



The Sustainable Supply Chain Game Research of Joint Inventory Decisions Considering Fairness Concern

Xuan Chen*

School of Economics and Management, Beijing University of Technology, Beijing 100124, China

{1967492306@qq.com*}

Abstract. The deterioration of the natural environment has attracted the attention of a wide range of scholars, and there has been a gradual increase in research on sustainable supply chains. In this paper, we consider the supply chain of joint decision making for inventory and sustainability technology investment. First of all, increasing demand variability reduces the optimal profitability of the centralized supply chain. Second, in a fair concern manufacturer-dominated Stackelberg game model, equilibrium results are derived and it is shown that manufacturer will adopt more conservative measures. Finally, the above results are verified by Matlab calculation examples.

Keywords: Sustainable supply chain; Fairness concern; Stochastic demand; Dynamic games; Inventory decisions.

1 Introduction

Sustainable supply chain will be the main direction of supply chain development, and the establishment and practice of sustainable supply chain has become a strategic task for the industrial development of all countries [1]. Due to the pressure from government regulations and international green barriers, as well as the increase of consumers' green awareness, enterprises actively take measures to invest in sustainability and provide the market with resource-saving and environmentally friendly green products.

There are numerous studies devoted to sustainable supply chain. Zhang et al. [2] focused on the impact of consumer environmental awareness and retailer fairness issues. However, marketing research has shown that equity factors play a critical role in developing and maintaining channel partnerships. Cui et al. [3] first introduced equity concerns into the supply chain environment and studied supply chain coordination. Wu and Niederhoff [4] improves the traditional fair preference utility function by expanding the reference system. Wang et al. [5] studies manufacturer who faces high cost pressures, while retailer with altruistic preferences. Therefore, we consider dynamic game of supply chain consisting of a fairness concern manufacturer and retailer.

The main points of contribution of this paper are threefold: (i) based on the deterministic demand function of Cui et al [3], we propose a stochastic demand that counts

on the sustainable investment; (ii) it constructs the fairness concern manufacturer-dominated Stackelberg game model and gives the equilibrium solutions; (iii) it is found that the fair concern manufacturer’s decision tends to be more conservative.

2 Model description

Supply chain of a sustainable manufacturer and retailer are modeled. Suppose that consumers in the marketplace have positive preference for sustainability, the level of investment in sustainable technologies is denoted as $e, e \geq 0$, the demand function is given as follows,

$$D(e) = \beta(e) + X_\kappa \tag{1}$$

where $\beta(e)$ is increasing with $e, 0 \leq \kappa \leq 1, X$ is random variable in $[\underline{L}, \bar{L}]$, with distribution function $F(x)$, density function $f(x)$, finite mean μ and inverse function $F^{-1}(x)$.

Remark 1. X_κ is the mean-preserving transformed of $X, X_\kappa = \kappa X + (1 - \kappa)\mu$, and μ is the mean value of X . The greater κ is, the greater the range of change in demand [6]. According to Dong et al. [7], denote the investment cost as $I(\tau) = \eta e^2 (e \geq 0, \eta > 0)$. The order quantity is q , unit cost is c , wholesale price is w . When q exceeds market demand, the remainder will be depreciated at the price v . When q is less, there is no out of stock penalty. Assuming the initial inventory is 0, the following equation holds, $p > c > v$.

Denote $\pi_m(q, w, e)$ and $\pi_r(q, w, e)$ as the manufacturer’s and retailer’s expected profits. Considering the function in Equation (1), the manufacturer’s profit is as follows:

$$\pi_m(q, w, e) = (w - c)q - \eta e^2 \tag{2}$$

And the retailer’s expected profit is as follows:

$$\pi_r(q, w, e) = E[p \min(q, D(e)) + v(q - D(e))^+ - wq] \tag{3}$$

where $D(e)$ is given in Equation (1), $(x)^+ = \max\{x, 0\}$.

Adding up the profits of two members is the profit of the supply chain integrator,

$$\pi_{sc}(q, e) = E[p \min(q, D(e)) + v(q - D(e))^+ - cq - \eta \tau^2] \tag{4}$$

Drawing on Du et al.’s study [8], we utilize the Nash bargaining scheme of both as a fairness reference point, which is more in line with reality.

