



Application of Game Theory in Distribution Networks

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Abstract . Game theory has become a powerful tool for analyzing and optimizing the complex interactions among multiple stakeholders in various fields, including distribution networks. This paper discusses the basic concepts of game theory and the specific application of game theory in distribution networks, proposes different game theory models and solutions. It provides some reference and guidance to help understand and apply game theory to solve problems related to power grid optimization, resource allocation and cooperation between different stakeholders in distribution networks.

Keywords: game theory models, distribution networks, power grid optimization, resource allocation

1 INTRODUCTION

The background and motivation for the use of game theory in distribution networks stems from the increasing complexity and interaction among multiple stakeholders in the evolving energy landscape. The emergence of distributed energy, the integration of renewable energy, the cooperation of demand response and the demand for efficient grid operation make it necessary to apply game theory to deal with various challenges.

Game theory generally includes non-cooperative games and cooperative games. In non-cooperative games, players act independently and compete with each other. In cooperative games, participants form alliances and work together to achieve certain goals.

By applying game theory to distribution networks, researchers and practitioners can gain insights into the strategic interactions among stakeholders, optimize decision-making processes, and address complex challenges in grid operations, resource allocation, and cooperation among diverse entities.

2 Non-cooperative game models in Distribution Networks

2.1 Resource allocation and bidding strategies in energy markets

Non-cooperative game models enable stakeholders to analyze and optimize their decision-making processes.

1) Generation Investment and Capacity Expansion: Non-cooperative game models can be used to analyze the strategic behavior of generation companies in making investment decisions and expanding their capacity. These models consider factors such as costs, market conditions, and regulatory constraints^[1]. By modeling the interactions among generation companies, game theory helps predict investment patterns, optimize capacity expansion strategies, and understand the competition dynamics in the energy market.

2) Generation Dispatch and Unit Commitment: Non-cooperative game models aid in analyzing the strategic behavior of generation companies in dispatching their generation units and committing them to the grid. These models consider factors such as fuel costs, generation capacities, and demand patterns^[2]. By modeling the interactions among generation companies, game theory helps predict generation dispatch patterns, optimize unit commitment decisions, and understand the strategic behavior of market participants.

3) Wholesale Electricity Market Bidding Strategies: Non-cooperative game models can be applied to analyze the strategic behavior of market participants in wholesale electricity markets. These models consider factors such as production costs, demand forecasts, and market rules. By modeling the interactions among generation companies, retailers, and market operators, game theory helps optimize bidding strategies, predict market outcomes, and understand the dynamics of price formation in the energy market^[3].

4) Renewable Energy Integration and Bidding: Non-cooperative game models are useful in analyzing the strategic behavior of renewable energy developers and their bidding strategies in energy markets. These models consider factors such as renewable resource availability, generation costs, and policy incentives. By modeling the interactions among renewable energy developers, grid operators, and market participants, game theory helps optimize bidding strategies, predict renewable energy integration patterns, and understand the competitive dynamics of renewable energy markets^[4].

2.2 Demand response and load management strategies

Non-cooperative game models capture the strategic behavior of consumers, utilities, and grid operators, allowing stakeholders to understand the dynamics of demand response programs and design effective strategies.

1) Demand Response Participation: Non-cooperative game models can be used to analyze the strategic behavior of consumers in deciding whether to participate in demand response programs. These models consider factors such as price elasticity, consumer preferences, and incentives^[5]. By modeling the interactions among consumers,

utilities, and grid operators, game theory helps predict participation rates, understand consumer decision-making processes, and design effective incentive mechanisms to encourage participation.

2) Load Curtailment Decision-Making: Non-cooperative game models aid in analyzing the strategic behavior of consumers in making load curtailment decisions during peak demand periods. These models consider factors such as electricity prices, comfort levels, and appliance scheduling. By modeling the interactions among consumers, game theory helps optimize load curtailment strategies, predict consumer response patterns, and design efficient mechanisms for load shedding or shifting^[6].

