



# Research Progress on Multi-attribute Decision Making Methods

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**Abstract.** As an important branch of decision direction, multi-attribute decision making has a very wide application prospect in the fields of economics, management, engineering and so on. Based on the reviews of multi-attribute decision theory and methods, the development background and definition of multi-attribute decision making are firstly introduced. The research and development status of multi-attribute decision making is reviewed from three aspects: information acquisition, weighting method and information fusion. Secondly, many hot issues of multi-attribute decision making in complex environment are analyzed. Finally, the multi-attribute decision making method is reviewed and prospected.

**Keywords:** Multi-attribute decision making · uncertain information · group decision making · social network

## 1 Introduction

Decision analysis is a process that decision makers (DMs) select and determine decision plans based on decision issues. Generally speaking, a complete decision-making process includes the steps of clarifying research objectives, determining research objects and attributes, reasonably assigning weights, and using scientific techniques to prioritize the alternatives. However, due to the complexity of process and problem, and usually involves multiple different attributes, which calls multi-attribute decision making (MADM). MADM [1] mainly selects the optimal alternative or ordering by using certain technologies with the existing known alternatives in the case of multiple conflicting and non-multiplicative attributes.

In this paper, the theory and methods of MADM are systematically summarized, so that relevant scholars can have a clearer understanding and promote its application in practice. The main work of this paper is as follows: Firstly, the MADM problem is described. Secondly, the research and development status of MADM is reviewed from three aspects: information acquisition, weighting method and information fusion. Then, many hot issues of MADM in complex environment are analyzed. Finally, the MADM method is reviewed and prospected.

## 2 Research on MADM in Traditional Environment

### 2.1 Development of MADM Information Form

Early MADM problems mainly focused on deterministic environment research, and decision information was mainly presented in the form of real numbers. Yager [1] proposed the ordered weighted aggregation operator (OWA) in 1988, which solves the global decision function problem of multiple real number information integration. However, decision information gradually develops to the trend of imprecision, fuzzy, uncertain and even incomplete. Fuzzy processing has become a focus of decision-making theory research. Zadeh [2] proposed the concept of membership function and built the fuzzy set theory, which made up for the deficiency of using probabilistic statistical methods to establish stochastic models. Atanassov [3] introduced the non-membership degree into the traditional fuzzy set and proposed the concept of intuitive fuzzy set. Subsequently, triangular fuzzy numbers, trapezoidal fuzzy numbers, Pythagorean fuzzy numbers and other fuzzy information are gradually introduced into the MADM problem [4, 7, 20]. Since then, on the basis of language evaluation information, 2-Tuple Linguistic has gradually become the research object of scholars [5, 6].

### 2.2 Weighting Method for MADM

The weight allocation strategy is different or brings profound influence to the decision result. Weight methods mainly include: subjective weighting method, objective weighting method, combination weighting method and interactive weighting method.

Subjective weighting method: Weight often comes from the DMs' own experience or preferences to give evaluation. Common methods include [1, 7, 8]: linguistic quantification method, Delphi method, Best and Worst method, Analytic Hierarchy Process, etc. However, the traditional subjective weighting method is based on the DMs' practical experience that lacks rigor and objectivity. In order to dig into the relative importance of attributes, attribute values and other related information in the description of decision problems, information entropy, programming method, minimum variance method and its variants are also widely used in practical decision problems [9, 10].

Objective weighting method focus on the relationship between data information and has strong data theory basis. However, scholars found that the empowerment criteria ignored the participation degree and preference of DMs, so the combination empowerment method came into being. In simple terms, combinatorial weighting method organically combines the above two kinds of weights, taking into account DMs' subjective preferences for attribute values as well as the internal logic and inherent laws of decision data information itself.

The above weighting methods all have certain applicable conditions and are obtained by DMs at one time. However, the attribute weight information is accompanied by uncertainty, multiple cycles, adjustment and correction, in fact. In this context, the interactive weighting method is more consistent with the realistic decision-making process [10, 11].