Assuming the Nash bargaining fair reference is $(\tilde{\pi}_m, \tilde{\pi}_r)$. The manufacturer cares fair and the retailer is fair neutral ($\lambda_m = \lambda > 0, \lambda_r = 0$). Therefore, the utility functions are as:

$$u_m = \pi_m(q, w, e) - \lambda(\tilde{\pi}_m - \pi_m(q, w, e)) \tag{5}$$

$$u_r = \pi_r(q, w, e) \tag{6}$$

According to Binmore et al. [9], the Nash bargaining fair reference $(\tilde{\pi}_m, \tilde{\pi}_r)$ is the optimal profit (π_m^*, π_r^*) , which can make the following model enhancement.

$$\psi = u_m u_r \tag{7}$$

where $\pi_m + \pi_r = \pi_{sc}(q, e)$, $\pi_m, \pi_r \in [0, \pi]$, $u_m, u_r > 0$.

Combined the Equations $\tilde{\pi}_m + \tilde{\pi}_r = \pi_{sc}(q, e)$ and $\pi_m + \pi_r = \pi_{sc}(q, e)$, we get the u_m ,

$$u_m = (1 + \lambda)(\pi_{sc} - \pi_r) - \lambda(\pi_{sc} - \tilde{\pi}_r) \tag{8}$$

Substituted u_m in Equation (7), we get that,

$$\psi(\pi_{sc}, \pi_r) = (1 + \lambda)(\pi_{sc} - \pi_r) - \lambda(\pi_{sc} - \tilde{\pi}_r) u_r \tag{9}$$

Find the second order partial derivative of $\psi(\pi_{sc}, \pi_r)$ with π_r , we get $\frac{\partial^2 \psi(\pi_{sc}, \pi_r)}{\partial (\pi_r)^2} = -2(1 + \lambda) < 0$, thus $\psi(\pi_{sc}, \pi_r)$ is strictly concave with π_r , so there exists an optimal π_r^* satisfying,

$$\frac{\partial \psi(\pi_{sc}, \pi_r^*)}{\partial \pi_r} = \pi_{sc} - 2(1 + \lambda)\pi_r^* + \lambda\tilde{\pi}_r = 0 \tag{10}$$

According to the immovable point theorem, the Nash bargaining solution is the desired fair reference, i.e. $\tilde{\pi}_r = \pi_r^*$, the retailer’s Nash bargaining fair reference solution is:

$$\tilde{\pi}_r = \frac{1}{2 + \lambda} \pi_{sc} \tag{11}$$

The manufacturer’s Nash bargaining fair reference solution is:

$$\tilde{\pi}_m = \frac{1 + \lambda}{2 + \lambda} \pi_{sc} \tag{12}$$

The utility function of the manufacturer is :

$$u_m = (1 + \lambda)\pi_m - \frac{\lambda(1 + \lambda)}{2 + \lambda} \pi_{sc} \tag{13}$$

3 Equilibrium results

This section analyzes the equilibrium results under a centralized system and a dynamic decentralized system consisting of the manufacturer fairness concerning and retailer.

3.1 Centralized supply chain system

Consider the problem in Equation (4), denote the optimal solution of the centralized system by the superscript c and ρ as the stock factor, $\rho = (p - c)/(p - v)$.

Proposition 1. Consider the profitability of supply chain integrator in Equation (4),

- i. If for any $e \geq 0$, $\beta''(e) \leq 0$ holds, then $\pi_{sc}(q, e)$ is a joint concave function of (q, e) and there exists the optimal solution (q^c, e^c) , given by the following Equations,

$$q^c = \kappa F^{-1}(\rho) + \beta(e^c) + (1 - \kappa)\mu \tag{14}$$

$$(p - c)\beta'(e^c) - 2\eta e^c = 0 \tag{15}$$

where

$$A(q, e) = [q - \beta(e) - (1 - \kappa)\mu] / \kappa \tag{16}$$

- ii. The optimal profitability $\pi_{sc}(q^c, e^c)$ of the supply chain integrator is given by,

$$\pi_{sc}(q^c, e^c) = (p - v)[\kappa T_X(\rho) + \rho(\beta(e^c) + (1 - \kappa)\mu)] - \eta(e^c)^2 \tag{17}$$

where $T_X(\rho)$ is given by Equation (18),

$$T_X(\rho) = \int_{\underline{l}}^{F_X^{-1}(\rho)} (\rho - F_X(x)) dx - \rho \underline{l}, 0 < \rho < 1 \tag{18}$$

- iii. For given demand variability κ , q^c increases as the level of sustainable investment effort e increases; for given e , q^c increases as the mean value μ of demand increases.
- iv. For given e , q^c varies with κ . When $0 < \rho \leq F(\mu)$, q^c of low-profitability products decreases with the increase of κ ; when $F(\mu) < \rho < 1$, q^c of high-profit products is opposite.
- v. The optimal profitability decreases with increasing demand variability κ .

