Discussion and Thinking in Course Design and Teaching of Numerical Analysis

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Abstract. In teaching numerical analysis courses to science and engineering students, we adopt different course design ideas and use blended learning methods to enable students to have a comprehensive understanding and grasp of course knowledge. Through the use of the combination of theory and practice and active student participation, we aim to stimulate students' thirst for knowledge of the course and enhance their willingness to more readily accept the teaching content, thereby improving their ability to apply what they learn. Overall, our approach is designed to create an engaging and effective learning environment for students.

Keywords: blended learning methods · Curriculum design · Number form association

1 Introduction

In the teaching process of science and engineering courses, mathematics courses often contain a large number of mathematical definitions, theorems, and associated proofs and derivations. This can make the courses tedious for most students, resulting in most of the content being taught by teachers in the classroom, and students forgetting it in a very short time. This is a major problem that has puzzled both teachers and students: how to define and understand the nature of theorems in a way that is more engaging for students and enables them to apply what they have learned. Although many courses have begun to adopt some methods to improve this teaching method, further investigation is needed to improve the current method of teaching and learning.

With the development of computer science and technology, its related technologies have been applied to various fields, and related models often rely on mathematical theory for support. Numerical analysis is the science of studying how to solve various mathematical problems with computers. Its core is to propose and study efficient and accurate algorithms for solving various mathematical problems. The importance of efficient computers cannot be overstated. The content of the course 'Numerical Analysis' covers some of the most basic concepts, calculation methods, and theoretical analysis used to solve various scientific and engineering problems [1–4].

In the current 'Numerical Analysis' curriculum, teachers typically start with theoretical concepts and principles, such as theorem proving, and rarely demonstrate real-world

P. Qi and Z. Chen (Eds.): ICBDIE 2023, AISR 178, pp. 419–425, 2024.
https://doi.org/10.2991/978-94-6463-238-5_59

applications of the course content. This approach leads to a lack of awareness among students about the practical significance of the course, with many believing that the content is simply a set of knowledge points to memorize for exams. As a result, students may struggle to apply what they have learned. Additionally, there are concerns that some curriculum content is outdated or inadequate to meet current societal needs. There is a growing call for curriculum reform to address these issues.

In recent years, the reform of curriculum teaching has gained momentum, with more and more schools and teachers focusing on curriculum reform under the guidance of policy guidelines. Liu [5] analyzed the necessity of adopting the flipped classroom teaching method for the Numerical Analysis course. By combining the characteristics of the flipped classroom teaching mode with the particularity of graduate numerical analysis courses, the main process and details of implementing flipped classroom teaching were designed, using the knowledge points of the least square method as an example. Yuan et al. [6] summarized several suggestions for teaching reform in the course design of Numerical Analysis based on common problems such as student characteristics, course teaching content, and teaching methods. Hao [7] has explored video assignments in key chapters of the numerical analysis course for three consecutive years and explained the positive impact of video assignments in enhancing teaching effectiveness. Despite these studies conducting curriculum reforms or explorations from different aspects, the teaching reform for science and engineering courses is still not perfect.

This article proposes innovative changes to the teaching sequence of the course content in the syllabus of the course 'Numerical Analysis'. By re-arranging the order of the content and placing detailed topics in the front and back positions for teaching, the aim is to make it easier for students to accept the course knowledge. Additionally, a combination of online and offline teaching methods is utilized to enable absent students to learn through online platforms.

