

Management Strategies for the Conservation of Wildlife Based on the Analytic Hierarchy Process and Gray Prediction Models

Minghuan Piao¹(⊠), Wanting Zhang², and Chaofeng Cheng¹

¹ Department of Computer Science and Technology, Yanbian University, Yanji, Jilin Province, China

2252196536@qq.com

² Department of Mathematics and Applied Mathematics (Teacher Training), Yanbian University, Yanji, Jilin Province, China

Abstract. This paper investigates resource management methods in the Masai Mara Wildlife Reserve, proposing management strategies for the conservation of wildlife and natural resources. We utilize the Analytic Hierarchy Process (AHP) model and gray relational analysis to determine the most effective management strategies and predict their long-term trends. We optimize weight values using a stepwise quality house method and establish an AHP hierarchical analysis model. Finally, we construct a gray forecasting model to predict the data for the next 12 years of the management strategies, enabling a long-term projection. Our findings can provide insights for conservationists and policymakers for effective resource management in the Masai Mara Wildlife Reserve.

Keywords: gray correlation analysis \cdot AHP \cdot management strategies \cdot gray prediction model

1 Introduction

Protected areas have been established in various regions around the world to preserve biodiversity. However, in many cases, local people have been displaced with little or no compensation and are unable to access natural resources within these areas, leading to numerous negative impacts. In our study, we conducted a survey around the Masai Mara National Reserve in southwestern Kenya, where people living near the reserve are expected to have negative interactions with wildlife, and communities in close proximity to the reserve incur higher economic costs to mitigate these negative effects. Nonetheless, local residents can also derive various economic benefits from the protected areas, and if the gains outweigh the losses, they may support conservation efforts.

This paper investigates management strategies for resources within and outside the boundaries of the Masai Mara Reserve, with the aim of balancing the interests of the local people living in the region. We then predict the economic impacts arising from humanwildlife interactions both inside and outside the reserve, to determine which management strategies will yield the most favorable outcomes and discuss how the results of these approaches can be ranked and compared. Lastly, we project the long-term trends that the management strategies will generate, analyze and estimate the potential long-term consequences and impacts, and describe how this methodology can be applied to other wildlife management domains.

Based on the current state of research utilizing the gray Prediction Model for assessing management strategies in the Masai Mara Reserve, scholars have achieved notable progress in the areas of ecological environment, biodiversity, and community participation [1]. However, challenges such as data quality and availability, ecosystem complexity, and socio-economic factors still need to be overcome. Previous studies have primarily employed the gray Prediction Model to evaluate various aspects, such as the impact of tourism development on the ecological environment, the fluctuating growth of biodiversity in the reserve, and the implementation of management strategies based on community involvement.

Firstly, a gray relational analysis model is established to determine the degree of association among wildlife, natural resources, and human interests. The Pearson correlation coefficient is utilized to represent the strength of their correlations, revealing the relationships among these factors. Based on this, management strategies for different regions within the protected area are proposed. Next, factors affecting the choice of wildlife protection area management strategies are identified, analyzed, and compared. Appropriate scales are assigned to pairwise comparisons of these factors, and a judgment matrix is constructed [2]. The Analytic Hierarchy Process (AHP) is employed to analyze the data, [3] and the square root method is applied to obtain eigenvectors for each influencing factor's weight. A consistency test is performed on the judgment matrix, indicating that the obtained weight results possess high credibility, thereby ranking and comparing the selected policies. Finally, using the gray prediction model, historical data for GDP, tourism, and species numbers are analyzed. A GM(1,1) model is established and solved to obtain the GDP, tourism, and species number forecasts for the next 12 years. The fitting effect of the GM(1,1) model is verified through the method of level ratio deviation. Long-term trend predictions are generated, and potential factors influencing these long-term results are analyzed.

2 Method

2.1 Gray Correlation Analysis

In Kenya, species count, per capita GDP, PPP, employment, forest area, tourism revenue, and cultivated land area are considered as reference sequences and sub-sequences. The strength, magnitude, and order between these sequences are determined by calculating the gray correlation degree between the main factor sequence and the sub-sequence. The higher the gray correlation degree between the main factor sequence and the sub-sequence on the main factor sequence.

