Equipment reliability assessment based on a two-parameter Weibull distribution

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Abstract. A computational model for assessing equipment reliability is developed using the two-parameter Weibull parameter, fitting the probability density function, probability distribution function and reliability function for the occurrence of equipment failures, relying on the least binomial to fit the shape and size parameters of the model, and using the KS test to check whether the extracted samples conform to the theoretical distribution. Finally, based on the analysis of equipment failure examples by the developed model, the variation law of equipment reliability with motorbike hours is calculated and the fitted probability distribution function can be matched with the original data points of equipment failure. The results show that the proposed method is important for targeted reliability assessment and fault prediction.

Keywords: two-parameter Weibull; equipment reliability; KS test; probability distribution function

1 Introduction

Traditional reliability assessment is based on the failure data of a large number of similar devices to calculate the reliability of the equipment, which is a key consideration in equipment engineering projects. An accurate and effective reliability assessment model is an important tool to ensure the stability, safety and reliability of equipment operation[1-3], and can provide a basis for decision making on the use, maintenance assurance and even design modification of equipment, as well as effectively promoting the assessment of generic quality characteristics and in-service assessment of equipment[4-7]. A large amount of failure data, degradation data, usage data, environmental data, etc. are generated by the equipment in a real service environment. Making full use of these data and using appropriate calculation methods, the quality characteristics of the product or system can be analyzed [8].

A. Rauf et al. (eds.), *Proceedings of the 3rd International Conference on Management Science and Software Engineering (ICMSSE 2023)*, Atlantis Highlights in Engineering 20,
https://doi.org/10.2991/978-94-6463-262-0_56

The Weibull distribution is a continuous distribution that can be used to infer probability distribution parameters, etc. It has the function of describing the distribution pattern of various types of mechanical component failure data, and is the theoretical basis for reliability analysis and life testing, therefore, the Weibull distribution is widely used in data processing of various life tests, equipment reliability design, fatigue reliability analysis, maintenance decision and warranty strategy. The Weibull distribution can take many forms, including one-parameter Weibull distributions, two-parameter Weibull distributions, three-parameter Weibull distributions or mixed Weibull distributions. The two-parameter Weibull analysis is used as a continuous distribution where the required parameters are slope and characteristic life. The advantage of the two-parameter Weibull distribution is that it can be used to fit correct and reasonable failure analyses and failure predictions based on small samples of data, and is widely used in various fields, especially in the analysis of data in the field of reliability[9-12].

The KS test is a goodness-of-fit test that determines whether a single sample is from a particular distribution by analyzing the difference between two sample data. The test can be performed directly on the n observations of the original data, allowing a more complete use of the sample data.

2 Principle of the algorithm

2.1 Two-parameter Weibull distribution

The probability density function of the two-parameter Weibull distribution is:

$$\lambda(t) = \frac{\beta t^{\beta-1}}{\eta^{\beta}}, t \geq 0 \tag{1}$$

The probability distribution function is:

$$F(t) = \begin{cases} 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{2}$$

The reliability function is:

$$R(t) = \begin{cases} e^{-\left(\frac{t}{\eta}\right)^{\beta}}, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{3}$$

Where β is the shape parameter, which determines the basic shape of the curve, and η is the scale parameter, which determines the size of the scale of the curve.

2.2 Parameter estimation

Probability distribution function from the Weibull distribution:

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}, t \geq 0 \tag{4}$$

Get:

$$1 - F(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{5}$$

$$\frac{1}{1 - F(t)} = e^{\left(\frac{t}{\eta}\right)^{\beta}} \tag{6}$$

Take logarithm:

$$\ln\left(\frac{1}{1-F(t)}\right) = \left(\frac{t}{\eta}\right)^{\beta} \tag{7}$$

Again taking logarithms:

$$\ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right) = \beta \ln\left(\frac{t}{\eta}\right) = \beta \ln t - \beta \ln \eta \tag{8}$$

Make:

$$y = \ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right)$$

$$x = \ln t \tag{9}$$

then

$$y = \beta x - \beta \ln \eta \tag{10}$$

The parameters β and η were fitted by least squares.

2.3 Kolmogorov-Smirnov test

The Kolmogorov-Smirnov (KS) test is a non-parametric test based on a cumulative distribution function and is used to test whether an extracted sample conforms to a certain theoretical distribution. The advantage of the KS test is that it does not require knowledge of the distribution of the test data, it is a non-parametric test. The one-sample KS test is used to test whether the observed empirical distribution of a data conforms to a known theoretical. The KS test is based on a cumulative distribution function and is used to test whether a distribution conforms to a theoretical distribution or to compare two empirical distributions to see if they are significantly different.