2 Differences and Similarities Between Online and Offline Teaching

Online teaching is a rapidly growing form of distance education that uses information technology as a carrier. Through online teaching, academic and teaching teachers can complete their teaching tasks without being in the same location as their students. This method provides more flexibility, as students can participate in classes at any time and from anywhere, without being restricted by time and region. Compared to traditional offline teaching, online teaching offers numerous advantages, including the potential to supplement and enhance the learning experience. During the COVID-19 epidemic, online teaching played a critical role in ensuring that various courses could continue to be taught and students could continue their education. Additionally, online teaching can increase learning flexibility, reduce congestion in teaching venues, improve enrollment rates, and reduce learning costs, which makes it an attractive alternative to traditional teaching methods. However, there are also several challenges associated with online teaching. Network connectivity limitations are a significant issue that can affect the quality of online teaching. Moreover, students in different regions may face unfair access to education due to hardware limitations, which is a problem that cannot be ignored. Additionally, online teaching requires teachers to have a certain level of technical expertise, which can be a barrier to entry for some teachers. Therefore, extensive training is needed to ensure that teachers are prepared to deliver quality online courses. Despite these challenges, online teaching has great potential to transform education at all levels, and it is expected to become increasingly popular in the future. At the same time, due to the inability of teachers to interact directly with students in online teaching, this has greatly affected the efficiency of students' acceptance of course content. Numerical Analysis is a course that requires calculation and theoretical derivation. If students are distracted while listening online, it is likely that they will not learn the content and will not be able to provide timely feedback to the instructor. This mismatch in information not only leads to insufficient mastery of knowledge by students but also significantly affects teachers' teaching progress. During offline teaching, teachers can grasp the teaching progress based on real-time feedback from students: conduct faster teaching at knowledge points where students' feedback is easy and conduct more detailed analysis at knowledge points where students' feedback is difficult, striving to enable students to comprehensively learn and master the course knowledge. Considering that some students may not be able to attend class on time due to some irresistible reasons (such as illness), this course is designed to be taught online and offline simultaneously to enable students to learn the course in any situation. Simultaneous online and offline teaching can solve the space constraints of offline teaching, as well as the problem of delayed feedback in online teaching. The two complement each other and should become a trend in future course teaching.

3 Adjustment of Teaching Content in Course Design

As higher education systems continue to evolve, it is important to ensure effective curriculum design to meet the demands of talent cultivation. Curriculum design is not just a teaching plan but also considers the links between various elements within the curriculum to meet the needs of student learning. In the course 'Numerical Analysis', innovative adjustments have been made to the teaching sequence to improve effectiveness and engagement. The course content has been restructured to make it more easily understandable for students. Chapters on relatively easy topics like direct methods, iterative methods, and numerical solutions are introduced after the initial introduction, enhancing students' acceptance of the course content. The curriculum design aims to dynamically adjust teaching based on real-time feedback to ensure the successful development of the course. In the teaching process, the Gaussian elimination method is used to solve linear equations. This method eliminates variables one by one until all variables are obtained. It is extended to solve equations with multiple variables, transforming the original set into a linear equation set in upper triangular form. By understanding and mastering these methods, students can solve problems encountered in compiling computer programs, and cultivate their ability to engage in scientific and engineering calculations using computers. This course provides a necessary foundation for future studies and work. In each iteration, unknown variables are solved one by one until all variables are obtained. To prevent rounding errors, the column pivot elimination method is introduced as an understanding of the full pivot elimination method. Additionally, the triangular decomposition method is taught to solve linear equations in upper or lower triangles. This step-by-step approach enables students to receive the course content well. For the iterative method, systems of linear equations with two variables are used as the starting point to help students understand the concept of iteration. Through this approach, students can learn and understand the basic concepts of numerical analysis, numerical calculation, and master commonly used numerical calculation methods. By mastering these methods, students can solve problems encountered when compiling computer programs and cultivate their ability to engage in scientific and engineering calculations using computers. This course lays the necessary foundation for future study and work.

For a system of binary linear equations $\begin{cases} a_1x + b_1y = c_1① \\ a_2x + b_2y = c_2② \end{cases}$, the general solution is to transform equation ① into $x = \frac{c_1 - b_1y}{a_1}, a_1 \neq 0$ ③ and then bring Eq. ③ into Eq. ② to obtain y and then according to equation ③, it can be calculated the value of x. According to this basic idea, for a complex system of linear equations $\begin{cases} n_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \\ n_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2 \\ \ldots \\ n_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n \end{cases}$ ⑤. , for each row, we can set the corresponding variable x_i use aernate variables $x_j, j = 1, 2, \ldots, i-1, i+1, \ldots, n$ to represent: $x_i = (b_i - n_{i1}x_1 - \ldots - a_{ii-1}x_{i-1} - -a_{ii+1}x_{i+1} - \ldots - a_{in}x_n)/n_{ii}$, which can be expressed as $\begin{cases} x_1 = (b_1 - a_{12}x_2 - \ldots - a_{1n}x_n)/n_{11} \\ x_2 = (b_2 - n_{21}x_1 - \ldots - a_{2n}x_n)/n_{22} \\ \ldots \\ x_n = (b_n - n_{n1}x_1 - \ldots - a_{nn-1}x_{n-1})/a_{nn} \end{cases}$ $n_{ii} \neq 0$ ⑥. The general iterative method for solving equation group ⑤ is the form of equation group ⑥.