Assuming the per capita GDP as the reference sequence and the other six indicators as sub-sequences, each sub-sequence collects a total of 29 samples. The formula can be

represented as follows: Reference Sequence:

$$e_0 = (e_0(1), e_0(2), \dots, e_0(29))^{\mathrm{T}}$$
 (1)

Sub-Sequence:

$$F_1 = (f_1(1), f_1(2), \dots, f_1(29))^T$$
(2)

$$f_2 = (f_2(1), f_2(2), \dots, f_2(29))^T$$
 (3)

$$\mathbf{F}_3 = (\mathbf{f}_3(1), \mathbf{f}_3(2), \dots, \mathbf{f}_3(29))^{\mathrm{T}}$$
(4)

$$f_4 = (f_4(1), f_4(2), \dots, f_4(29))^T$$
 (5)

$$f_5 = (f_5(1), f_5(2), \dots, f_5(29))^T$$
 (6)

$$\mathbf{f}_6 = (\mathbf{f}_6(1), \mathbf{f}_6(2), \dots, \mathbf{f}_6(29))^{\mathrm{T}}$$
 (7)

Assuming " α " as the extreme minimum difference and " β " as the extreme maximum difference, we have:

$$\alpha = \min(i)\min(k)|e_0(k) - f_i(k)| \tag{8}$$

$$\beta = \max(i) \max(k) |e_0(k) - f_i(k)|$$
(9)

The formula for the correlation coefficient E (Gamma value) is:

$$E(e_{0}(k), f_{i}(k)) = \frac{\alpha + \beta * \rho}{|e_{0}(k), f_{i}(k)| + \beta * \rho} (\rho \ \mathbb{R} 0.5)$$
(10)

Assuming the gray correlation degree as "E(e_0 (k),f_i (k)), ", we have:

$$E(e_0(j), f_i(j)) = \frac{1}{n} \sum_{j=1}^{29} E(e_0(j), f_i(j))$$
(11)

The arithmetic mean root of each column represents the degree of association between each sub-sequence and the reference sequence. Similarly, using the number of people employed in industry, arable land (as a percentage of land area), forestry area, tourist income, and number of species as reference sequences and the remaining six indicators as sub-sequences, the above model repeated.

2.2 Pearson Correlation Analysis

The correlation analysis is used to determine the correlation between species count, per capital GDP, PPP, employment, forest area, tourism revenue, and cultivated land area in Kenya, so as to understand the relationship between wild animals, natural resources, and

people's interests. The Pearson correlation coefficient is used to represent the strength of the correlation relationship.

Assuming species count as Z and other variables as Y, the total correlation coefficient between each variable is:

$$\rho = \frac{\operatorname{cov}\left(Z, Y\right)}{\sqrt{\operatorname{var}\left(Z_{i}\right)\sqrt{\operatorname{var}\left(Y\right)}}}$$
(12)

where cov(Z, Y) is the covariance between the two variables, $var(Z_i)$ and var(Y) is the variance between variables Z_i and Y.

Since the total correlation coefficient is variable, the sample correlation coefficient is needed for calculation. Assuming $Z_i = (z_{i1}, z_{i2}, ..., z_m)$ and $Y = (y_1, y_2, ..., y_n)$ are from two samples of w_i and T, respectively, then the sample correlation coefficient is:

$$\mathbf{r} = \frac{\sum_{j=1}^{n} \left(z_{ij} - \overline{z}_i \right) \left(y_j - \overline{y} \right)}{\sqrt{\sum_{i=1}^{n} \left(z_{ij} - \overline{z}_i \right)^2 \sum_{j=1}^{n} \left(y_j - \overline{y} \right)^2}}$$
(13)

2.3 Analytic Hierarchy Process (AHP)

An analytic hierarchy model is established and the progressive weight method is applied to optimize the weight values of each indicator [4]. Then, the comprehensive evaluation is carried out using the gray correlation degree, and finally, the policies that will produce the best results are determined. Five standard factors are extracted for the impact factors of wild animal protection area management strategies: ecological value, management cost, community participation, legal and policy environment, and scientific research. Emphasis is placed on analyzing the ranking of more effective management strategies in different regions and determining the weights of design requirements.

