarakan pendapatnya dengan jawaban yang berbeda. Tidak menutup kemungkinan, peserta lainnya juga memiliki ide dan jawaban yang berbeda. Setiap siswa memiliki tingkat pemahaman yang berbeda serta ide yang berbeda juga. Sehingga, variasi jawaban yang muncul juga hampir selalu berbeda tiap siswanya. Setiap perbedaan jawaban dari siswa harus diterima sehingga menjadi dasar untuk membangun pemahaman konsep terhadap suatu materi. Selain itu, guru perlu memberikan konteks masalah matematika yang lebih sulit sehingga siswa dapat membangun pemahaman mengenai konsep peluang.

[Each student conveys opinion with different answer. It is possible that other students also have different ideas and answer. Each student has different level of understanding, as well as different ideas. Therefore, the variation of the answers is also different for each student. Each different answer from the students must be accepted to be the foundation of their conceptual understanding of a topic. Aside from that, teachers also have to provide contextual mathematics problems that are more difficult so that the students can build their understanding on the concept of probability]

Fig. 3. General interpretations of student's mathematical thinking

Yang saya pelajari berdasarkan video 2 dari pemahaman peserta didik adalah bahwa kebanyakan peserta didik berfikir bahwa peluang kejadian tersebut "sangat kecil" atau "kecil". Tentunya mereka menemukan jawaban tersebut karena didasari oleh beberapa hal, salah satunya didasari nilai peluang yang mereka temukan. Berdasarkan video, kebanyakan peserta didik menemukan nilai peluangnya 23/365 atau 0,0630136986, dimana 23 merupakan jumlah orang dalam tim. Saya melihat bahwa dalam video peserta didik kebanyakan memisalkan jumlah hari yang memungkinkan ulang tahun anggota tim sebagai 23 hari. Tentunya 23 itu didapatkan melalui jumlah anggota tim sebanyak 23. Berdasarkan hal ini saya dapat memahami bahwa dalam menyelesaikan permasalahan terdapat

[what I learn based on the two videos of the students' understanding is that the majority of the students think that the probability is "small" or "really small". Obviously, there are the reasons behind those answers, such as the value of the probability they calculated. Based on the video, the majority of the students found the value to be 23/365 or 0.0630136986, where 23 is the number of the people in the team. I saw that in the video the majority of the students assume that there would be 23 possible dates for the team members' birthday. Obviously, they got the number 23 from the number of people in the team. Based on this I can understand that in solving the pro lem there is …]

Fig. 4. Example of root interpretations

Memberikan penjelasan kepada peserta didik mengenai materi.
Memberikan penjelasan mengenai jawaban yang sesuai.

[Give explanation to the students about the topic. Give explanations on the correct answers]

Fig. 5. Example of the participants' response categorized as no clear connection.

There are three subcategories of preservice mathematics teachers' written report in *responding* student's mathematical thinking, namely *no clear connection*, *elaborated*, and *facilitated*. The first subcategory is illustrated by preservice mathematics teachers' report in Fig. 5. In the report, the preservice mathematics teachers do not provide connection between student's mathematical thinking and their planned response to the thinking.

The second subcategory is exemplified by a written report in Fig. 6. In the report, the preservice mathematics teachers respond the student's mathematical thinking by focusing on mathematics aspect of student's misconception.

The last subcategory, facilitated response, is exemplified by Fig. 7. The report in the figure does not only focus on mathematics, but also focus on how to facilitate further discussion so that students are aware about their misconception.

