

Volatility Analysis of Hong Kong Stock Hang Seng Index Based on GARCH Model

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Abstract. Financial time series models are one of the commonly used methods to conduct stock market analysis and have good forecasting effects. In this paper, we use the monthly return data of Hong Kong stock Hang Seng Index as the research sample and calculate the volatility of low-frequency data based on high-frequency data returns as a basis for comparative exploration of the optimal model applicable to fitting the volatility forecasts of Hong Kong stocks to provide reference for stock investors. After the normality test, ADF test, white noise test, and ARCH effect test, it was determined that the log return series of the sample had conditional heteroskedasticity and was suitable for constructing a GARCH model. The GARCH model, EGARCH model, APARCH model, and TGARCH model are established respectively; furthermore, this paper also forecasts the future volatility scenario and calculates the estimates of actual volatility using high-frequency data, and after comparison, it is determined that the APARCH model has the best accuracy in estimating volatility for the sample data.

Keywords: Stock volatility; GARCH model; EGARCH model; APARCH model; TGARCH model

1 Introduction

Stock volatility forecasting is of great importance in stock market research. Volatility represents the magnitude of stock price changes, the greater the stock price increase or decrease, the more intense the price action back and forth, the greater the volatility, and vice versa, the smaller the volatility. Therefore, volatility can be used to measure the risk of stock investment, the same expected return conditions, the less volatility the less risk, the more valuable investment.

Hong Kong is the world's leading international financial center and a hub for offshore RMB business, handling over 70% of the world's RMB trade settlements, and has great potential for development in RMB investment and risk management products. The reform and opening up of the mainland and the reunification of Hong Kong have led to closer financial interactions between the mainland and Hong Kong, leaving investors with more and more investment options, of which the Hang Seng Index of the

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Hong Kong stock market is a representative investment research object. The study of stock prices and volatility has received increasing attention as it helps people predict trends and make decisions.

A common method used to perform stock volatility forecasting is to explore the ARCH effect on the data, so modeling using GARCH-like models is a common research approach.

Yingchao Zhang et al. studied the SSE index based on ARIMA model to further predict the future SSE index situation and obtained better prediction results. Using ARMA-GARCH family models with different hypothetical distributions, Tu Li-Ming et al. analyzed the volatility of the CSI 500 index and found that the ARMA(1,1)-GARCH(1,1) model based on the t distribution could better describe the volatility clustering phenomenon of the return series; the optimal model was the ARMA(1,1)-EGARCH(1,1) model based on the GED distribution. In the process of analyzing stock prices, Qi Yang et al. first fitted the ARMA model to the time series, and then eliminated the conditional heteroskedasticity by adding the GARCH term to finally obtain the ARMA-GARCH fitted model. Shuya Xu et al. used a time series model to predict the closing price of Yutong Bus stock, and after comparing ARMA, ARIMA and ARIMA-GARCH models, the ARIMA-GARCH model was chosen for the next step. Fancheng Yin et al. used GARCH-M model to simulate the trend of volatility and make predictions, and their results were simulated to obtain a more realistic stock index. Some empirical studies have found that since there is often an asymmetric effect of positive and negative information on stock information in the financial field, this asymmetry is difficult to be described by a pure GARCH model. Based on this, in addition to considering the effect of the general GARCH model on volatility analysis, this paper further considers its effect on volatility using asymmetric models, such as EGARCH model, APARCH model, and TGARCH model, to compensate for market situations that cannot be described by the GARCH model.

In many cases, the log returns of stock indices exhibit the characteristics of a white noise series, which is consistent with the theory of efficient market hypothesis, and therefore stock forecasting using ARMA models is inefficient. And the difficulty for GARCH-like models is that the actual volatility is not available. Based on this, this paper refers to the processing of high-frequency data and calculates the variance of daily stock price return data in a month to obtain the estimated value of the true volatility of monthly data so that comparisons between models can be made.

2 Model Review and Data Selection

2.1 Model Overview

2.1.1 ARMA Model.

If a sequence is identified as a stationary non-white noise sequence after preprocessing, it indicates that the sequence is a stationary sequence containing relevant information. A more refined and accurate algorithm for analysis and forecasting of time series data is the Box-Jenkins (Box-Jenkins) method [1], whose common models include: autoregressive model (AR model), sliding average model (MA model), mixed

autoregressive-sliding average model (ARMA model), and differentially integrated moving average autoregressive model (ARIMA model). In ARMA(p,q), AR is "autoregressive" and p is the number of autoregressive terms; MA is "sliding average" and q is the number of sliding average terms.