Generally, assuming n policies are determined as $C_I(C_1, C_2, C_3, ...)$, and m policy criteria $C_{R1}, C_{R2}, ..., C_{Rm}$ are obtained, in order to calculate the comprehensive ranking weights $a_1, a_2, ..., a_m$ of n policies with respect to the target, which are recorded as $a = (a_1, a_2, ..., a_m)$, the AHP method under a single criterion is used first to calculate the ranking weights of p criteria with respect to the target, recorded as $v_1, v_2, ..., v_p$, recorded as $v = (v_1, v_2, ..., v_p)$. Then, the relative weights of n policies with respect to the policy criteria $p_{j1}, p_{j2}, ..., p_{jm}$ are calculated, where the weight of the policy criteria not dominated by criterion C_j is zero. After determining the comprehensive ranking weights of m, it is also necessary to carry out consistency checks on the ranking hierarchy of n criteria and their ranking structure according to Saaty's experience rule to ensure the consistency of all judgments and analysis in the process of determining the ranking weights.

In the design requirement ranking hierarchy, the expansion precision of the Analytic Hierarchy Process is defined as the consistency ratio C·R·k between the design requirement layer and the overall goal.

$$\mathbf{C} \cdot \mathbf{R} \cdot \mathbf{k} = \frac{\mathbf{C} \cdot \mathbf{I} \cdot^{\mathbf{k}}}{\mathbf{R} \cdot \mathbf{I} \cdot^{\mathbf{k}}} \tag{14}$$

where

$$\mathbf{C} \cdot \mathbf{I}^{\mathbf{k}} = \left(\mathbf{C} \cdot \mathbf{I}^{\mathbf{k}}_{1}, \mathbf{C} \cdot \mathbf{I}^{\mathbf{k}}_{2}, \dots, \mathbf{C} \cdot \mathbf{I}^{\mathbf{k}}_{m} \right) \boldsymbol{\alpha}^{\mathrm{T}}$$
(15)

$$\mathbf{R} \cdot \mathbf{I}^{\mathbf{k}} = \left(\mathbf{R} \cdot \mathbf{I}_{1}^{\mathbf{k}}, \mathbf{R} \cdot \mathbf{I}_{2}^{\mathbf{k}}, \dots, \mathbf{R} \cdot \mathbf{I}_{m}^{\mathbf{k}} \right) \boldsymbol{\alpha}^{\mathrm{T}}$$
(16)

where CR is the consistency index based on the policy standard layer in the design requirement ranking hierarchy and the average random consistency index, a is the composite ranking weight vector of the policy standard layer for the overall goal. In fact, using C·R·k as a measure of the expansion precision of the Analytic Hierarchy Process is reasonable, as it comprehensively measures the consistency of all judgments and analyses made in the process of converting policy standards into design requirements [5]. Therefore, according to Saaty's empirical rule, when C·R·k < 0.1, the expansion accuracy of the hierarchical quality house for the overall policy standard goal can be considered acceptable.

Step 1: Develop the model and Establish the matrix.

In the fuzzy matrix $R = (r_{ij})_{n \times n}$, $0 \le r_{ij} \le 1$ (i = 1, 2, ..., n; j = 1, 2, ..., n), the fuzzy complementary matrix R is not only a fuzzy matrix but also satisfies:

$$r_{ii} = 0.5 (i = 1, 2, ..., n)$$
 (17)

The significance of comparing element i with element j and the complementarity of the significance of comparing element j with element i can be stated as follows: A fuzzy consistency matrix is not only a fuzzy complementarity matrix, but also satisfies the property of $\forall i, j, k$.

$$r_{ij} = r_{ik} - r_{jk} + 0.5 \tag{18}$$

where $r_{ij} = 0.5$ represents the equal importance of elements i and j, $0 < r_{ij} < 0.5$ represents the greater importance of element j compared to element i, with smaller values of r_{ij} indicating greater importance of element j over element i. $0.5 < r_{ij} < 1$ represents the greater importance of element i compared to element j, with larger values of r_{ij} indicating greater importance of element i over element j. $r_i = \sum_{k=1}^{n} r_{ik}$, $i = 1, 2, \ldots, n$ involves the following mathematical transformation:

$$\mathbf{r}_{ij} = \frac{\mathbf{r}_i - \mathbf{r}_j}{2\mathbf{n} + 5} \tag{19}$$

The transformed matrix is a fuzzy consistency matrix. Any submatrix obtained by deleting an arbitrary row and its corresponding column from the fuzzy consistency matrix R is still a fuzzy consistency matrix. The fuzzy consistency matrix R satisfies the property of mid-valuation transitivity, meaning:

Given a real number $\lambda \ge 0.5$, if $r_{ij} \ge \lambda$, $r_{jk} \ge \lambda$, then it follows that: $r_{ik} \ge \lambda$. When $\lambda \le 0.5$, if $r_{ij} \le \lambda$, $r_{ik} \le \lambda$, then $r_{ik} \le \lambda$ follows.