The test statistic is: $D_n = \sup_t |F(t) - \hat{F}(t)|$, where $F(t)$ is the observed series value and $\hat{F}(t)$ is the theoretical series value. For the two-parameter Weibull distribution, the test steps are:

1) Formulate hypothesis H_0: $F(t) = \hat{F}(t)$

2) Calculate the absolute difference between the cumulative frequency of the sample and the cumulative probability of the theoretical distribution, such that the maximum absolute difference is D_n:

$$D_n = \max | F(t) - \hat{F}(t) | \tag{11}$$

3) Use the sample size n and the significance level α to find out the critical value D_c:

$$D_c = \frac{1.63}{\sqrt{n}} \tag{12}$$

4) If $D_n < D_c$, then H_0 is accepted and the fit is considered satisfactory.

The flowchart is shown in Figure 1.

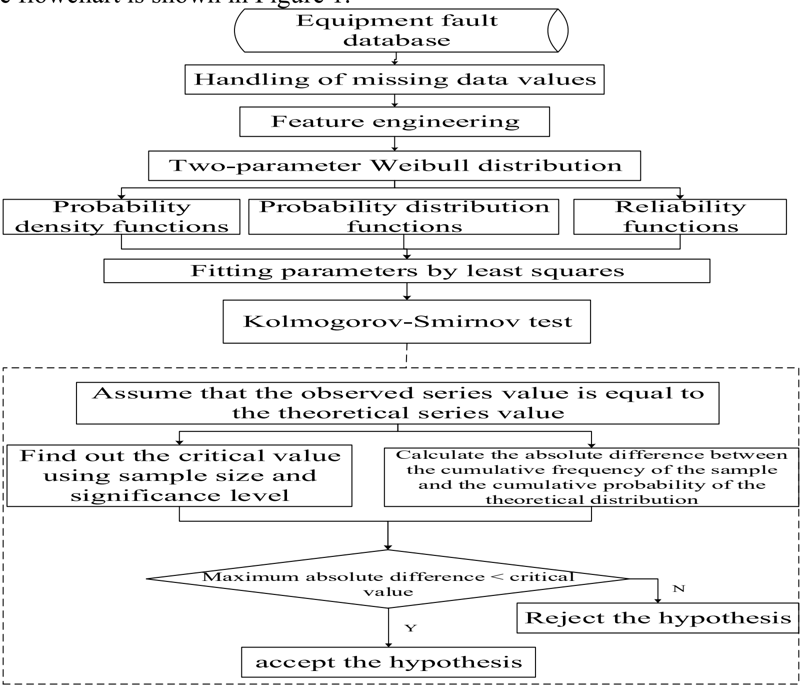

Fig. 1. Flow chart of the equipment reliability assessment model

3 Example analysis

Collect a certain type of listed equipment, 500 units in total, and record the motorbike hours when each unit fails in turn, recorded as $t_i (i = 1,2, \cdots, n)$.

3.1 Calculate the median rank

The median rank is used to obtain a measure of unreliability estimates and is calculated as follows:

$$F(t_i) = \frac{i - 0.3}{n + 0.4} \tag{13}$$

3.2 Draw Weibull probability diagram

Using t_i as the horizontal coordinate and $F(t_i)$ as the vertical coordinate, a graph is drawn on Weibull probability paper as shown in Figure 2.

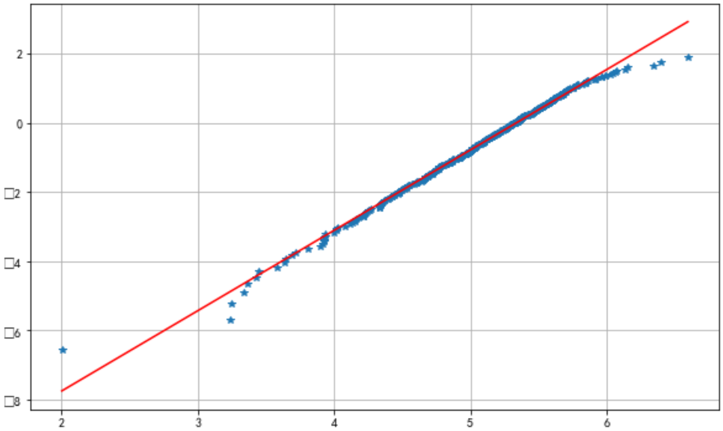

Fig. 2. Weibull probability diagram

It can be roughly seen from the Weibull probability plot that the majority of the data points are distributed around the perimeter of the line and the sample can be considered to obey the Weibull distribution.