The ARMA model can be expressed as follows:

$$\begin{cases} Y_{t} = \varphi_{0} + \varphi_{1}Y_{t-1} + \varphi_{2}Y_{t-2} + \dots + \varphi_{p}Y_{t-p} + e_{t} - \theta_{1}e_{t-1} - \dots - \theta_{q}e_{t-q} \\ \varphi_{p} \neq 0, \theta_{q} \neq 0 \\ E(e_{t}) = 0, Var(e_{t}) = \sigma_{e}^{2}, E(e_{t}e_{s}) = 0, s \neq t \\ E(Y_{s}e_{t}) = 0, \forall s < t \end{cases}$$

2.1.2 ARCH Model.

The ARCH model is an autoregressive conditional heteroskedasticity model for the variance of time series changes first proposed by Engle [2], formally a regression model with the dependent variable being the conditional volatility and the covariate being the squared value of prior period returns. The ARCH model can effectively describe the stock market volatility situation and has attracted a lot of attention and research from scholars after it was proposed.

The conditional mean equation and conditional variance equation described by ARCH (q) process are :

$$\begin{cases} y_t = \sigma_t \varepsilon_t \\ \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \end{cases}$$

where ε_t is independently and identically distributed and is usually assumed to obey a normal, t-distribution; σ_t^2 is the conditional variance of ε_t when it is smooth, $\omega > 0$, $\alpha_i > 0$ and $\sum_{i=1}^{q} \alpha_i < 1$. The parameter α_i denotes the coefficient of the effect of external shocks.

2.1.3 GARCH model.

Although ARCH models can describe some properties of financial time series, in practice, larger autoregressive periods are often required to estimate time series using ARCH models. An extension of ARCH models is the generalized autoregressive heteroskedasticity (GARCH) model, proposed by Bollerslev [3], which addresses this shortcoming well.

The GARCH(p,q) process describes the conditional variance equation as:

$$\begin{cases} y_t - \sigma_t \varepsilon_t \\ \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \end{cases}$$

where σ_t^2 is the conditional variance of ε_t when it is smooth, $\omega > 0$, $\alpha_i > 0$, $\beta_j \ge 0$, and $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$. The parameter α_i denotes the impact coefficient of external shocks, and the parameter β_j denotes the long-term memory of external shocks, and the sum of the two expresses the persistence of volatility. On the surface of a large number of actual data, GARCH (1,1) model can well describe the volatility characteristics of financial time series.

2.1.4 EGARCH Model.

For the situation of asymmetry of positive and negative information, such as profit and loss, which often occurs in the financial field, improved GARCH-like models have been proposed to solve the problem, one of the common model of which is the EGARCH(p,q) model proposed by Nelson [4], which effectively responds to the impact of the situation of positive and negative information asymmetry on volatility.

$$\begin{cases} y_t = \sigma_t \varepsilon_t \\ ln \ \sigma_t^2 = \lambda_0 + \sum_{j=1}^p \eta_j ln \sigma_{t-j} + \sum_{i=1}^q \lambda_i g(\varepsilon_{t-i}) \\ g(\varepsilon_t) = \theta \varepsilon_t + \gamma[|\varepsilon_t| - E|\varepsilon_t|] \end{cases}$$

where ε_t is independently and identically distributed, $E|\varepsilon_t| = \sqrt{\frac{2}{\pi}}$, and usually taken as $\gamma = 1$.

2.1.5 APARCH Model.

ARCH and GARCH models can construct unconditional distributions with fat-tailedness, but they cannot eliminate the fat-tailedness of the conditional distribution, that is, the deviation of the disturbance terms from the normal distribution, and it is difficult to describe the leverage effect of shocks accurately. In this regard, the improvement of the conditional mean equation and conditional variance equation becomes an optimization direction for the GARCH model. Ding, Z., C. W. J. Granger and R. F. Engle proposed the asymmetric power APARCH(p,q) model [5].

The model can be written as follows:

$$\begin{cases} y_t = \sigma_t \varepsilon_t \\ \sigma_t^{\delta} = \omega + \sum_{j=1}^q \beta_j \sigma_{t-j}^{\delta} + \sum_{i=1}^p \alpha_i (|y_{t-i}| + \gamma_i y_{t-i})^{\delta} \end{cases}$$

where δ is a positive real number, $\varepsilon_t \sim IID(0,1)$, and the coefficients ω , α_i , γ_i and β_i satisfy certain regularity conditions such that the volatility is positive.