### 2.3 Information Fusion Theory of MADM

(1) Information fusion theory based on weighted average operator

Yager [1] proposed the ordered weighted average (OWA) operator, which solved the global decision function problem of multiple real number information integration. Yager [12] introduced a data aggregation operator to continuous intervals to aggregate continuous data information. Considering that the weighted average may be similar, Torra [13] proposed a weighted OWA (WOWA) operator. Liu [14] improved the expression of continuous OWA operators and extended the properties of WOWA operators from the discrete case to the continuous case. Xu et al. [15] further extended OWA operator and proposed the ordered weighted geometric average operators. Subsequently, aggregation operators of ordered weighted average are widely used in decision-making problems [16, 17], such as generalized ordered weighted average operator and ordered weighted harmonic average operator.

(2) Choquet integral under association relation

The traditional weighted average method does not measure the priority relationship or correlation degree among data information or attribute objects. Choquet integral solves the problem among attributes with correlation relations well and provides the possibility for the construction of non-probabilistic models. As an important aggregation function, Choquet integral can model dependent events. Some scholars also extended Choquet integral to other decision information environments [18–20], such as intuitionistic fuzzy environment, linguistic information environment, interval value environment, etc.

(3) Bonferroni mean and Heronian operator

Bonferroni mean and Heronian operator are also effective means to reflect the correlation of fused decision information, and can capture the correlation between different attribute values or parameters. Bonferroni mean is a kind of integration operator between maximum and minimum, originally proposed by Bonferroni. However, compared with other types of parameters, the parameters of generalized Bonferroni mean aggregation apply to the real number form. This contradicts the fact that clear data cannot be obtained in the complex social environment in reality, so Xu et al. [18] proposed the intuitionistic fuzzy Bonferroni mean and discussed its various special cases.

The Heronian mean operator is an average type of aggregation operator used to aggregate values in a set. Unlike Choquet integrals, the Heronian mean operator focuses on the aggregation parameter, while the Choquet integral focuses on changing the weight vector of the aggregation operator. Yu [19] defined geometric Heronian mean operators, and then applied them to fuzzy decision environment and proposed intuitive fuzzy geometric Heronian operators and Heronian operators in hesitant environment.

### 3 Research on MADM in Complex Environment

(1) Research on MADM from group to large group decision making

With the need of practical decision-making problems, decision-making groups gradually become a kind of multi-attribute large group. The research focus turns to how to construct a group consensus decision model to further integrate the views of multiple DMs, so as to achieve internal consistency (or compromise) of DMs. The key lies in how to feedback adjustment, which can be roughly divided into

the following two aspects: (1) identify DMs, alternatives and evaluation/preference elements with poor consensus level, and generate the direction of opinion adjustment (increase, decrease or unchanged) [21]; (2) Considering that consensus resources (such as time, etc.) in group consensus decision-making problems are limited, then minimize the distance before and after opinion adjustment or the cost of opinion adjustment [22, 23]. Some scholars further improve the consensus level by using clustering method based on similarity.

(2) Theoretical research on MADM considering behavior

Considering the complexity and uncertainty of real decision-making problems, it is necessary for multiple members to participate in the decision-making process. Due to the possible conflicts of interest among different members or the limited rationality of DMs, it is inevitable to consider the impact of their psychological behavior on the evaluation results. Common behavioral theories mainly include regret theory, TODIM method, prospect theory and three-branch decision. Compared with the previous three behavioral decision-making theories, which take gain or loss as the main research fulcrum, scholars have also studied non-cooperative and manipulative behaviors among DMs [24, 25].

(3) Theoretical research on MADM under social networks

With the rapid development of information technology, the rise of social networks has changed the individual and promoted more frequent social activities between people. Considering that some decision-making problems often require the participation of experts with different knowledge backgrounds, which indirectly leads to the formation of a relationship network with certain social characteristics among decision-making participants, the research on MADM under the analysis of social network has become one of the hot spots of many scholars [26, 27]. For example, Du et al. [26] proposed a clustering method based on trust similarity analysis to manage clustering operations of large-scale group decision events in social network environment. Influenced by the concept of viewpoint dynamics, research on MADM considering opinion dynamics has gradually become an important research topic in the current complex environment [27, 28].