To ensure the success of the iterative method, it is essential to focus on its convergence. Only when the iterative method is convergent, can it produce the expected solution results through iteration. The concept of convergence is emphasized repeatedly, especially in the part where nonlinear equations and equations are solved. Following the teaching of linear and nonlinear equations and equations, the course content can be further enriched by introducing interpolation methods, function approximation, numerical integration, and numerical differentiation.

4 Numerical Combination

During the teaching process, we always adhere to the principle of "student-centered" with students as the main focus, and teachers as guides to inspire and facilitate students' active participation in the course. Classroom participation is a crucial indicator of students' mastery of curriculum knowledge. High participation in specific course content indicates that students have a good understanding of that part of the material. Moreover, classroom participation also reflects students' willingness to learn the course. To encourage student participation, teachers should motivate and urge students to actively participate in classroom activities. The combination of numbers and shapes is a fundamental feature of mathematics courses. Some course content cannot be merely presented as knowledge; it must be combined with graphics and images to help students understand the true meaning of the content they have learned. By making students visually aware of the course material, they are more likely to develop an interest in the subject matter, and comprehend the knowledge points more thoroughly. For instance, in the section on the numerical solution of nonlinear equations, we present the following examples through a combination of numbers and shapes.

Example 1. At the xOy plane, find the intersection of two parabolas $y = x^2 + a$ as well as $x = y^2 + a$. Considering the opening directions of the two parabolas, the following illustration can be made through a combination of numbers and shapes:

From Fig. 1, it can be clearly seen whether there is a point of intersection between the two parabolas, i.e., a set of nonlinear equations $\begin{cases} y - x^2 - a = 0 \\ x - y^2 - a = 0 \end{cases}$ whether there is a solution depends on a the value size of when $a > 0.25$ when the two parabolas do not intersect, the nonlinear equation system has no solution; When $a = 0.25$ when two parabolas have an intersection point, the nonlinear equation system has a unique solution; When a as the value of decreases gradually, two parabolas will have two or four intersections, and a nonlinear equation system will have two or four solutions. Through this combination of numbers and shapes, students can better understand the concept of nonlinear equations and the existence of solutions to nonlinear equations. Try to have students draw their own graphics in class without having to a the value of is excessively accurate, only the parabola needs to be known $y = x^2 + a$ will follow a the decreasing curve of the value of will follow y axis translation downward; parabola $x = y^2 + a$ will follow a the decreasing curve of the value of will follow x axial left translation is sufficient.

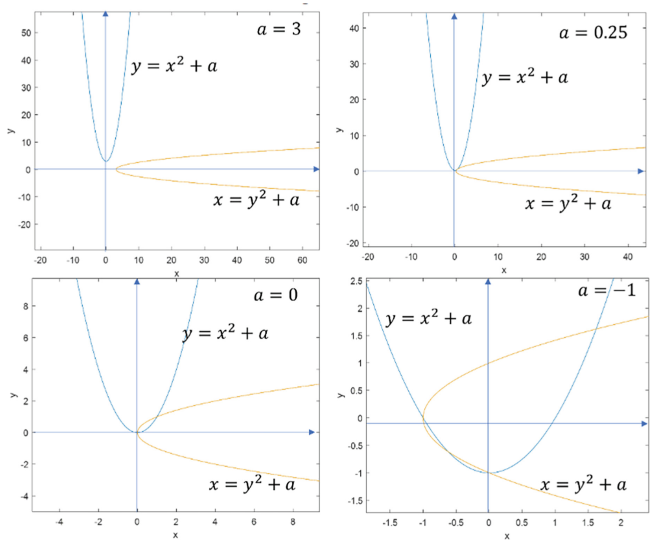

Fig. 1. Visual Diagram of Different Intersection Points of Parabola

5 Conclusion

The course design and teaching process of numerical analysis aim to help students correctly understand the basic concepts and theories related to numerical analysis, grasp the basic ideas of numerical calculation, and become proficient in classic and commonly used basic numerical calculation methods. By compiling computer programs and using these methods, students can solve various problems encountered in scientific and engineering calculations, which will help to cultivate their ability to engage in scientific and engineering calculations using computers. This will provide them with a necessary foundation for future study and work. Through this course, students will master the basic concepts of numerical calculation for scientific problems, the ideas and rules of calculation method design, and the requirements and approaches from mathematical models to practical calculations. They will also become proficient in programming methods for basic problems in scientific computing and be able to solve problems through practical programming and calculation.

Acknowledgement. This study was supported by the Fundamental Research Funds for the Central Universities (Grant No. XJS222503), and by the Foundation Project of Guangzhou Institute of Technology, Xidian University (Grant No. 01131002).

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