2.1.6 TGARCH Model.

In addition to the EGARCH model, the TGARCH model (or GJR-GARCH model) is another volatility model that captures the leverage effect and was proposed by Glosten,Jagannathan and Runkle [6].

The TGARCH(p,q) process describes the conditional variance equation as:

$$\begin{cases} y_{t} = \sigma_{t}\varepsilon_{t} \\ \sigma_{t}^{2} = \omega + \sum_{i=1}^{q} \alpha y_{t-i}^{2} + \sum_{j=1}^{p} \beta_{j}\sigma_{t-i}^{2} + \sum_{k=1}^{q} \gamma_{k}y_{t-k}^{2}d_{t-k} \end{cases}$$

where $d_t = \begin{cases} 1, \ \varepsilon_{t-k} < 0 \\ 0, \ \varepsilon_{t-k} \ge 0 \end{cases}$, $\omega > 0$, $\alpha_i > 0$, $\beta_j \ge 0$, and d_t is a dummy variable. When bad news appears, $\varepsilon_{t-k} < 0$ and $d_t = 1$; conversely, good news appears, $\varepsilon_{t-k} \ge 0$ and $d_t = 0$. When $\gamma_k > 0$ and significant, there is an asymmetric effect and the shock from good news is larger than that from bad news.

2.2 Data Selection

This paper examines the volatility of listed stocks in the Hong Kong stock market based on monthly closing index data of the Hong Kong Hang Seng Index (HSI) from January 1987 to April 2021, and daily closing index data of the Hong Kong Hang Seng Index (HSI) from January 2, 1987 to April 30, 2021.

Since the monthly data of financial data are heavy-tailed, while the daily data are extremely heavy-tailed. Some studies show that in the case of extreme fat-tailedness, the parameter estimates of many models tend to be less satisfactory, compared to monthly data, which are less heavy-tailed and can avoid the effect of fat-tailedness to some extent. Based on this, this paper selects the monthly closing index data of Hong Kong Hang Seng Index (HSI) for analysis.

At the same time, this paper selects daily closing index data that are in the same time period as the monthly closing index data, from January 1987 to April 2021, and calculates the variance of daily returns of all trading days in each month with reference to the processing of high frequency data to obtain the actual volatility estimates for that month. This treatment can provide judgment criteria for comparing the accuracy of volatility estimated by different model fits.

3 Experimental Steps

(1) Transformation of stock price data into log-return data using the formula $lnX_t - lnX_{t-1}$;

(2) Performing a normality test on the log-return data;

(3) Performing a stationarity test on the log-return data using the ADF test;

- (4) White noise test for autocorrelation of log-return data;
- (5) Conducting ARCH effect tests to determine whether they are heteroskedastic;

(6) Establishing GARCH, EGARCH, APARCH and TGARCH models, respectively;

(7) Perform model comparison based on the obtained results to determine the optimal model for estimating the volatility of this series.

4 Data Analysis and Research

4.1 4.1. Analysis of the Basic Statistical Characteristics of the Hang Seng Index Returns

A. Data processing

The monthly increase and decrease data of Hang Seng Index from January 1987 to April 2021 are selected and the logarithmic return series is obtained by using equation $lnX_t - lnX_{t-1}$.

B. Normality test

The next step examines whether the log return data conforms to the normality distribution, for which this paper directly observes the image situation of the QQ plot of the data(figure 1).



Fig. 1. Log-return series QQ chart

According to the results, the images pass from bottom to top via a normal straight line, which is not normally distributed, and the data show a heavy-tailed distribution.

C.Stationary Test

In order to avoid the phenomenon of "pseudo-regression", a stationary test should be performed before model estimation. The ADF test (Augmented Dickey-Fuller test) is used to determine whether there is a unit root in the time series at a set significance level.

Table 1. ADF Test Results

Lag	P-value
1	0.01
2	0.01
3	0.01
4	0.01
5	0.01

The ADF test was performed on the log-return series and the original hypothesis of the ADF test was the existence of a unit root in the data, as can be seen from the table 1, for different lags, P < 0.05, the original hypothesis was rejected and the time series was a stationary time series.

D.White Noise Test

Next, a white noise test is performed on the log-return data to determine whether the data need to be analyzed by testing the ARMA model. In this paper, the Box-Pierce test and the Ljung-Box test are selected to analyze the data.