## 4 Conclusion

By collating and studying the literature in the field of MADM, it can be found that the theory and method of MADM has formed a relatively complete research system, and has been widely applied to supply chain management, energy system optimization and other fields. But at the same time, there are some weak points in the research that need further study. Future research may be carried out from the following aspects:

- (1) Decision methods for complex and heterogeneous information. DMs' subjective judgment information and real objective information constitute complex and heterogeneous information. How to represent such information and the decision-making methods of such information still need to be further discussed.
- (2) Group decision-making method with hierarchical structure. The decision-making group has become a complex social network, in which decision-making individuals often form a certain hierarchical structure. How to design a fair and reasonable

decision-making mechanism should not only reflect the information transmission between the upper and lower levels, but also keep decision-making individuals at different levels relatively independent, which still needs to be further studied.

- (3) Manipulation of large-scale group decision making. In large-scale group decision-making problems, opinions of opinion leaders tend to be polarized, and some opinion leaders may be able to manipulate the results of decision-making. How to design a reasonable consensus mechanism to overcome the manipulation of some opinion leaders may be a research hotspot in the future.

## References

1. Yager R.R. (1988) On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on Systems, Man and Cybernetics*, 18(1):183–190. <https://doi.org/10.1016/B978-1-4832--1450-4.50011-0>.
2. Zadeh, L.A. (1965) Fuzzy sets. *Information and Control*, 8(3):338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X).
3. Atanassov, K.T.(1986) Intuitionistic fuzzy sets. *Fuzzy Sets & Systems*, 20(1):87–96. [https://doi.org/10.1016/S0165-0114\(86\)80034-3](https://doi.org/10.1016/S0165-0114(86)80034-3).
4. Wang, F. (2021) Preference degree of triangular fuzzy numbers and its application to multi-attribute group decision making. *Expert Systems with Applications*, 178:114982. <https://doi.org/10.1016/J.ESWA.2021.114982>.
5. Xu, T, Zhang H, Li B. (2021) Pythagorean fuzzy TOPSIS method based on 2-tuple probability weight. *Journal of Intelligent and Fuzzy Systems*, 40(12):1–14. <https://doi.org/10.3233/JIFS-201533>.
6. Naghizadeh, V.A., Ansari, R., Khalilzadeh, M, et al. (2021) An Integrated Decision Support Model Based on BWM and Fuzzy-VIKOR Techniques for Contractor Selection in Construction Projects. *Sustainability*, 13(12):28. <https://doi.org/10.3390/SU13126933>.
7. Lee, S. W., Xue, K. (2021) An Integrated Importance-Performance Analysis and Modified Analytic Hierarchy Process Approach to Sustainable City Assessment. *Environmental science and pollution research international*, 28(44). <https://doi.org/10.1007/S11356-021-15235-0>.
8. Zhang, Y. M., Jia, X., Tang, Z. (2021) Information-theoretic measures of uncertainty for interval-set decision tables. *Information Sciences*, 577. <https://doi.org/10.1016/J.INS.2021.06.092>.
9. Caador, S., Dias, J. M., Godinho, P. (2020) Global minimum variance portfolios under uncertainty: a robust optimization approach. *Journal of Global Optimization*, 76(2): 267–293. <https://doi.org/10.1007/s10898-019-00859-x>.
10. Lee, D. H., Kim, K. J. (2012) Interactive weighting of bias and variance in dual response surface optimization. *Expert Systems with Applications*, 39(5):5900–5906. <https://doi.org/10.1016/j.eswa.2011.11.114>
11. Ma, X.J., Gong, Z.W., Wei, G., et al. (2021) A New Consensus Model Based on Trust Interactive Weights for Intuitionistic Group Decision Making in Social Networks. *IEEE transactions on cybernetics*, <https://doi.org/10.1109/TCYB.2021.3100849>.
12. Yager, R. R. (2004) Owa aggregation over a continuous interval argument with applications to decision making. *IEEE Transactions on Systems Man & Cybernetics Part B Cybernetics A Publication of the IEEE Systems Man & Cybernetics Society*, 34(5): 1952–1963. <https://doi.org/10.1109/tsmcb.2004.831154>.
13. Torra, V. (1997) The weighted OWA operator. *International Journal of Intelligent Systems*, 12(2). [https://doi.org/10.1002/\(SICI\)1098-111X\(199702\)12:2<153::AID-INT3>3.0.CO;2-P](https://doi.org/10.1002/(SICI)1098-111X(199702)12:2<153::AID-INT3>3.0.CO;2-P)