Step 2: Rank and Compare the results.

Next, based on the relative importance of the factors at each level towards the factors at the higher level, a priority relationship matrix is constructed. The factor weights are then calculated based on the relationship between the elements and weights of the fuzzy consistency judgment matrix that is transformed from the priority relationship matrix. The formula for calculating the weight S_i^k of factor Ai under the objective Ok is:

$$S_{i}^{k} = \frac{1}{n} - \frac{1}{2\alpha} + \frac{\sum_{j=1}^{n} r_{ij}}{n\alpha} (i = 1, 2, \dots, n)$$
(20)

Step 3: Evaluation trade off.

In the above formula, the parameter α satisfies that arranging $\alpha \geq \frac{n-1}{2}$ from large to small S_i^k (i = 1, 2, ..., n) shows the importance order of the factors Ok relative to the objective S_i^k . wk represents the weight of the factors for each solution, and the formula for calculating the overall superiority of each solution relative to the overall objective is as follows: T_i .

$$T_i = \sum_{k=1}^{n} w_k S_i^k \tag{21}$$

2.4 Gray Prediction Model (GPM)

The gray Prediction Model (GM (1,1)) uses the original discrete data column, generates a new discrete data column with weakened randomness and more regularity through a cumulative process, and then establishes a differential equation model to obtain solutions at discrete points [6]. The approximation estimate of the original data generated through the cumulative subtraction is used to predict the subsequent development of the original data. Let $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$ be the original original data column, and we perform one cumulative operation to obtain the new generated data column as:

$$\mathbf{x}^{(1)} = \left(\mathbf{x}^{(1)}(1), \mathbf{x}^{(1)}(2), \dots, \mathbf{x}^{(1)}(n)\right)$$
(22)

where:

$$\mathbf{x}^{(m)} = \sum_{i=1}^{m} \mathbf{x}^{(0)}(i), \quad \mathbf{m} = 1, 2, \dots, \mathbf{n}$$
 (23)

Let $z^{(1)}$ be the neighboring average generated sequence of the sequence $x^{(1)}$, i.e.

$$\mathbf{z}^{(1)} = \left(\mathbf{z}^{(1)}(2), \, \mathbf{z}^{(1)}(3), \, \dots, \, \mathbf{z}^{(1)}(n) \right) \tag{24}$$

where $z^{(1)}(m) = \delta x^{(1)}(m) + (1 - \delta)x^{(1)}(m - 1)$, m = 2, 3, ..., n and $\delta = 0.5$ are constants. We refer to the equation as the basic form of the GM(1,1) model. In this equation, "b" represents the gray effect amount and "-a" represents the development coefficient. The first "1" in GM(1,1) indicates that the equation is first-order and the second "1" indicates that there is only one variable. Next, we introduce the matrix form:

$$\mathbf{u} = (\mathbf{a}, \mathbf{b})^{\mathrm{T}}, \ \mathbf{Y} = \begin{bmatrix} \mathbf{x}^{(0)}(2) \\ \vdots \\ \mathbf{x}^{(0)}(\mathbf{n}) \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} -\mathbf{z}^{(1)}(2) \ \mathbf{1} \\ \vdots \\ -\mathbf{z}^{(1)}(\mathbf{n}) \ \mathbf{1} \end{bmatrix}$$
(25)

Thus, the GM (1,1) model $x^{(0)}(k) + az^{(1)}(k) = b$ can be represented as:

$$Y = Bu \tag{26}$$

We can use the least squares method to obtain the estimated values of the parameters a. b as:

$$\hat{\mathbf{u}} = \begin{pmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{pmatrix} = \left(\mathbf{B}^{\mathrm{T}} \mathbf{B} \right)^{-1} \mathbf{B}^{\mathrm{T}} \mathbf{Y}$$
 (27)

