Enterprise Credit Rating Method Based on Stochastic Dominance Under Linguistic Distribution Assessments Context

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Abstract. In the process of enterprise risk management, credit rating is an important and effective method, which has been widely used in many fields. However, current credit rating methods rarely consider linguistic distribution assessment, which is often given by many experts. Inspired by this, in this paper, we developed a corporate credit rating method based on the stochastic dominance theory in the context of linguistic distribution assessment. In this method, the stochastic dominance theory and the minimum adjustment model are combined to establish a minimum adjustment cost model to achieve consensus in the process of credit rating. Then, we propose a dominance method to calculate the dominance degree of the distribution evaluation of any two languages, and then determine the ranking results of enterprises.

Keywords: Credit rating · Consensus reaching · Linguistic distribution assessment · Stochastic dominance

1 Introduction

Credit rating is a basic method of credit risk management, which is usually used to eliminate the potential risks of commercial banks or other investors by assessing the repayment ability of enterprises [1, 2].

To date, a large amount of research on credit rating methods has been developed. For example, Li et al. (2020) employed the machine learning techniques to predict the credit ratings for the banks in GCC [3]. Chen and Chen (2022) developed an approach to forecasting corporate credit ratings by analyzing public opinion toward corporations on social media [4]. Chen et al. (2023) employed a K-means clustering algorithm to obtain credit rating results [5]. Guo et al. (2023) proposed a novel network to model the enterprise credit rating problem using DNNs and attention mechanisms [6]. Liang et al. (2023) determined the credit rating result of enterprises by calculating the dominance degree between the candidate enterprises and the representative enterprises [7].

In addition, in recent years, the consensus reaching methods have been developed in the context of linguistic distribution assessment. For instance, for multi-criteria group decision making problems with multi-granular unbalanced linguistic information, Zhang

S. H. B. D. M. Zailani et al. (Eds.): ICMSEM 2023, 259, pp. 302–308, 2024.
https://doi.org/10.2991/978-94-6463-256-9_32

et al. (2021) developed two optimization models to generate adjustment advice for decision makers who have to change his/her opinions in consensus reaching process [8]. Wu et al. (2023) provided a consensus model with distribution linguistic preference relations that controls both the cardinal consistency and the ordinal consistency [9]. Zou et al. (2023) proposed a new social network driven consensus reaching process method for probabilistic linguistic multi-criteria group decision making problems [10].

Although existing research has made significant contributions to corporate credit rating and group consensus. However, there are still some problems that need to be studied: (i) Most existing credit rating methods are established on the premise of numerical evaluation of enterprises, but in some cases, experts may provide some linguistic distribution evaluation. (ii) Although the existing group consensus methods are widely used, they are not included in the scope of corporate credit rating. In order to fill in the above gaps, the goal of this paper is to propose a consensus approach to support corporate credit rating by combining stochastic dominance and minimum adjustment models.

2 Preliminaries

2.1 Stochastic Dominance

Stochastic dominance theory is a method of ranking two random variables based on their distribution function. The types of stochastic dominance involved in this paper are as follows:

Definition 1. Let Y_1 and Y_2 be two random variables defined on the interval [0, 1], and $F_{Y_1}(x)$ and $F_{Y_2}(x)$ be their cumulative distribution functions respectively. If $F_{Y_1}(x) - F_{Y_2}(x) \leq 0$ for all $x \in [0, 1]$, then we call $F_{Y_1}(x)$ FSD (First-order Stochastic Dominance) $F_{Y_2}(x)$.

Definition 2. Let Y_1 and Y_2 be two random variables defined on the interval [0, 1], and $F_{Y_1}(x)$ and $F_{Y_2}(x)$ be their cumulative distribution functions respectively. If $\int_0^x (F_{Y_1}(t) - F_{Y_2}(t))dt \leq 0$ for all $x \in [0, 1]$, then we call $F_{Y_1}(x)$ SSD (Second-order Stochastic Dominance) $F_{Y_2}(x)$.

2.2 The Minimum Adjustment Consensus Model

Let $I = \{I_1, I_2, \cdots, I_m\}$ be the set of individuals, and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$ be the associated weight vector where $\lambda_k \geq 0 (k = 1, 2, \cdots, m)$ and $\sum_{k=1}^m \lambda_k = 1$. Let $O = \left\{o^1, o^2, \cdots, o^m\right\}$ and $\overline{O} = \left\{\overline{o}^1, \overline{o}^2, \cdots, \overline{o}^m\right\}$ be two sets of the original and adjusted individuals' preferences. Then, let $\overline{o}^c$ be the collective adjusted preference where $\overline{o}^c = \sum_{k=1}^m \lambda_k \overline{o}^k$. In addition, α is a threshold value. In order to reach an acceptable consensus for all individuals, some adjustments need to be made until $\left|\overline{o}^k - \overline{o}^c\right| < \alpha, k = 1, 2, \cdots, m$. In addtion, in the consensus reaching process, $\sum_{k=1}^m d\left(o^k, \overline{o}^k\right)$ denotes the distance between $\overline{O}$ and O, which should be as small as possible. To obtain optimal adjusted preferences of individuals, the minimum adjustment consensus model can be built as follows [11, 12]:

$$\begin{cases} \min \sum_{k=1}^{m} d\left(o^k, \overline{o}^k\right) \\ s.t. \begin{cases} \overline{o}^c = \sum_{k=1}^{m} \lambda_k \overline{o}^k \\ \left|\overline{o}^k - \overline{o}^c\right| < \alpha, k = 1, 2, \cdots, m \end{cases} \end{cases} \tag{1}$$

3 Problem Description and Model Establishment

3.1 Problem Description

Let $I = \{I_1, I_2, \cdots, I_m\}$ be the set of individuals and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$, $\mu = (\mu_1, \mu_2, \cdots, \mu_m)^T$ be the associated weight vector and cost rate vector of individuals respectively. Let $E = \{E_1, E_2, \cdots, E_n\}$ be the set of enterprises. Let $C = \{C_1, C_2, \cdots, C_q\}$ be the set of attributes and $\omega = (\omega_1, \omega_2, \cdots, \omega_q)^T$ be the associated weight vector of attributes. Let $Y^k = (Y_{ij}^k)_{n\times q}$ be the individual decision matrix provided by I_k, where $Y_{ij}^k = \left\{\left(s_g, p_{ij}^{k,g}\right) | g = 0, 1, \ldots, T\right\}$ denotes I_k's linguistic distribution assessment on the enterprise E_i with respect to the attribute C_j, and $p_{ij}^{k,g}$ is the probability that Y_{ij}^k equals to s_g where $s_g = \frac{g}{T}$. Let $Y^c = (Y_{ij}^c)_{n\times q}$ be the collective decision matrix, where $Y_{ij}^c = \left\{\left(s_g, p_{ij}^{c,g}\right) | g = 0, 1, \ldots, T\right\}$ and $p_{ij}^{c,g} = \sum_{k=1}^{m} \lambda_k p_{ij}^{k,g} \geq 0$.

3.2 Model Establishment

By combining the theory of stochastic dominance with the minimum adjustment consensus model, a consensus reaching method is proposed as follows:

Firstly, the objective function of the model is to minimize the adjustment cost of all the individuals, which can be represented as follows:

$$\min \sum\nolimits_{k=1}^{m} \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{q} \sum\nolimits_{t=0}^{T} \mu_k \left|p_{ij}^{k,t} - \overline{p}_{ij}^{k,t}\right| \tag{2}$$

Then, the constraint for reaching the acceptable group consensus level can be expressed as follows:

$$\overline{GCL} = 1 - \frac{1}{n \times q \times T} \sum\nolimits_{k=1}^{m} \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{q} \sum\nolimits_{g=0}^{T} \lambda_k \left|\overline{p}_{ij}^{k,g} - \overline{p}_{ij}^{c,g}\right| \geq \alpha \tag{3}$$

Let $\overline{Y}^c = (\overline{Y}_{ij}^c)_{n\times q}$ be the adjusted collective decision matrix, where $\overline{Y}_{ij}^c = \left\{\left(s_g, \overline{p}_{ij}^{c,g}\right) | g = 0, 1, \ldots, T\right\}$ and $\overline{p}_{ij}^{c,g}$ is determined as follows:

$$\overline{p}_{ij}^{c,g} = \sum\nolimits_{k=1}^{m} \lambda_k \overline{p}_{ij}^{k,g} \tag{4}$$

Let $F_{\overline{Y}_{ij}^c}(x)$ be the cumulative distribution function of $\overline{Y}_{ij}^c$. To ensure that decision-maker can provide accurate preferences, it is required that the SSD relationship between any two enterprises E_i and E_v on each attribute should be identified, i.e.,

$$\int_0^x \left(F_{\overline{Y}_{ij}^c}(y) - F_{\overline{Y}_{vj}^c}(y)\right)dy \le 0, or \int_0^x \left(F_{\overline{Y}_{ij}^c}(y) - F_{\overline{Y}_{vj}^c}(y)\right)dy \ge 0x \in [0, 1] \tag{5}$$