Proof: (i) We rewrite Equation (4),

$$\pi_{sc}(q, e) = (p - c)q - \kappa(p - v) \int_{\underline{l}}^{A(q, \tau)} F(x) dx - \eta \tau^2 \tag{19}$$

Find the first and second partial derivatives of q and e of the Equation (19), we can get

$\partial \pi_{sc}(q, e) / \partial q = p - c - (p - v)F(A(q, e))$, $\partial^2 \pi_{sc}(q, e) / \partial q^2 = -(p - v)f(A(q, e)) / \kappa$, $\partial \pi_{sc}(q, e) / \partial e = (p - v)F(A(q, e))\beta'(e) - 2\eta \tau$, $\partial^2 \pi_{sc}(q, e) / \partial e^2 = (p - v)F(A(q, e))\beta''(e) - (p - v)f(A(q, e))[\beta'(e)]^2 / \kappa - 2\eta$, $\partial^2 \pi_{sc}(q, e) / \partial q \partial e = (p - v)f(A(q, e))\beta'(e) / \kappa$. When $\beta''(e) \leq 0$ holds for any $e \geq 0$, the hessian matrix is negative, $\pi_{sc}(q, e)$ is a joint concave function of (q, e) . Let $\partial \pi_{sc}(q, e) / \partial q = 0$, $\partial \pi_{sc}(q, e) / \partial e = 0$, the optimal solutions can be deprived. (ii) Replace (q^c, e^c) in Equations (14) and (15) into (19). (iii) It is easily to prove from (14). (iv) From (14), $\partial q^c / \partial \kappa = F^{-1}(\rho) - \mu$. When $F^{-1}(\rho) > \mu$, q^c is increasing of κ ; when $F^{-1}(\rho) \leq \mu$, q^c is decreasing of κ . (v) From (17), we obtain $\partial \pi_{sc}(q^c, e^c) / \partial \kappa = (p - v)(T_X(\rho) - \rho\mu)$, denote $M_X(\rho) = T_X(\rho) / \rho$, $0 < \rho < 1$, we obtain $\partial M_X(\rho) / \partial \rho = \int_{\underline{l}}^{F_X^{-1}(\rho)} F_X(x) dx / \rho^2 > 0$, $M_X(\rho)$ is increasing of ρ and, $M_X(\rho) = \underline{l}$, $M_X(\rho) = \mu$, then $\underline{l} < M_X(\rho) < \mu$ holds for any $0 < \rho < 1$, so $T_X(\rho) < \rho\mu$ holds for any $0 < \rho < 1$. \square

Remark 2. From Proposition 1(iii), the optimal order quantity increases with the level of investment in sustainable technologies as consumers prefer sustainable products.

Remark 3. Theorem 1(iv) indicates that the optimal order quantity of high profit products ($F(\mu) < \rho \leq 1$) is greater than the average demand, while the low profit products ($0 < \rho \leq F(\mu)$) is opposite. This finding is in line with Schweitzer and Cachon [10].

3.2 Dynamic decentralized supply chain system with fairness concern

Firstly, the manufacturer decides wholesale price w and e to maximize its utility function u_m . Secondly, the retailer optimizes q to maximize its desired profit u_r . Denote $h(x)$ as the failure rate function of X , $h(x) = f(x)/(1 - F(x))$, $g(x) = xh(x)$. Denote the optimal decisions by the superscript d .

Proposition 2. Consider the dynamic Stackelberg game model with fairness concern,

i. If X satisfies a uniform distribution on $[0, 1]$, $\beta''(e) \leq 0$ and Equation

$$(2 + \lambda)(\beta'(e))^2 + \kappa(4 + \lambda)(\beta(e) + (1 - \kappa)\mu) \leq 0 \tag{20}$$

for any $e \geq 0$ holds, then (q^d, e^d, w^d) exists and derived by following Equations,

$$2\kappa[(p - c) - (p - \nu)F(A(q^d, e^d))] - (2 + \lambda)(p - \nu)f(A(q^d, e^d))q^d = 0 \tag{21}$$

$$(p - \nu)\beta'(e^d)[(2 + \lambda)f(A(q^d, e^d))q^d - \lambda\kappa F(A(q^d, e^d))] - 4\kappa\eta e^d = 0 \tag{22}$$

$$w^d = p - (p - \nu)F(A(q^d, e^d)) \tag{23}$$

ii. The manufacturer's optimal profit $u_m^d = u_m(q^d, e^d, w^d)$ is given by Equation (24),