Additionally, if we consider the moment $x^{(0)}(m)$ for m = 2, 3, ..., n as a continuous variable t, and if $x^{(1)}$ is regarded as a function of time t and recorded as RR, then its derivative corresponds to $\frac{d\hat{x}^{(1)}(t)}{dt}$, $z^{(1)}(k)$ which corresponds to $x^{(1)}(t)$. We can then establish the white differential equation relative to the gray equation GM (1,1), denoted as $\frac{d\hat{x}^{(1)}(t)}{dt} + \hat{x}^{(1)}(t) = b$, which we refer to as the whitening equation of GM (1,1). If we take the initial value $\hat{x}^{(1)}(t) | t = 1 = x^{(0)}(1)$, we can calculate the corresponding solution as:

$$\hat{\mathbf{x}}^{(1)}(\mathbf{t}) = \left[\mathbf{x}^{(0)}(1) - \frac{\mathbf{b}}{\mathbf{a}}\right] \mathbf{e}^{-\mathbf{a}(\mathbf{t}-1)} + \frac{\mathbf{b}}{\mathbf{a}}$$
 (28)

Furthermore, we can obtain the solution of GM(1,1) model $x^{(0)}(k) + az^{(1)}(1)$ (k) = b as:

$$\hat{\mathbf{x}}^{(1)}(\mathbf{m}+1) = \left[\mathbf{x}^{(0)}(1) - \frac{\mathbf{b}}{\mathbf{a}}\right] e^{-\mathbf{a}\mathbf{m}} + \frac{\mathbf{b}}{\mathbf{a}}, \mathbf{m} = 1, 2, \dots, \mathbf{n} - 1$$
 (29)

From the above formula, we can obtain the simulated value of the original data column x⁽⁰⁾ as:

$$\hat{x}^{(0)}(m+1) = \hat{x}^{(1)}(m+1) - \hat{x}^{(1)}(m)$$

= 1 - e^a $\left[x^{(0)}(1) - \frac{b}{a} \right] e^{-am}, m = 1, 2, ..., n-1$ (30)

If we want to predict the original data, we only need to take m > n in the above formula. Using the coefficient of variation method for verification, we first calculate the coefficient of variation $\sigma(k) = \frac{x(k)}{x(k-1)}$, x = 2,3,...,n using the original data, x = 2, 3, ..., n, and then calculate the corresponding coefficient of variation deviation $\beta(k) =$ $1 - \frac{1 - 0.5a}{1 + 0.5a} \frac{1}{\sigma(k)}$. It is generally believed that when $\beta(k) < 0.1$ is met, it reaches a high requirement.

Results and Discussion 3

In this study, we conducted a comprehensive analysis of the relationship between wildlife, natural resources, and human interests using the gray correlation analysis model and Pearson correlation coefficient. We also evaluated the effectiveness of various policies and management strategies in wildlife conservation areas.



Fig. 1. Pearson correlation coefficient visualization

3.1 Results of Gray Correlation Analysis

The gray correlation analysis model showed that per capita GDP is highly correlated with tourism revenue (correlation coefficient = 1), while the number of species is closely related to forestry area (correlation coefficient = 0.995). The correlation between employment (data from Herrendorf et al.) and the number of species is 0.747. In addition, there is a significant negative correlation between forestry area and per capita GDP, as well as per capita GDP PPP (2017 international constant dollars).

3.2 Results of Pearson Correlation Analysis

In the present study, Fig. 1 reveals a lack of correlation between the number of species, GDP per capital and forestry area, Number of people employed in industry (Herrendorf etal. Data) and forestry area, tourist income and forestry area, and Arable land (as a percentage of land area) and forestry area among the 1 item, and there is a correlation between them and the other 6 item-s. There is a significant negative correlation between forestry area and GDP per capital, PPP (constant 2017 international \$), and there is no correlation between forestry area and the other 6 items.

3.3 Specific Policies and Management Strategies

We analyzed the relationship between wildlife, natural resources, and human interests in Kenyan conservation areas and proposed a series of management policies, including strengthening the promotion and implementation of wildlife protection laws and regulations, regularly monitoring the environmental and wildlife populations of conservation areas, regulating tourist behavior, limiting the number of visitors, and developing sustainable tourism.

3.4 Results of Analytic Hierarchy Process

A comparison of the importance of the five criteria of ecological value, management cost, community participation, legal and policy environment, and scientific research to construct a subjective 5-level evaluation matrix and use the analytic hierarchy process to obtain the weight results of each index is shown in the following Table 1:

Then, by comparing the importance of 14 sub-factors among the five standard factors, repeat the above process and obtain the weight results of each index as shown in the Fig. 2.