3) Pricing Mechanism Design: Non-cooperative game models can be applied to analyze the strategic behavior of utilities and grid operators in designing pricing mechanisms for demand response programs. These models consider factors such as demand elasticity, cost structures, and market conditions. By modeling the interactions among utilities, consumers, and regulators, game theory helps optimize pricing strategies, predict market outcomes, and design incentive mechanisms that encourage load management and maximize social welfare^[7].

4) Incentive Design and Behavioral Response: Non-cooperative game models can be used to analyze the strategic behavior of consumers in responding to different incentive structures and behavioral interventions. These models consider factors such as social norms, peer effects, and psychological biases. By modeling the interactions among consumers, game theory helps optimize incentive design, predict consumer responses, and understand the behavioral aspects of demand response and load management strategies^[8].

2.3 Grid optimization and congestion management

Non-cooperative game models capture the strategic behavior of market participants, grid operators, and regulators, allowing for the design of effective strategies to enhance grid performance and mitigate congestion.

1) Grid Congestion Management: Non-cooperative game models aid in analyzing the strategic behavior of market participants in managing transmission congestion. These models consider factors such as transmission capacity limits, congestion costs, and transmission rights. By modeling the interactions among market participants, grid operators, and regulators, game theory helps optimize congestion management strategies, predict congestion pricing patterns, and design effective mechanisms for congestion relief, such as redispatching generation or utilizing flexible demand^[9].

2) Grid Expansion Planning: Non-cooperative game models can be applied to analyze the strategic behavior of power system operators and investors in planning grid expansions. These models consider factors such as investment costs, revenue allocation, and regulatory frameworks. By modeling the interactions among system operators, investors, and regulators, game theory helps optimize grid expansion strategies, predict investment patterns, and design mechanisms for efficient and cost-effective grid expansion^[10].

3) Ancillary Service Provision: Non-cooperative game models aid in analyzing the strategic behavior of market participants in providing ancillary services to ensure grid

stability and reliability. These models consider factors such as service costs, capacity constraints, and payment mechanisms. By modeling the interactions among service providers, grid operators, and market participants, game theory helps optimize the provision of ancillary services, predict market outcomes, and design effective mechanisms for incentivizing participation and ensuring system reliability^[11].

3 Cooperative game models in Distribution Networks

3.1 Coalition formation and resource sharing among stakeholders

Cooperative game models allow stakeholders to analyze and optimize their cooperative behaviors, negotiate agreements, and distribute resources in a fair and efficient manner.

1) **Microgrid Formation:** Microgrids are small-scale energy systems that integrate distributed energy resources and can operate autonomously. Cooperative game models consider factors such as the generation capacities, load profiles, and network constraints of different stakeholders. By modeling the interactions among stakeholders, game theory helps optimize microgrid formation, predict coalition structures, and design mechanisms for resource sharing and coordination^[12].

2) **Renewable Energy Sharing:** Cooperative game models consider factors such as the generation capacities, intermittency patterns, and demand profiles of different participants. By modeling the interactions among renewable energy producers, consumers, and grid operators, game theory helps optimize resource sharing strategies, predict energy sharing patterns, and design mechanisms for efficient utilization of renewable energy^[13].

3) **Demand Response Cooperation:** Cooperative game models consider factors such as the flexibility of consumer loads, the costs of load curtailment, and the benefits of demand response. By modeling the interactions among consumers, utilities, and grid operators, game theory helps optimize demand response cooperation, predict participation levels, and design mechanisms for load shedding or shifting^[14].

4) **Cost Allocation and Revenue Sharing:** Cooperative game models consider factors such as investment costs, operational costs, and revenue generation from energy sales. By modeling the interactions among stakeholders, game theory helps optimize cost allocation mechanisms, predict revenue sharing patterns, and design mechanisms that incentivize efficient investment and operation of the distribution network^[15].