Memberikan konfirmasi mengenai materi peluang yaitu dengan memberikan pemahaman kepada peserta didik mengenai peluang munculnya suatu mata dadu. Ketika dadu dilempar satu kali maka peluang muncul mata dadu ke… adalah 1/6. Ketika dadu dilempar 2 kali maka peluang muncul mata dadu ke … adalah komplemen peluang dari munculnya mata dadu ke … di percobaan pertama dikali dengan peluang munculnya mata dadu ke.. di percobaan kedua.
Pemahaman peserta didik di video 3 menurut saya masih kurang tepat. Memang peluang muncul mata dadu bisa di pelemparan berapapun. Peserta didik belum menggunakan konsep peluang pada saat mencari peluang muncul mata dadu 5 di lemparan ke… jadi menurutnya mau di pelemparan berapapun peluang muncul mata dadu 5 adalah sama yaitu 1/6 tanpa

[giving confirmation about the topic of probability, specifically the probability of the appearance of a side of a die. When a die is tossed once, the probability of the side … is 1/6. If a die is tossed twice, the probability of the side … is a complement of the probability of the appearance of side … in the first trial multiplied by the probability of the appearance of side .. in the second trial. The understanding of the students in Video 3, according to me, is still incorrect. Indeed, the appearance of a certain side of a die can be in any trial. The students have not used the concept of probability in looking for the probability of the appearance of 5 in trial number .., so, according to them, in any trial the probability of the appearance of 5 is the same which is 1/6 without …]

Fig. 6. Elaborated response

Yang akan saya lakukan setelah menanyakan pendapat siswa mengenai manakah yang benar dari 2 pendapat yang muncul ketika diskusi jika saya menjadi guru dalam video tersebut adalah dengan mencoba membimbing/ memberikan pertanyaan yang akan membantu siswa dalam menemukan jawaban yang benar. Hal ini dilakukan seperti dengan menanyakan kepada siswa jika dadu dilempar sekali maka peluang mata dadu 5 yang muncul berapa, kemudian siswa akan menjawab 1/6. Setelah itu saya akan kembali bertanya. Jika pada lemparan pertama mata dadu 5 belum muncul maka dilakukan lemparan kedua, berapa peluang munculnya mata dadu 5 pada lemparan kedua. Dan pertanyaan yang sama akan saya munculkan pada lemaparan berikutnya hingga nanti mulai terlihat bahwa ketika semakin banyak kita melakukan lemaparan maka semakin sedikit peluang munculnya mata dadu 5. Setelah itu saya kembali menanyakan pertanyaan awal yang ada pada permasalahan yaitu dari nilai peluang yang telah ditemukan, pada lemparan berapakah peluang terbesar munculnya mata dadu 5. Dari kegiatan ini siswa akan mulai sadar jika semakin banyaknya kemungkinan yang mungkin terjadi maka peluang terjadinya suatu kejadian dalam hal ini adalah munculnya mata dadu 5 akan semakin kecil.

[what I would do after asking for the students' opinion about which one is correct between the two opinions during the discussion, if I was the teacher in the video, I would try to guide them/giving prompts that would help the students in finding the correct answers. This can be done by, for example, asking the students if a die is thrown once what would be the probability of 5 comes up, then the students would answer 1/6. Then I would ask again. If in the first toss 5 does not appear then if we toss for the second time, what would be the probability of 5 coming up. I will ask same question for the subsequent trials until it is observable that the more tosses we do, the smaller the probability of 5 appears. Afterward, I will ask the initial question in the problem which is the value of the probability that has been found, during which toss is the largest probability of 5 appears. From this activity the students will realize that the more trials [sic] then the probability of something happens, in this case the appearance of 5, will also be smaller.]

Fig. 7. Facilitated response

The preservice mathematics teachers participating in the present study still lack the evidence in interpreting students' thinking and responding to facilitate further student understanding of mathematics and engagement in mathematics lesson, as shown in other studies [7, 13, 14]. More programs are needed to support the participants in interpreting and responding students' mathematical thinking.

The present study also has implications for teacher educators. Since noticing skills are important but the preservice mathematics teachers in the present study still lack of them, teacher educators should promote these skills. This can be acquired by various methods and tools. Using video, as in the present study, is one method for developing noticing skills [15]. Furthermore, an animation approach [16], interview process [17], and using students' written work [18] have the potential to help preservice mathematics teachers learn to notice.

4 Conclusion

To conclude, analysis and discussions of classroom videos enabled the preservice mathematics teachers to analysis of classroom videos and supported them to notice classroom instruction in more substantive ways. However, the preservice mathematics teachers should be supported in interpreting and responding student's mathematical thinking.

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