ItemI	Feature Vector	Weight Value	Maximum Eigenvalue	CI Value
Ecological value	1.747	34.94%	5.000	0
Management cost	0.751	15.02%	-	
Community participation	1.000	20.00%		
Legal and policy environment	0.502	10.05%		
Scientific research	1.000	20.00%		

Table 1. Results of AHP analysis for ecological value, management cost and scientific research



3.5 Consistency Check of the Judgment Matrix

The maximum eigenvalue of the standard factors is 5.311, and the RI value is 1.12. Thus, CR = CI/RI = 0.069 < 0.1. The maximum eigenvalue of the sub-factors is 14, and the RI value is 1.58. Thus, CR = CI/RI = 0 < 0.1, passing the one-time test. This indicates that there is no logical error in the judgment matrix, and the value of CR is much smaller than 0.1, thus the weight results obtained have a high degree of credibility.

3.6 Best Strategies for Wildlife Protection Area Policy and Management

Fully protecting the core conservation areas, restricting human entry, cracking down on illegal hunting and destructive behavior, developing sustainable tourism while limiting the number of tourists, regulating tourist behavior, cooperating with local communities, providing job opportunities for local residents to participate in wildlife conservation and management, and providing other ways to sustain their livelihoods, protecting ecological restoration areas and wildlife corridors to promote the sustainable development of wildlife.

3.7 Results of Gray Prediction Model

The prediction results of GDP, tourist, and number of species data are obtained through gray prediction as shown in the following Table 2:

year	GDP (in ten million dollars)	Tourist (in ten thousand)	Number of species (in millions)
2019	3253.06	17.749	154082.828
2020	3326.325	17.826	161055.174
2021	3400.399	17.903	168077.125
2022	3475.292	17.981	175149.031
2023	3551.012	18.058	182271.25
2024	3627.57	18.137	189444.139
2025	3704.973	18.215	196668.058
2026	3783.231	18.294	203943.371
2027	3862.355	18.373	211270.443
2028	3942.352	18.453	218649.642
2029	4023.234	18.533	226081.34
2030	4105.009	18.613	233565.909

Table 2. Predicted GDP, tourist, Number of species data from 2019 to 2030

4 Conclusion

This study investigated resource management approaches in the Masai Mara Wildlife Reserve, adopting an integrated methodology that considered wildlife, natural resources, and human interests. By employing the Analytic Hierarchy Process (AHP), gray relational analysis, and gray prediction models, we proposed effective management strategies and forecasted their long-term trends. Our analysis indicated that the recommended policies and management strategies could potentially result in increased wildlife populations, improved ecosystem health, enhanced management efficiency, and boosted economic benefits for local communities. However, when implementing these strategies, potential uncertainties and negative impacts, such as climate change, poaching, illegal activities, and external factors, should be thoroughly considered. To ensure the long-term success of the proposed strategies, continuous monitoring, timely adjustments, and collaboration among various stakeholders are essential. This study offers valuable insights into the management of the Masai Mara Wildlife Reserve and provides a methodological framework that can be applied to other wildlife management domains.

References

- Homewood, K., Lambin, E. F., Coast, E., Kariuki, A., Kikula, I., Kivelia, J., ... & Thompson, M. (2009). Staying Masai: livelihoods, conservation and development in East African rangelands. Springer Science & Business Media.
- Saaty, T. L. (2008). Decision making with the analytic hierarchy process. International Journal of Services Sciences, 1(1), 83–98.
- 3. Bana e Costa, C. A., & Vansnick, J. C. (2008). A critical analysis of the eigenvalue method used to derive priorities in AHP. European Journal of Operational Research, 187(3), 1422–1428.
- 4. Wang, Y. M., & Elhag, T. M. (2006). Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. Expert Systems with Applications, 31(2), 309–319.
- 5. Saaty, T. L., & Vargas, L. G. (2012). Models, Methods, Concepts & Applications of the Analytic Hierarchy Process. Springer Science & Business Media.
- Deng, J. L. (1982). Control problems of gray systems. Systems & Control Letters, 1(5), 288– 294.

Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (http://creativecommons.org/licenses/by-nc/4.0/), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