5) **Risk Management and Insurance:** Cooperative game models consider factors such as grid reliability, asset failure probabilities, and insurance premiums. By modeling the interactions among stakeholders, game theory helps optimize risk management strategies, predict risk-sharing patterns, and design mechanisms for efficient risk mitigation and insurance coverage^[16].

3.2 Cooperative strategies for grid stability and reliability

Cooperative game models enable stakeholders to understand the interactions and dependencies among different entities in the network and develop cooperative strategies to enhance the stability and reliability of the distribution system.

1) Voltage and Frequency Control: Cooperative game models can be used to analyze the collaboration among generators, grid operators, and consumers in voltage and frequency control. These models consider factors such as generation capacities, load demands, and network constraints. By modeling the interactions among stakeholders, game theory helps optimize the coordination of control actions, predict stability outcomes, and design mechanisms for efficient voltage and frequency regulation^[17].

2) Fault Detection and Diagnosis: Cooperative game models aid in analyzing the collaboration among sensors, control systems, and grid operators in fault detection and diagnosis. These models consider factors such as sensor placements, fault detection algorithms, and communication constraints. By modeling the interactions among stakeholders, game theory helps optimize the deployment of sensors, predict fault detection performance, and design mechanisms for reliable fault identification and localization^[18].

3) Demand-Side Management: Cooperative game models can be applied to analyze the collaboration among consumers, utilities, and grid operators in demand-side management programs. These models consider factors such as load profiles, demand response capabilities, and grid constraints. By modeling the interactions among stakeholders, game theory helps optimize load curtailment strategies, predict demand response participation, and design mechanisms for efficient demand-side management and load balancing^[19].

4) Distributed Energy Resource Integration: Cooperative game models aid in analyzing the collaboration among distributed energy resource (DER) owners, aggregators, and grid operators in integrating DERs into the distribution network. These models consider factors such as generation capacities, grid integration constraints, and economic incentives. By modeling the interactions among stakeholders, game theory helps optimize DER dispatch strategies, predict DER integration patterns, and design mechanisms for coordinated DER operation and grid support^[20].

5) Resilience and Restoration: Cooperative game models can be used to analyze the collaboration among utilities, grid operators, and other stakeholders in resilience and restoration planning. These models consider factors such as outage scenarios, restoration priorities, and resource availability. By modeling the interactions among stakeholders, game theory helps optimize restoration strategies, predict restoration times, and design mechanisms for efficient resource allocation and system recovery^[21].

4 Solution Method and Optimization of game theory Model

4.1 Nash equilibrium

The solution algorithm for finding Nash equilibrium depends on the type of game being analyzed, as there are different methods for different game structures. See Figure 1 for details.

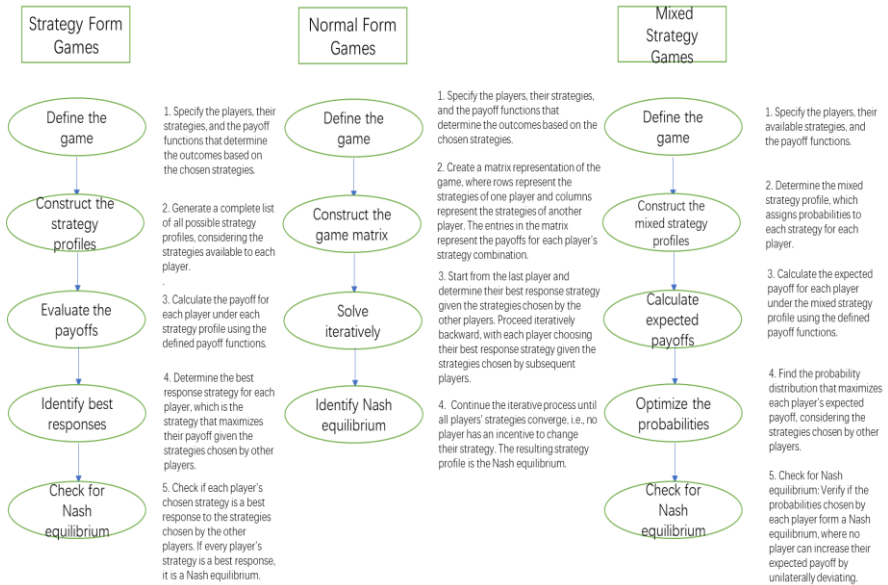


Fig. 1. Solution algorithms for Nash Equilibrium with Different Game Structures.

