Design and experiment of complex Trajectory Cam based on MATLAB

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Abstract. A cam design method is proposed in this study that utilizes mathematical modeling and MATLAB simulation analysis. The objective is to construct cam profiles that are suitable for complex trajectories with precision and efficiency. Initially, mathematical modeling is employed to determine the projected running trajectory. Subsequently, the corresponding cam contour is derived through MATLAB simulation analysis. To demonstrate the effectiveness of the proposed method, a carbon-free car navigating a complex path is chosen as an illustrative example for cam design. Experimental results confirm that the car successfully follows the predetermined path using the cam steering mechanism, with the walking trajectory closely aligning with the preset trajectory. This outcome substantiates the reliability of the cam design method.

Keywords: Cam profile design; complex trajectory; MATLAB; simulation analysis

1 INTRODUCTION

1.1 Practical application of complex cam

A cam serves as a mechanical element that facilitates motion transmission through direct surface contact [1]. Its primary function involves converting rotational motion into linear or nonlinear motion while controlling the trajectory and timing of other mechanical components [2]. Cam holds significant importance as a mechanical component widely employed in various systems. By manipulating cam geometry and motion parameters, an array of complex motion modes can be achieved [3]. For example, in an internal combustion engine, a cam on the camshaft transforms rotational motion into the up and down movement of the valve through the valve rod and connecting rod, facilitating gas intake and exhaust [4]. In the field of industrial automation, modular cam-controlled automatic assembly equipment [5] ensures efficient and stable repetitive operations by regulating the trajectory and speed of robotic arms or other moving parts. This equipment finds extensive use in automated assembly lines and packaging machinery. Furthermore, cam mechanisms can realize complex motion modes in a robot's end-effector, as demonstrated by bionic jumping robots that sys-

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tematically store and release energy through cam rotation [6]. Additionally, the cam structure used in unpowered exoskeletons [7] achieves more natural and accurate motion control by predicting the mechanism's dynamic characteristics, thereby enhancing mobility and quality of life for prosthetic users.

1.2 Cam design method

The design of the cam's contour curve is crucial as it determines the transmission relationship, which directly affects the mechanism's vibration and reliability. Therefore, optimizing the contour curve design is of utmost importance. Qiu et al. [8] employed CAD(Computer Aided Design) technology to describe the control value of the cam-optimized B-spline curve using uniform B-spline curves. They proposed a method that considered and satisfied multiple evaluation criteria. Halit et al. [9] simplified the dynamic equation of the high-speed cam follower system's behavior using the dimensionless method. By finding the optimal cam model, they successfully reduced system component vibrations. Chan et al. [10] utilized the Monte Carlo method, an exploratory search method, to generate random points within the variable range and identify feasible solutions to multi-objective problems based on Pareto optimality criteria. Nguyen et al. [11] employed the finite element method to design the cam profile based on the desired follower displacement. BL MacCarthy et al. [12] demonstrated how the standard cam law could be accurately approximated using a small number of points and appropriate boundary conditions. Gao et al. [13] utilized the least square method to fit cam curves using the Zemax programming language. Zhang et al. [14] proposed a method for analyzing and simulating the geometric characteristics of fit surfaces in globular cam mechanisms. They illustrated their approach using a globular cam mechanism in an automatic tool-changing device for CNC(Computer Numerical Control) machine tools. This paper aims to explore the design method for utilizing the cam mechanism, which leverages its unique boundary conditions to control the complex motion characteristics of the follower. The study utilizes MATLAB to investigate the cam's potential for achieving complex motion trajectories.

This paper focuses on the design of a steering mechanism based on the cam steering principle for complex trajectories. The design involves establishing the mathematical model and motion equation of the cam, followed by conducting simulation analysis using MATLAB. The performance and trajectory of the cam can be evaluated by adjusting and optimizing the design parameters in accordance with the specified geometry, motion parameters, and constraints. To validate the practical application of the complex trajectory, experiments are conducted on the designed cam steering mechanism using a carbon-free car as the test platform.