$$u_m(q^d, e^d, w^d) = (1 + \lambda)\pi_m(q^d, e^d, w^d) - \lambda(1 + \lambda)/(2 + \lambda)\pi_{sc}(q^d, e^d) \tag{24}$$

where $\pi_m(q^d, e^d, w^d)$ is given in Equation (25) and $\pi_{sc}(q^d, e^d, w^d)$ is given in (26),

$$\pi_m(q^d, e^d, w^d) = (2 + \lambda)(p - \nu)(q^d)^2 f(A(q^d, e^d)) / (2\kappa - \eta(e^d)^2) \tag{25}$$

$$\pi_{sc}(q^d, e^d) = (p - c)q^d - (p - \nu)\kappa \int_L^{A(q^d, e^d)} F(x) dx - \eta(e^d)^2 \tag{26}$$

iii. The retailer's optimal profitability $\pi_r^d = \pi_r(q^d, e^d, w^d)$ is obtained from Equation (27),

$$\pi_r(q^d, e^d, w^d) = (p - \nu)[\kappa GL_X(\rho^d) + (\beta(e^d) + (1 - \kappa)\mu)] \tag{27}$$

where $\rho^d = (p - w)/(p - \nu)$ and $T_X(\rho)$ is given by equation (18).

Proof: (i) Using inverse induction, we rewrite the Equation (3),

$$\pi_r(q, e, w) = (p - w)q - \kappa(p - \nu) \int_L^{A(q, \tau)} F(x) dx \tag{28}$$

Take the first and second partial derivatives of q of the Equation (28), respectively, we get $\partial \pi_r(q, e, w) / \partial q = p - w - (p - \nu)F(A(q, e))$, $\partial^2 \pi_r(q, e, w) / \partial q^2 = -(p - \nu)f(A(q, e)) / \kappa < 0$, $\pi_r(q, e, w)$ is strictly concave of q , let $\partial \pi_r(q, e, w) / \partial q = 0$, we obtain $w(q, e) = p - (p - \nu)F(A(q, e))$. We rewrite the manufacturer's utility, $u_m(q, e, w(q, e)) = (1 + \lambda)[(p - c - (p - \nu)F(A(q, e)))q - \eta e^2]$

$-\lambda(1+\lambda)/(2+\lambda)[(p-c)q-\kappa(p-v) \int_L^{A(q,e)} F(x)dx]-\eta e^2]$. Take the first and second partial deriva-

tives of q and e , respectively, we can get

$$\begin{aligned} \partial u_m(q,e,w(q,e))/\partial q &= 2(1+\lambda)[p-c-(p-v)F(A(q,e))]/(2+\lambda)-(1+\lambda)(p-v)f(A(q,e))q/\kappa, \\ \partial^2 u_m(q,e,w(q,e))/\partial q^2 &= (1+\lambda)(4+\lambda)(p-v)f(A(q,e))/[\kappa(2+\lambda)]-(1+\lambda)(p-v)f'(A(q,e))q/\kappa^2, \\ \partial u_m(q,e,w(q,e))/\partial e &= (1+\lambda)(p-v)q\beta'(e)f(A(q,e))/\kappa-\lambda(1+\lambda)(p-v)F(A(q,e))\beta'(e)/(2+\lambda)-4(1+\lambda)\eta e/(2+\lambda), \end{aligned}$$

$$\partial^2 u_m(q,e,w(q,e))/\partial e^2 = (p-v)\beta''(e)(1+\lambda)[f(A(q,e))q/\kappa-\lambda F(A(q,e))/(2+\lambda)]-(p-v)[\beta'(e)]^2[(1+\lambda)f'(A(q,e))q/\kappa-\lambda(1+\lambda)f(A(q,e))/(2+\lambda)]/\kappa-4(1+\lambda)\eta/(2+\lambda),$$

$\partial^2 u_m(q,e,w(q,e))/\partial q \partial e = (1+\lambda)(p-v)\beta'(e)[f(A(q,e))+f'(A(q,e))q/\kappa-\lambda(1+\lambda)(p-v)\beta'(e)f(A(q,e))/(2+\lambda)]/\kappa$. If X satisfies uniform distribution on $[0,1]$, $\beta''(e) \leq 0$ and Equation (20) holds for any $e \geq 0$, the Hessian matrix is non-negative and $u_m(q,e,w(q,e))$ is joint concave. Let $\partial u_m(q,e,w(q,e))/\partial q = 0$, $\partial u_m(q,e,w(q,e))/\partial e = 0$, we get the equilibrium solutions. (ii) Substituting Equations (21), (22), and (23) in (13). (iii) Substituting Equations (21), (22), and (23) in (28).