Common algorithms used to find Nash equilibrium is as follows:

1. **Best Response Dynamics:** Players iteratively update their strategies to choose the best response to the strategies chosen by others until a Nash equilibrium is reached.

2. **Lemke-Howson Algorithm:** A method for finding Nash equilibria in mixed strategy games by iteratively traversing the vertices of the game's polytope.

3. **Linear Programming:** In some cases, Nash equilibrium can be found by formulating the game as a linear programming problem and solving the linear program to obtain the optimal strategy profile.

4. **Fictitious Play:** Players estimate the strategy probabilities of their opponents based on observed past play and adjust their own strategies accordingly, converging to a Nash equilibrium over time.

5. **Evolutionary Dynamics:** Simulating the evolution of strategies through repeated interactions, where successful strategies are more likely to be adopted by other players, leading to the emergence of a Nash equilibrium.

Overall, the calculation and solution algorithm for Nash equilibrium depend on the specific game structure and the available information about the players' strategies and payoffs.

4.2 Stackelberg game solution methods

The Stackelberg game is a sequential game where one player, known as the leader, makes decisions first, and the other players, known as followers, make their decisions afterward. The leader's decision affects the followers' payoffs, and the objective is for the leader to choose their strategy in a way that maximizes their own payoff while taking into account the followers' reactions. As shown in Figure 2.

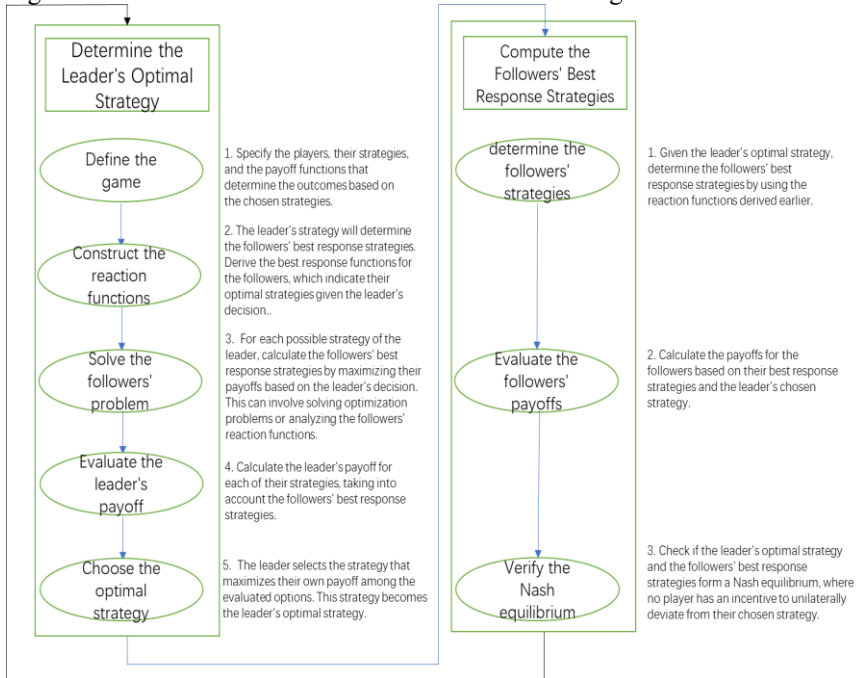


Fig. 2. Solution algorithms for Stackelberg game solution methods.