2 CAM DESIGN FOR COMPLEX TRAJECTORIES

2.1 Design principle of cam swing-rod mechanism

To control the motion along complex trajectories, the deflection of the guide wheel using the cam is essential. Each bump on the cam corresponds to a turn in the path. Figure 1 illustrates the fundamental principle of steering with disk cams. In the figure, O represents the center of curvature for the turning radius. AB denotes the pendulum rod responsible for the swing of the front wheel, while BD represents the disk cam. Position B denotes the contact position between the AB pendulum rod and the BD cam, while position D corresponds to the center of the cam base circle.

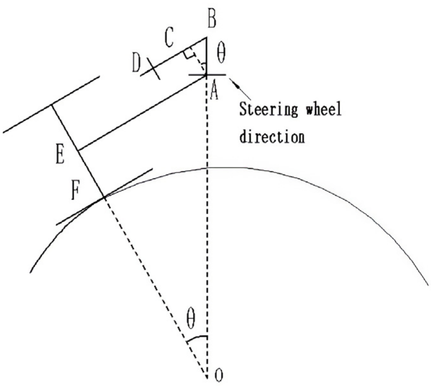

Fig. 1. General principles of steering with disk cams

In the above figure, *BC* represents the increment relative to the base circle. When the *AB* pendulum rod deflects clockwise, the front wheel deflects to the right, achieving right steering. Conversely, when the increment is negative, the *AB* pendulum rod deflects counterclockwise, causing the front wheel to deflect to the left, achieving left steering. By solving the increment of each point along the complex trajectory relative to the base circle of the cam, the complete contour of the cam can be obtained as it traverses the corresponding trajectory.

Based on the aforementioned general principle, further design of the cam profile can be conducted. By considering the operational needs of the complex trajectory, the desired curve for the trajectory is fitted. The objective is to identify the trajectory that possesses the shortest cycle, utmost stability, and highest reliability as the ideal trajectory. Typically, the fitting process involves determining the trajectory equation through mathematical analysis of the motion law and generating the trajectory graph using software. The methodology for this fitting process is elaborated in the subsequent article.

2.2 Design method of cam contour according to running trajectory

By analyzing the steering of each stage of the running track, it is required to run smoothly and smoothly. The rotation angle of the front wheel is equal to the variation arc of the running track, the path is differentiated, the running track of each stage is analyzed, and the equation is established to obtain the running track, and the appropriate relationship between cam push and rotation is designed. The geometric relationship of the running track is shown in Figure 2 below. a is the distance between the main driving wheel and the center of the front wheel, *b* is the distance between the driving wheel and the center of the front wheel, θ is the deflection angle of the guide wheel, ρ is the distance between the center of the rear wheel axis and the rotation center of the motion tracking.

To ensure a smooth and seamless operation at each stage of the running track, it is necessary to analyze the steering process. The front wheel's rotation angle corresponds to the variation in the track's arc, and the differentiation of the path allows for analysis of each stage of the running track. By establishing equations and examining the appropriate relationship between cam push and rotation, a running track can be obtained. Figure 2 below illustrates the geometric relationship of the running track, where a represents the distance between the main driving wheel and the center of the front wheel, *b* denotes the distance between the driving wheel and the center of the front wheel, θ signifies the deflection angle of the guide wheel, and ρ represents the distance between the center of the rear wheel axis and the center of rotation for motion tracking.