3.3 Parameter estimation

Fitting a probability distribution function using the least squares method.

$$F(t) = \begin{cases} 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{14}$$

The shape parameter β and the scale parameter η by calculating that:

$$\eta = 205.61$$

$$\beta = 2.39$$

Then the probability distribution function for the failure of this type of equipment is:

$$\hat{F}(t) = \begin{cases} 1 - e^{-\left(\frac{t}{205.61}\right)^{2.39}}, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{15}$$

The raw data points and probability distribution function graph for this type of equipment failure are shown in Figure 3.

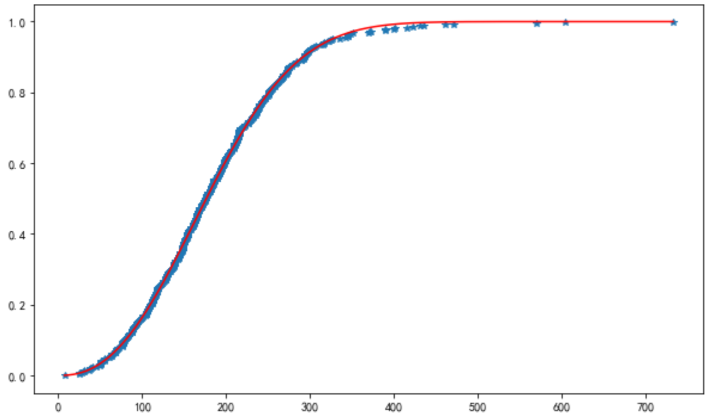

Fig. 3. Plot of the original data points and probabili-ty distribution function of the fault

The reliability function for this type of equipment is:

$$\hat{R}(t) = \begin{cases} e^{-\left(\frac{t}{205.61}\right)^{2.39}}, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{16}$$

The reliability function curve for this type of equipment is shown in Figure 4 and the reliability variation table is shown in Table 1.

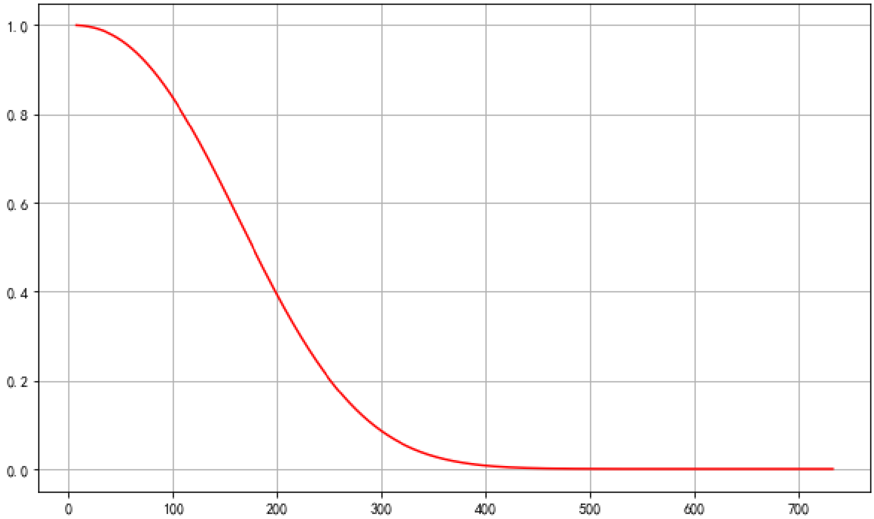

Fig. 4. Plot of the reliability function of the equipment

Table 1. Table of changes in reliability of equipment

Motorbike hours	Reliability	Motorbike hours	Reliability
10	0.9993	260	0.1736
20	0.9962	270	0.1472
30	0.9899	280	0.1237
40	0.9801	290	0.1031
50	0.9664	300	0.0851
60	0.9485	310	0.0696
70	0.9265	320	0.0565
80	0.9003	330	0.0453
90	0.8701	340	0.0361
100	0.8361	350	0.0284
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It can be obtained that the reliability of this type of equipment has fallen to 0.50 at around 180 motor hours, and to less than 0.01 after more than 390 motor hours.

3.4 Model testing

Calculation gives:

$$D_n = 0.0163$$

$$D_c = 0.0729$$

Therefore $D_n < D_c$, accepts the two-parameter Weibull model as the appropriate model for the fault distribution of this type of equipment.

4 Conclusion

This paper uses the two-parameter Weibull distribution, establishes a reasonable calculation model, obtains the probability distribution function of equipment failure according to the equipment's previous failure database, fits the shape and size parameters of the model by the least squares term method, and after KS test, relies on the equipment's historical failure data to estimate the change of equipment reliability with motorbike hours, etc. The fitted function obtained is in good agreement with the actual data. The two-parameter Weibull model is used to estimate the distribution of equipment failures, which provides a methodological basis for improving the reliability of military equipment and accurately assessing military equipment failures.

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