4.3 Evolutionary game dynamics

Evolutionary game dynamics is a solution method that models the evolution of strategies in a population of players over time. It simulates how strategies spread or decline based on their performance and interaction with other strategies. As shown in Figure 3.

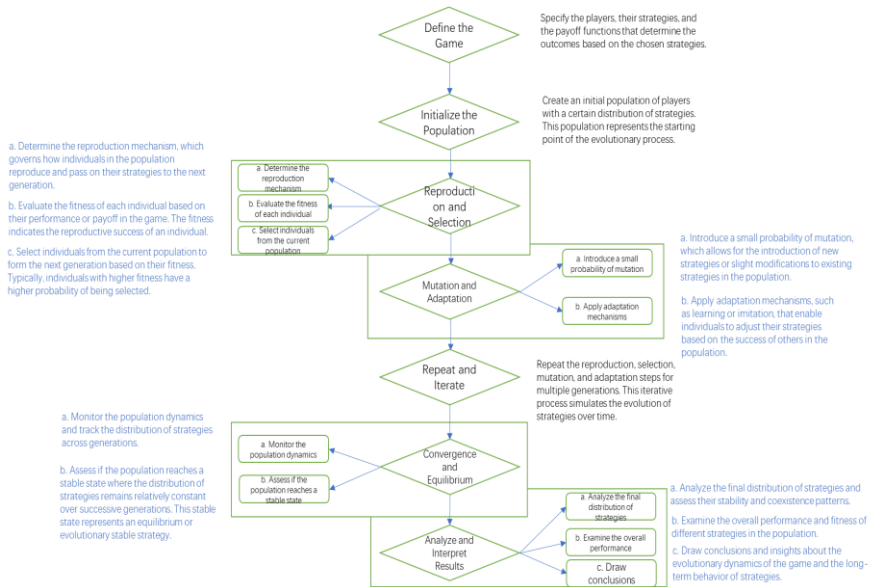


Fig. 3. Solution algorithms for Evolutionary game dynamics.

4.4 Cooperative game solution concepts

The solution concepts aim to allocate the total value or payoff of a cooperative game among the players. These concepts provide guidelines for distributing the benefits or costs of cooperation in a fair and efficient manner.

1. **Shapley Value:** The Shapley value assigns a unique allocation to each player based on their marginal contribution to every possible coalition. It considers all possible permutations of players and calculates the average contribution of each player to each coalition. The Shapley value satisfies desirable properties such as efficiency, symmetry, and additivity.

2. **Core:** The core is a solution concept that ensures stability by prohibiting any subgroup of players from improving their payoff through cooperation without compensating others. It represents the set of allocations that cannot be blocked by any coalition of players. An allocation is in the core if it is individually rational and there is no coalition that can obtain a more beneficial outcome for themselves by excluding or compensating other players.

3. **Nash Bargaining Solution:** The Nash bargaining solution seeks to find a distribution that maximizes the product of players' utilities, subject to the constraints imposed by their disagreement points (i.e., the minimum utility they are willing to accept in the absence of agreement). It provides a compromise solution that reflects the relative bargaining power of the players. The Nash bargaining solution is based on the idea of maximizing the joint gains from cooperation.

4. **Cooperative Game with Transferable Utility (TU Games):** In TU games, players' utilities are transferable, meaning players can negotiate and transfer their shares of the

total payoff among themselves. Solution concepts for TU games include the Shapley value, core, nucleolus, and various other value functions that allocate the total value to players based on different criteria and fairness principles.

5. Nucleolus: The nucleolus is a solution concept that identifies an allocation that minimizes the envy players may have towards each other. It represents the most stable and efficient allocation among all possible allocations. The nucleolus is characterized by considering the excess or shortfall of each player relative to their worth in all possible coalitions.

6. Bargaining Sets: Bargaining sets are solution concepts that capture the possible outcomes of a bargaining process. They consist of allocations that satisfy certain axioms of fairness, such as individual rationality and no-envy. Different bargaining sets can yield different allocations based on the specific fairness principles employed.