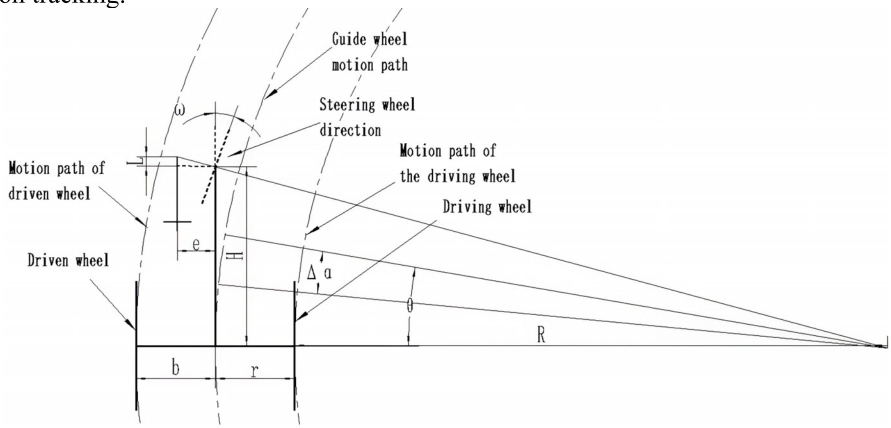

Fig. 2. Motion trajectory analysis

Based on geometric relations, the following quantities can be determined: In a unit of time *t*, the rear wheel center movement trajectory experiences a rotational angle increment of $\Delta\alpha$ and a displacement increment of ΔX. Additionally, the driven wheel movement undergoes a trajectory increment of $\Delta X1$, while the driven wheel movement experiences a trajectory increment of $\Delta X2$. When the driving wheel's rotation angle increment is $\Delta\alpha$, the rear wheel axis experiences a rotation angle increment of $\Delta\beta$, resulting in a rotation arc length of $R\Delta\beta$.

$$\tan\theta = \frac{H}{R+\mathrm{r}} \tag{1}$$

$$\Delta X = \rho * \Delta\alpha = \frac{H}{\tan\theta} * \Delta\alpha = \frac{LR\Delta\beta}{(\rho-\alpha)\tan\theta} \tag{2}$$

$$\Delta X1 = (\rho+b) * \Delta\alpha = \frac{(\rho+b)R\Delta\beta}{(\rho-\alpha)} \tag{3}$$

$$\Delta X2 = R * \Delta\beta = (\rho-a)\Delta\beta \tag{4}$$

The running trajectory is subsequently analyzed using the differential method in MATLAB software. Based on geometric analysis, equation (5) is derived.

$$\theta = \arctan\frac{H}{\rho} \tag{5}$$

For each section, the front wheel angle is obtained, recorded, and utilized in cam design. The rotation angle of the driving wheel in stage *i* is denoted as *βi*, while the rotation angle of the driving wheel in stage *i-1* is *βi-1*. Similarly, the center direction angle of the front wheel in stage *i* is represented as *γi*, and the center direction angle of the front wheel in stage *i-1* is *γi-1*. By employing the differential method and considering the running trajectory, the following equation can be derived:

$$\gamma_i = \gamma_{i-1} + \Delta\alpha \tag{6}$$

$$\Delta\sigma = \frac{R\Delta\beta}{\rho-a} \tag{7}$$

$$\beta_i = \beta_{i-1} + \vartriangle\beta \tag{8}$$

Considering the motion law, let's assign the coordinates of the *i*-th section of the carbon-free car's running track as (*xi*, *yi*), the track coordinates of the (i-1)-th section as (*xi-1*, *yi-1*), and the initial position coordinates as (*x0*, *y0*). Based on these parameters, the following relationship can be derived:

$$x_i = x_{i-1} + \frac{\mathrm{LR}\Delta\beta}{(\rho-\alpha)\ \tan\theta}\cos\gamma_{i-1} \tag{9}$$

$$y_i = y_{i-1} + \frac{\mathrm{LR}\Delta\beta}{(\rho-\alpha)\ \tan\theta}\sin\gamma_{i-1} \tag{10}$$

By considering the motion relationship between the wheels of the carbon-free car, we can obtain the relevant coordinates, which in turn allow us to determine the operational simulation trajectory of each wheel.

2.3 Realize the contour design of the cam with MATLAB

To achieve the desired curve as described above, several parameters of the front wheel angle curve need to be set. These parameters include the amplitudes (*a, b, c, d, e, f, g, h*) of the main sine curve, which determine the extent of the car's left and right turns. Additionally, the number of nodes in each section of the curve (*N1, N2,..., N13*) determines the distance covered by the car during left and right turns. The sum of the nodes multiplied by L represents the total distance traveled by the car in one cycle.