Based on Eqs. (2)-(5), the minimum adjustment cost model P is established as follows:

$$min \sum\nolimits_{k=1}^{m} \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{q} \sum\nolimits_{t=0}^{T} \mu_k \left| p_{ij}^{k,t} - \overline{p}_{ij}^{k,t} \right|$$

$$\begin{cases} \overline{p}_{ij}^{c,t} = \sum_{k=1}^{m} \lambda_k \overline{p}_{ij}^{k,t} \\ \int_0^x \left(F_{\overline{Y}_{ij}^c}(y) - F_{\overline{Y}_{vj}^c}(y)\right)dy \le 0, or \int_0^x \left(F_{\overline{Y}_{ij}^c}(y) - F_{Y_{vj}^c}(y)\right)dy \ge 0x \in [0, 1] \\ \overline{GCL} = 1 - \frac{1}{n \times q \times T} \sum_{k=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{q} \sum_{t=0}^{T} \lambda_k \left| \overline{p}_{ij}^{k,t} - \overline{p}_{ij}^{c,t} \right| \ge \alpha \\ \sum_{t=0}^{T} \overline{p}_{ij}^{k,t} = 1 \\ 0 \le \overline{p}_{ij}^{k,t} \le 1 \end{cases} \tag{6}$$

3.3 Ranking Method on the Credit Score of Enterprises

Step 1. Determine the stochastic dominance relationship between two enterprises on each attribute by solving model P.

Step 2. Calculate the dominance degree between two enterprises on each attribute.

Let $\Omega = \left\{x | F_{\overline{Y}_{ij}^c}(x) \le F_{Y_{vj}^c}(x), x \in [0, 1]\right\}$, $\Theta = \{x | F_{\overline{Y}_{ij}^c}(x) \ge F_{\overline{Y}_{vj}^c}(x), x \in [0, 1]\}$, and

$$\beta_{iv}^j = \frac{\int_\Omega \left(F_{\overline{Y}_{ij}^c}(x) - F_{\overline{Y}_{vj}^c}(x)\right)dx}{\int_\Omega \left(F_{\overline{Y}_{ij}^c}(x) - F_{\overline{Y}_{vj}^c}(x)\right)dx + \int_\Theta \left(F_{\overline{Y}_{vj}^c}(x) - F_{\overline{Y}_{ij}^c}(x)\right)dx} \tag{7}$$

Let D_{iv}^j be the dominance degree for enterprise E_i over enterprise E_v with respect to attribute C_j. If $F_{\overline{Y}_{ij}^c}(y)$ SSD $F_{\overline{Y}_{vj}^c}(y)$, $D_{iv}^j = \beta_{iv}^j$. Otherwise, $D_{vi}^j = 1 - \beta_{iv}^j$.

Step 3. Determine the overall dominance degree matrix.

Let $D^j = \left(D_{iv}^j\right)_{m \times m}$ be the dominance degree matrix on attribute C_j, and $D = (d_{iv})_{m \times m}$ be the overall dominance degree matrix, where d_{iv} denotes the dominance degree for enterprise E_i over enterprise E_v. d_{iv} can be determined by

$$d_{iv} = \sum_{j=1}^{q} \omega_j D_{iv}^j \tag{8}$$

Step 4. Determine the ranking result of the credit rating of enterprises.

Let d_i^+ and d_i^- be the overall dominance degree for enterprise E_i over other enterprises and other enterprises over enterprise E_i respectively, where

$$d_i^+ = \sum_{v=1}^{m} d_{iv}, i = 1, 2, \cdots, m \tag{9}$$

$$d_i^- = \sum_{v=1}^{m} d_{vi}, i = 1, 2, \cdots, m \tag{10}$$

Let d_i be the net dominance degree for enterprise E_i over other enterprises, where

$$d_i = d_i^+ - d_i^-, i = 1, 2, \cdots, m \tag{11}$$

Obviously, $d_i \in [-m, m]$. And the higher the value d_i is, the better the enterprise E_i is.

4 Conclusions

In the context of linguistic distribution assessment, this paper utilizes the stochastic dominance to support the credit rating of enterprises. The main research work of this paper is as follows:

(i) By considering the linguistic distribution assessment, this paper proposes an efficient consensus reaching framework, which provides a more reliable method for enterprise credit rating.
(ii) In the framework of stochastic dominance we develop a consensus reaching model by considering the minimal adjustment cost. Then, a dominance method is proposed to determine the credit rating results of different enterprises.

In the future, further research can be conducted as follows:

(i) The preferences or utilities under different types of risk attitudes usually exhibited the larger difference. In future research, the impact of decision-makers' risk attitude on rating results should be considered, such as risk preference, risk neutrality or risk aversion.
(ii) In some practical credit rating problems, the individuals (or experts) are always in a social network. Generally, the network structures often have greater influence on the adjustment of opinions and the weights of experts. Therefore, it would be interesting to integrate the proposed consensus reaching approach into the social network context.

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