4.5 Optimization techniques for large-scale game models

Optimization techniques aim to find optimal solutions that satisfy certain objectives or constraints. The choice of optimization technique depends on the specific characteristics of the game model, such as linearity, convexity, or complexity.

1. Linear Programming (LP): LP is a widely used optimization technique for solving large-scale game models. In LP, the objective function and constraints are linear. LP formulations can be used to model and solve various types of game problems, such as resource allocation and market equilibrium. LP solvers utilize efficient algorithms, such as the simplex method or interior-point methods, to find optimal solutions.

2. Mixed-Integer Linear Programming (MILP): MILP extends linear programming by allowing variables to take integer values. It is suitable for game models that involve discrete decisions or combinatorial optimization problems. MILP can handle complex game scenarios, including resource allocation with capacity constraints, facility location, and scheduling problems. Advanced algorithms like branch-and-bound or branch-and-cut are commonly employed to solve MILP problems.

3. Nonlinear Programming (NLP): NLP deals with optimization problems where the objective function or constraints are nonlinear. Game models with non-convex objectives or constraints often require NLP techniques for optimization. NLP algorithms, such as gradient-based methods, interior-point methods, or genetic algorithms, are used to find local or global optima in the game models.

4. Heuristic and Metaheuristic Algorithms: Heuristic and metaheuristic algorithms are employed when exact optimization techniques are computationally expensive or infeasible for large-scale game models. These algorithms provide approximate solutions by iteratively exploring the search space. Examples include genetic algorithms, simulated annealing, particle swarm optimization, and ant colony optimization. Heuristic algorithms are often used to solve complex game problems, such as portfolio optimization.

5. Decomposition Methods: Decomposition methods split a large-scale game model into smaller sub-problems that can be solved independently and then combined to obtain a solution for the overall problem. Examples include the Benders decomposi-

tion, Lagrange relaxation, or alternating direction method of multipliers (ADMM). Decomposition methods help reduce the computational complexity of solving large-scale game models by exploiting problem structure and parallel processing.

6. Approximation Algorithms: Approximation algorithms provide near-optimal solutions with a provable guarantee on the quality of the solution. Approximation algorithms are designed to deliver efficient and scalable solutions for large-scale game models, often sacrificing optimality for computational efficiency.

5 Conclusions

Game theory can help optimize the allocation of resources in distribution networks. It also can facilitate the coordination of demand response and load management strategies in distribution networks. Game theory can contribute to the design of market mechanisms and coordination strategies in distribution networks. And can assist in the integration of renewable energy sources into distribution networks. Game theory can help identify mechanisms and incentive structures that encourage collaboration, resource sharing, and joint investment. Meanwhile it can help identify strategies for mitigating risks, enhancing grid resilience, and adapting to changing market and environmental conditions.

Overall, this paper contributes to the understanding of game theory's applications and solutions in distribution networks, fostering the development of efficient, sustainable, and collaborative grid operations.

References

1. REN Zhiming, ZENG Ming, LI Chunxue, QAIN Qiqi., "Research on investment decisions for renewable energy generation under the condition of increasing carbon prices, " *hydro-power energy science* 33(11), 211-214+87(2015).
2. CHEN Jiejie., "Research on the Investment Model and Benefit Allocation of Distributed Photovoltaic Power Generation in Zhongshan City, " *South China University of Technology*, 67(2018).
3. LU Chenyu, JIANG Ting, DENG Hui, FANG Le, WANG Xu, JIANG Chuanwen., "Participation of clean energy power generation companies in spot market bidding strategies and profit distribution based on cooperative games, " *power construction* 41(12), 150-158(2020).
4. YIN Shuxin., "Research on the Strategy of Virtual Power Plants Participating in the Spot Electricity Market, " *Shandong University of Technology*, 56(2022).
5. CHENG Hongbo, LUO Tong, TANG Chen, TIAN Xu, GUO Yuanxi, WANG Xun., "Electric vehicle agents participate in the multi-level game bidding model of demand response, " *new electric energy technologies* 39(02), 46-56(2020).
6. LENG Zhaoying, CHEN Zhong, XING Qiang, CHEN Xuan, ZHANG Tian., "Non-cooperative game model for load aggregators bidding in advance based on load type segmentation, " *power grid and clean energy* 36(05), 17-28(2020).
7. ZHUANG Xiaoye, LI Ke., "Non-cooperative game pricing mechanism for optimizing cloud resource allocation, " *Journal of Xinxiang University* 38(03), 48-52(2021).