By adjusting the aforementioned parameters, we can obtain the optimal trajectory of the car. The node parameters and curve amplitudes used in the simulation are listed in Table 1 below:

Table 1. Parameters under the optimal trajectory

Node Parameters					
N1=1000	N2=5500	N3=15000	N4=1500	N5=16000	N6=5000
N7=16900	N8=1700	N9=12000	N10=12500	N11=1100	N12=10000
N13=2800	houl=0.06	N=2*(N1+N2+N3+N4+N5+N6+N7+N8+N9+N10+N11+N12+N13)			
The amplitude of each curve section					
a=0.135	b=-0.1993	c=0.175	d=-0.12010	e=-0.10300	f=0.140
g=0.022951	h=-0.18433235				

First, the angle curve of the front wheel is defined. In the initial paragraph, a sequence of angle values is created using the formula *t=pi/ (Ni): pi/ (N1): pi*, where *Ni* represents the corresponding node parameter. The corresponding angle value is then calculated using the formula *sita1= A-a*cos(t)*.

In the subsequent paragraph, a zero vector *sita2* of length *N2* is created, and a loop is used to set each element in the vector to *sita2 (i) =2*a*, representing the amplitude of the curve for this section. Similarly, in the next paragraph, a similar approach is followed to create the zero vector *sita3* of length *N3*, and the corresponding angle values are obtained using the formula *sita3=2*a+b-b*cos(t)*.

The fourth paragraph mirrors the second paragraph, creating a zero vector sita4 of length *N4*, where each element is set to *sita4 (i) =2*a+2*b*, defining the curve amplitude for this section. The remaining code follows a similar pattern, generating angle values for sita5 to sita13 using their respective angle sequences and calculation formulas.

By concatenating all these angle values in series, the complete front wheel angle curve is obtained.

An empty array, *s*, of length *N* is defined to store the lift of the cam. For each position *i*, the value of *p(i)* is evaluated. If *p(i)* is less than or equal to *0*, indicating a right turn, the lift in the right turn case is calculated as -*sita(i)* multiplied by the tangent value of *L2*. If *p(i)* is greater than *0*, indicating a left turn, the lift in the left turn case is calculated as *-sita(i)* multiplied by the tangent value of *L1*. The calculated lift is then stored in the array *s*.

Next, the push of the cam is calculated based on the cam radius, *R0*, and the lift, *s*. The push is equal to *R0* plus each element in the array *s*. An empty array, alpha, of length *N* is defined to store cam angles. For each position *i*, the angle *alpha(i)* is calculated as *2*pi*i/N*. By performing these calculations, the shape of the theoretical cam is obtained, as depicted in Figure 3.

After setting the drive ratio, the diameter of the rear wheel can be determined, which may result in a non-integer value. To obtain more concise and intuitive data, the number of nodes can be fine-tuned. By fine-tuning, it is determined that when the front wheel transmission ratio *(i)* is 24 and the rear wheel diameter is 130 mm, a relatively large transmission is achieved, and a two-stage transmission is selected. The gear module is set to *1*, with teeth *z1=z2=20*, *z3=80*, and *z4=120*.

However, the theoretical cam cannot be directly applied to the carbon-free car because the contact shaft and the theoretical contact point of the cam do not align with the actual contact point, except for the position where the cam push is the largest. To address this issue, the theoretical cam is enveloped with an ellipse, and the actual cam required is the inner envelope of the theoretical cam, as shown in Figure 4.