Remark 4. From Equation (14), wholesale price decreases as the level of investment in sustainable technology by manufacturer increases, implying that investment in sustainable technologies is beneficial to manufacturer.

4 Numerical example

Example 1. Assume $v=2$, $\beta(e)=2+0.2e$, $p=10$, $\eta=0.3$, and c takes 8 or 4. Suppose that X is a uniform random variable defined at $[0,1]$. Table 1 illustrates the impact of demand variability on supply chain integrator.

Table 1. Optimal decisions of the supply chain integrator of different demand variability

κ	$\rho = 0.75$			$\rho = 0.25$		
	q^c	e^c	$\pi_{sc}(q^c, e^c)$	q^c	e^c	$\pi_{sc}(q^c, e^c)$
0.10	2.9250	2.0000	16.3478	2.6083	0.6667	5.0831
0.30	2.9750	2.0000	16.5893	2.5583	0.6667	4.9766
0.50	3.0250	2.0000	16.6688	2.5083	0.6667	4.8521
0.70	3.0750	2.0000	16.4783	2.4583	0.6667	4.6976
0.90	3.1250	2.0000	15.9098	2.4083	0.6667	4.5011

Remark 5. According to Table 1, when κ increases, there is no effect on the e^c , while q^c increases for high-profit products and decreases for low-profit products. However, the profitability of the supply chain decreases as demand variability increases.

5 Conclusions

Based on the deterministic demand function of Cui et al [3], we propose a stochastic demand that counts on the sustainable investment. The main conclusions are as follows (i) In centralized system, the optimal order quantity increases with sustainability technology investment; low-profit product's order quantity decreases with the increase of κ , and the opposite is true for high-profit product. The optimal profit of the supply chain

integrator decreases with increasing κ . (ii) In the manufacturer-dominated Stackelberg game model, manufacturer tends to adopt more conservative decisions when considering equity concerns. Issues that merit further research include (i) Consider how retailer behavioral factors affect optimal decision-making. (ii) Consider the impact on the different power leadership.

Acknowledgments.

We sincerely thank all the reviewers for their constructive suggestions on this paper.

References

1. O'Rourke, D. (2014). The science of sustainable supply chains. *Science*, 344(6188), 1124–1127. doi:10.1126/science.1248526.
2. Zhang, L., Zhou, H., Liu, Y., & Lu, R. (2019). Optimal environmental quality and price with consumer environmental awareness and retailer's fairness concerns in supply chain. *Journal of Cleaner Production*, 213, 1063–1079. <https://doi.org/10.1016/j.jclepro.2018.12.187>.
3. Cui, T. H., Raju, J. S., & Zhang, Z. J. (2007). Fairness and channel coordination. *Management Science*, 53(8), 1303–1314. <https://doi.org/10.1287/mnsc.1060.0697>.
4. Wu, X., & Niederhoff, J. A. (2014). Fairness in Selling to the Newsvendor. *Production and Operations Management*, 23(11), 2002–2022. <https://doi.org/10.1111/poms.12222>.
5. Wang, Y., Yu, Z., Jin, M., & Mao, J. (2021). Decisions and coordination of retailer-led low-carbon supply chain under altruistic preference. *European Journal of Operational Research*, 293(3), 910–925. doi:10.1016/j.ejor.2020.12.060.
6. Gerchak, Y., & Mossman, D. (1992). On the Effect of Demand Randomness on Inventories and Costs. *Operations Research*, 40(4), 804–807. doi:10.1287/opre.40.4.804.
7. Dong, C., Shen, B., Chow, P.-S., Yang, L., & Ng, C. T. (2014). Sustainability investment under cap-and-trade regulation. *Annals of Operations Research*, 240(2), 509–531. doi:10.1007/s10479-013-1514-1.
8. Du, S., Nie, T., Chu, C., & Yu, Y. (2014). Newsvendor model for a dyadic supply chain with Nash bargaining fairness concerns. *International Journal of Production Research*, 52(17), 5070–5085. <https://doi.org/10.1080/00207543.2014.895446>.
9. Binmore, K., Rubinstein, A., & Wolinsky, A. (1986). The Nash Bargaining Solution in Economic Modelling. *The RAND Journal of Economics*, 17(2), 176. <https://doi.org/10.2307/2555382>.
10. Schweitzer, M. E., & Cachon, G. P. (2000). Decision Bias in the Newsvendor Problem with a Known Demand Distribution: Experimental Evidence. *Management Science*, 46(3), 404–420. <https://doi.org/10.1287/mnsc.46.3.404.12070>.

Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (<http://creativecommons.org/licenses/by-nc/4.0/>), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