Lag	Box-Pierce Test P-value	Ljung-Box Test P-value
1	0.8162	0.8155
2	0.9535	0.9531
3	0.8093	0.8065
4	0.6729	0.6671
5	0.518	0.5091
6	0.6385	0.6298
7	0.3109	0.2981
8	0.3974	0.383
9	0.4704	0.4548
10	0.2697	0.2526

Table 2. Box-Pierce test and Ljung-Box test results

Taking the logarithm of the original return series for differencing, and using Box-Pierce test and Ljung-Box test on the new series respectively, see table 2, the results were greater than 0.05, indicating that the series is a white noise series and does not require an autoregressive model test on the mean.

E.ARCH Effect Test

Next, this paper uses the LM test to determine whether the series has conditional heteroskedasticity, which enables the construction of a GARCH-like model to be judged.

Lag	P-value
1	0.8309
5	0.9523
10	0.7783
15	0.0763
20	0.01945
21	0.003551
22	0.003819

Table 3. LM Test Results

It can be seen from the results in the table 3 that when lag ≥ 20 , the p value is less than 0.05, which indicates that the data has ARCH effect.

4.2 Construction and Testing of GARCH-like Models

4.2.1. GARCH Model Construction and Testing.

The model suffers from heteroskedasticity, and the GARCH(1,1) model is chosen here.

The GARCH(1,1) model was fitted to the stock data using the GARCH(1,1) model to estimate the model parameters and the results are presented in the table 4.

	estimate	Std.error	T-value	P-value
Mu	0.009831	0.004281	2.2966	0.021641
omega	0.000334	0.000151	2.2072	0.027299
Alpha1	0.245530	0.064061	3.8327	0.000127
Beta1	0.734461	0.057772	12.7131	0.000000

Table 4. GARCH Model Fitting Parameter Results

The GARCH(1,1) model can be obtained from the fitted parameters: $\sigma_t^2 = 0.000334 + 0.245530\varepsilon_{t-1}^2 + 0.734461\sigma_{t-1}^2$

4.2.2. EGARCH Model Construction and Testing.

A characteristic of financial time series is that investors react more strongly to negative news than to positive news, i.e., the response of returns to positive and negative perturbations may be asymmetric, which is not reflected in the GARCH model. Models that can describe the asymmetric effect include the TGARCH model and the EGARCH model. The existence of asymmetric effects in return volatility is first tested by constructing an EGARCH(1,1) model.

The EGARCH(1,1) model is used to fit the stock data and estimate the model parameters, and the results are presented in the table 5.

	estimate	Std.error	T-value	P-value
Mu	0.011017	0.003259	3.3806	0.000723
omega	-0.290981	0.145157	-2.0046	0.045006
Alpha1	0.111037	0.037722	2.9436	0.003244
Beta1	0.940581	0.027375	34.3597	0.000000
Gammar1	0.358357	0.079072	4.5320	0.000006

Table 5. EGARCH Results of Model Fitting Parameters

From the tabular results, it is clear that there is a significant asymmetric effect of return volatility.

The EGARCH(1,1) model can be obtained from the fitted parameters:

$$ln \sigma_t^2 = -0.290981 + [0.111037\varepsilon_{t-1} + 0.358357(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|)] + 0.940581ln \sigma_{t-1}^2$$

4.2.3. APARCH Model Construction and Testing.

Using the APARCH model, introducing skewness coefficients in the symmetric distribution to portray sample skewness has a significant impact on model fitting and forecasting, especially forward multi-step forecasting, which is important for portraying typical facts of financial markets, accurate asset pricing, risk measurement and management.

The APARCH(1,1) model is used to fit the stock data and estimate the model parameters, and the results are presented in the table 6.

	estimate	Std.error	T-value	P-value	
Mu	0.007612	0.003077	2.4739	0.013367	
omega	0.000001	0.000003	0.4247	0.671057	
Alpha1	0.022024	0.009224	2.3876	0.016960	
Beta1	0.903898	0.026115	34.6125	0.000000	
Gammar1	-0.476687	0.171352	-2.7819	0.005404	
delta	3.500000	0.285711	12.2502	0.000000	

Table 6. APGARCH Results of Model Fitting Parameters

The APARCH(1,1) model can be obtained from the fitted parameters:

 $\sigma_t^{3.5} = 0.000001 + 0.022024(|y_{t-1}| + 0.476687y_{t-1})^{3.5} + 0.903898\sigma_{t-1}^{3.5}$

4.2.4. TGARCH Model Construction and Testing.

After the analysis using the EGARCH model, this paper continue using another model capable of describing asymmetric effects, the TGARCH model processing the data.