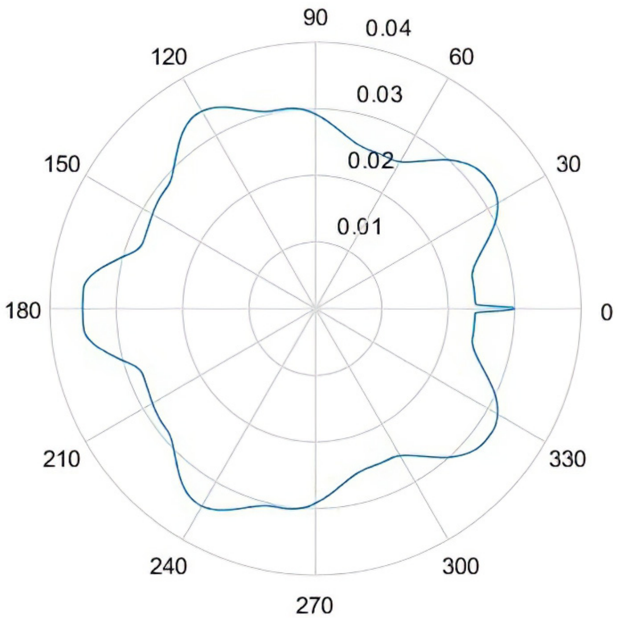

Fig. 3. Simulated cam profile

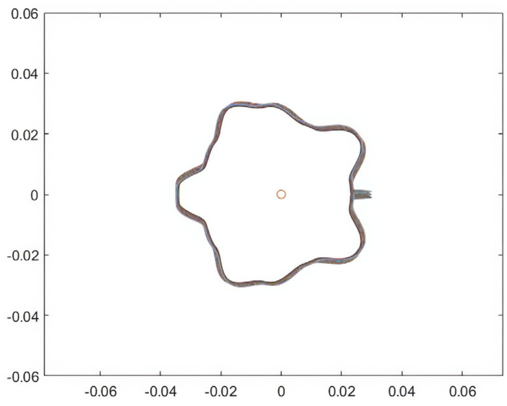

Fig. 4. The actual cam shape after the envelope

The cam is a critical component of the steering mechanism, and it must possess high accuracy to ensure trajectory correctness. Therefore, the design of the cam's structure and trajectory simulation involves the development of MATLAB programs to optimize the cam's contour and minimize abrupt changes in curvature. In the provided figure, the simulated cam exhibits local spikes, indicating the presence of abrupt contour curvatures. Consequently, corrective measures are required for the cam's contour.

Initially, the cam's contour coordinates are output as discrete points. Subsequently, using modeling software such as SolidWorks or UG, the cam trajectory is manually created by connecting the abrupt line segments with smooth curves. This procedure facilitates the generation of the desired cam model. Finally, additional processing and experimental verification can be performed.

3 EXPERIMENTAL VERIFICATION -- TAKE CARBON-FREE CARS AS AN EXAMPLE

3.1 Car Structure

To validate the aforementioned cam design method and ensure the reliable functionality of complex repetitive motions facilitated by the cam, a carbon-free car powered by gravitational potential energy has been developed. The car consists of three main components: the transmission part, the driving part, and the steering part.

In the driving part, the car's kinetic energy is entirely converted from the gravitational potential energy of a heavy weight. A plumb weight, positioned at a specific height, is released, allowing it to freely fall under the influence of gravity. A flexible rope attached to the weight alters the direction of the pulling force by passing it through a pulley mounted on the pulley seat. This, in turn, drives the rotation of the rear wheel, enabling the car to move forward.

Simultaneously, the steering part relies on the rotation of the front shaft to initiate the movement of the cam. The top column, fixed to the cam, rotates along with it. Through the connection of two joint bearings and an adjustment rod, the connecting shaft is pushed to execute a back-and-forth reciprocating motion. This motion, in turn, drives the swing of the front wheel support, which makes contact with the push plate, facilitating the steering action of the car.

The transmission part of the car features a three-wheel configuration, with the front wheel serving as the steering wheel. To address the issue of speed variation resulting from the unequal radii of the two rear wheels during the car's turning process, a single-wheel drive system is employed for the rear wheel. This involves the use of three drive shafts, a gear transmission, and a secondary transmission between the cam and the rear wheel. Specifically, when the cam completes one rotation, the car covers a distance of one week. The transmission ratios are 4 and 6, respectively, resulting in a total transmission ratio of 24. The car travels a distance of 10.7 meters in one week, and the rear wheel has a diameter of 130 mm. The structural parameters of the car are presented in Table 2.

Table 2. Car parameter

Parameter name	**Symbolic labeling**	**Specific data**
Gear ratio	i	24(4*6)
Gear module		1
The distance between the front wheel and the drive wheel axle	A/mm	90
The front wheel is offset from the right side of the drive wheel	eL/mm	94.75
Distance from front wheel to left side of cam	$L1$/mm	21.5
Distance from front wheel to right edge of cam	$L2$/mm	18.5
Cam thickness	$L0$/mm	3
Base radius	$R0$/mm	30
Rocker diameter	da/mm	2