14. Liu, X. W. (2006) Some properties of the weighted owa operator. *IEEE TRANSACTIONS ON CYBERNETICS*, 36(1): 118–127. <https://doi.org/10.1109/tsmca.2005.854496>.
15. Xu, Z.S., Da, W. L. (2002) The ordered weighted geometric averaging operators. *International Journal of Intelligent Systems*, 17(7). <https://doi.org/10.1002/int.10045>.
16. Ahmad, S., Alnowibet, K., Alqasem, L., et al. (2021) Generalized OWA operators for uncertain queuing modeling with application in healthcare. *Soft Computing*, 25(6):4951–4962. <https://doi.org/10.1007/S00500-020-05507-1>.
17. Liu, P.D., Wang, X., Teng, F. (2021) Online teaching quality evaluation based on multi-granularity probabilistic linguistic term sets. *Journal of Intelligent and Fuzzy Systems*, 40(2):1–20. <https://doi.org/10.3233/JIFS-202543>.
18. Xu, Z.S., Yager, R.R. (2011) Intuitionistic fuzzy bonferroni means. *IEEE Transactions on Systems Man & Cybernetics Part B*, 41(2): 568–578. <https://doi.org/10.1109/tsmcb.2010.2072918>.
19. Yu, D.J. (2013) Intuitionistic fuzzy geometric heronian mean aggregation operators. *Applied Soft Computing Journal*, 13(2):1235–1246. <https://doi.org/10.1016/j.asoc.2012.09.021>.
20. Herrera-Viedma, E., Herrera, F., Chiclana, F. (2002) A consensus model for multiperson decision making with different preference structures. *IEEE Transactions on Systems Man & Cybernetics Part A Systems & Humans*, 32(3):394–402. <https://doi.org/10.1109/tsmca.2002.802821>.
21. Dong, Y.C., Xu, Y.F., Li, H.Y., et al. (2010) The owa-based consensus operator under linguistic representation models using position indexes. *European Journal of Operational Research*, 203(2): 455–463. <https://doi.org/10.1016/j.ejor.2009.08.013>.
22. Ben-Arieh, D., Easton, T. (2007) Multi-criteria group consensus under linear cost opinion elasticity. *Decision Support Systems*, 43(3):713–721. <https://doi.org/10.1016/j.dss.2006.11.009>.
23. Chao, X. R., Kou, G., Peng, Y., et al. (2021) An efficient consensus reaching framework for large-scale social network group decision making and its application in urban resettlement. *Information Sciences*, 575. <https://doi.org/10.1016/J.INS.2021.06.047>.
24. Cao, M., Wu, J., Chiclana, F, et al. (2021) A bidirectional feedback mechanism for balancing group consensus and individual harmony in group decision making. *Information Fusion*, 76(2): 133–144. <https://doi.org/10.1016/j.inffus.2021.05.012>.
25. Tian, Z.P., Nie, R.X., Wang, J.Q., et al. (2018) A two-fold feedback mechanism to support consensus-reaching in social network group decision-making. *Knowledge-Based Systems*, 162(Dec.15): 74–91. <https://doi.org/10.1016/j.knosys.2018.09.030>.
26. Du, Z.J., Luo, H.Y., Lin, X.D., et al. (2020) A trust-similarity analysis-based clustering method for large-scale group decision-making under a social network. *Information Fusion*, 63. <https://doi.org/10.1016/j.inffus.2020.05.004>.
27. Dong, Y.C., Ding, Z.G., Herrera, F., et al. (2017) Managing consensus based on leadership in opinion dynamics. *Information Sciences an International Journal*, 397:187–205. <https://doi.org/10.1016/j.ins.2017.02.052>.
28. Li, Y.P., Liu, M., Cao, J., et al. (2021) Multi-attribute group decision-making considering opinion dynamics. *Expert Systems with Applications*, (5):115479. <https://doi.org/10.1016/J.ESWA.2021.115479>.

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