The TGARCH(1,1) model was used to fit the stock data and estimate the model parameters, and the results are presented in the table 7.

	estimate	Std.error	T-value	P-value
Mu	0.009204	0.003175	2.8991	0.003742
omega	0.000224	0.000119	1.8815	0.059898
Alpha1	0.313105	0.089260	3.5078	0.000452
Beta1	0.798521	0.056692	14.0854	0.000000
Gammar1	-0.225251	0.075004	-3.0032	0.002672

Table 7. TGARCH Results of Model Fitting Parameters

The TGARCH(1,1) model can be obtained from the fitted parameters:

 $\sigma_t^2 = 0.000224 + (0.313105 - 0.225251d_{t-1})y_{t-1}^2 + 0.798521\sigma_{t-1}^2$

4.3 Accuracy comparison

In this paper, we refer to French's method [7] and consider using high-frequency returns to calculate the volatility of low-frequency returns as follows: by calculating the variance of daily returns of all trading days within each month, we obtain the actual volatility estimate for that month, and based on this actual volatility estimate, we compare the mean squared and error (MSE) of the volatility estimated by different models with the actual volatility estimate, and the model with the best predicted volatility is selected[8-14].

The daily gain and loss data of the Hang Seng Index from January 2, 1987 to April 30, 2021 are selected, and the variance of daily returns of all trading days in each month is calculated to obtain an estimate of the true monthly volatility, which is compared with the volatility under each GARCH-like model fit in the same image, as shown in the figure below(figure 2).



Fig. 2. Comparison of the actual volatility estimated based on daily data with the volatility fitted by each model

As can be seen from the images, the fit of each GARCH class model is good. The volatility forecasts from May 2021 to October 2021 were performed using the established GARCH, EGARCH, APARCH, and TGARCH models.

Their forecasts are shown below(table 8):

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	2021.5	2021.6	2021.7	2021.8	2021.9	2021.10
GARCH	0.04833004	0.05121678	0.05389596	0.05639820	0.05874705	0.06096117
EGARCH	0.05025794	0.05190290	0.05349922	0.05504547	0.05654061	0.05798396
APARCH	0.04938039	0.05005250	0.05070214	0.05133099	0.05194055	0.05253214
ГGARCH	0.05023119	0.05238750	0.05445648	0.05644771	0.05836917	0.06022752

Table 8. Predicted volatility for each GARCH-like model

The GARCH model, EGARCH model, APARCH model, and TGARCH model predicted volatility values for the period from May 2021 to October 2021 are compared with the volatility estimates calculated based on daily returns. The mean squared and error (MSE) of the predicted volatility of each model and the volatility estimates calculated based on daily returns are taken separately and the results are presented in the table 9 below.

 Table 9. MSE of predicted volatility for each GARCH-like model vs. calculated volatility for HF

 data

	MSE
GARCH	0.002617478
EGARCH	0.002534511
APARCH	0.002217171
TGARCH	0.002655271

The smallest error is obtained for the APARCH model with an error of 0.002217171, which shows that for the prediction effect, the APARCH model has the best effect in predicting volatility.

5 Conclusion

Using a GARCH-like model, this paper analyzes and investigates the volatility of the log returns of the Hang Seng Index of the Hong Kong stock market using the monthly returns as a sample and draws the following conclusions:

First, the volatility of the Hang Seng Index returns of the Hong Kong stock market is more obvious, as shown by the QQ plot of the data, which shows that the log return series does not obey a normal distribution and has a thick-tailed nature.

Secondly, it is tested that the log return data obtained after processing the original sample data is a stationary white noise series, so the autoregressive model test on the data is not considered. And then, after ARCH effect test, it is known to have heteroskedasticity, which is suitable for constructing GARCH model for analysis.

Further, this paper uses the EGARCH model, APARCH model, and TGARCH model to compensate for market situations that cannot be described by the GARCH model, including situations based on the frequent positive and negative information asymmetry of stock market return volatility, and situations with asymmetric reflection of the leverage effect of shocks.

Finally, by comparing the mean square and error of the estimated volatility of each model and the actual volatility estimates, the model with the best predicted volatility is selected in this paper as the APARCH model.

For Hong Kong stock research, the method can be used for daily life applications, but more tests are needed such as studying the impact of different factors on stock prices for sophisticated investors.

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