3.2 Experimental process

To ensure structural stability of the car body, the principle of achieving the lightest overall weight was followed during the processing and production of various components such as wheels, transmission gears, cams, and drive shafts, which were made from aluminum alloy material. Additionally, parts like the car body, pulley, and support components were fabricated using ABS plastic through 3D printing. For testing purposes, a smooth and level plastic field was chosen, and the starting point was selected as the gentle position on the trajectory. Ink was applied to the front wheel to trace the actual movement trajectory. The results confirmed that the car started and moved smoothly, with an average driving speed of approximately 0.3 m/s. It was able to cover a maximum effective walking distance of nearly 33 meters before running out of energy. Figure 5 illustrates the planned travel route of the car, while Figure 6 depicts the car's operational state at a specific location.

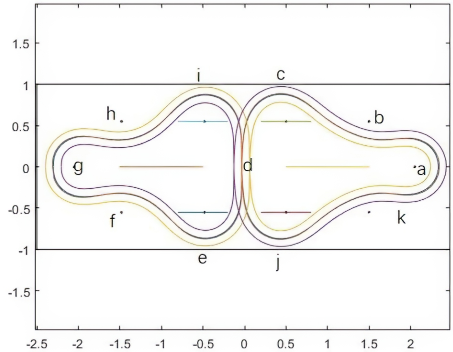

Fig. 5. The predetermined driving route of the car

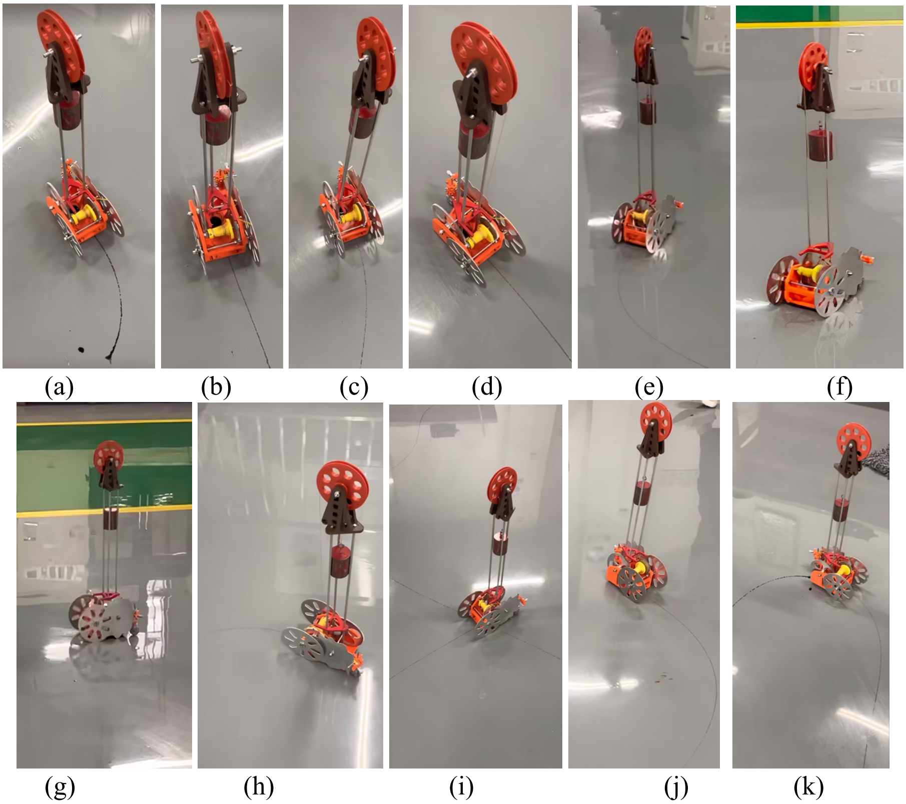

Fig. 6. The working status of the car at a specific location

4 CONCLUSION

This paper presents a design method utilizing cams to achieve complex trajectory motion and performs parametric design using MATLAB. Subsequently, the feasibility of this method is demonstrated through experiments conducted on a carbon-free car. These findings hold significant guiding implications for innovation and optimization in the design and application of cam mechanisms. The design concept proposed in this paper offers a convenient and efficient approach for cam profile design and serves as an illustrative example for the application of such cam pendulum mechanisms.

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