8. ZHANG Weiwei., "Research and Design of P2P Incentive Model Based on game theory, " Northwestern University, 61(2009).
9. LANG Gaiping, XU Yubin, MA Lin., "Heterogeneous network selection algorithm based on non-cooperative game theory, " Journal of South China University of Technology (Natural Science Edition) 42(05), 29-35(2014).
10. LIANG Longji, ZHANG Jing, HE Yu, ZENG Xixi, CHEN Chaokuan., "Game theory based multi-agent collaborative optimal configuration of interconnected microgrid, " electronic technology 34(07), 43-49+78(2021).
11. LI Bosong., "Research on Optimization and Bidding Strategies for High Permeability Distributed Energy Aggregation Operation, " Shanghai Jiao Tong University, 146(2020).
12. ZHAO Pengjie, WU Junyong, ZHANG Wenhao, HE Shan, ZHANG Hesheng., "A peer-to-peer cooperative scheduling strategy for multiple micro power grid points considering distribution network flow and source load uncertainty, " power grid technology 46(12), 4885-4896(2022).
13. WANG Qiaoqiao., "Research on Distributed Operation Optimization Algorithm of Microgrid Based on Potential Game Theory, " South China University of Technology, 67(2019).
14. ZHENG Shunlin., "Research on Optimization Model and Strategy of Integrated demand response for Multi energy Cooperation, " North China Electric Power University (Beijing), 112(2022).
15. CUI Hanqi., "Research on the Game Purchase and Sales Strategy of Electricity Selling Companies Based on the Negotiation and Bargaining Model, " Northeast Electric Power University, 62(2021).
16. ZHAO Min, SHEN Chen, LI Shunxin, Xiao Dong, YUE Yunli, Bo LI., "Research on the Cooperation Conditions of Multiple Microgrids Considering Power Outage Risk through Alliance Game Theory, " Control Theory and Application 35(05), 688-698(2018).
17. CHEN Jingfeng., "Research on Control and Optimal Dispatching Strategies for Independent Microgrid Systems with Hybrid Energy Storage, " South China University of Technology, 150(2020).
18. KE Ting, TIAN Songlin, ZHOU Hai., "Research on Fault Diagnosis Model for Small Substation Based on Cooperative Game and Cloud Model," Electrical Transmission (Recruitment Finalization), (27 Mar 2023).
<https://kns.cnki.net/kcms2/article/abstract?v=3uoqIhG8C45S0n9fL2suRadTyEVI2pW9UrhTDCdPD64YF-MTIOh1L1-ekuWwXasXQmIIV34ojp3tW5emHh6mqTdTniibsWou&uniplatform=NZKPT>
19. GUO Wenbo., "Research on Demand Side Management Pricing Strategy and Energy Scheduling for Energy Systems," Yanshan University, 69(2021).
20. LIU Dongran., "Research on Regional Distributed Energy System Participation in Power Trading Assistant Decision Model," North China University of Electric Power (Beijing), 166(2022).
21. YAO Weiqiang, ZHOU Jian, SHI Shanshan, GAO Shilin, CHEN Ying., "A Decision Model for Disaster Recovery of Multi microgrid Systems Based on Cooperative Games," New Technologies in Electrical Energy Engineering 40(03), 32-38(2021).

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