The Conversion of Anisotropic Materials from 3D to 2D in Limit Equilibrium

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Abstract. The consideration of anisotropic strength in limit equilibrium has become popular among practitioners in the field of slope stability. Rock masses exhibit failure planes in various directions which cannot be ignored in the analysis. The consideration of anisotropy in limit equilibrium by varying the shear strength directionally has been accomplished in slope stability software such as Slide2 and Slide3. The typical approach compares the angle of the sliding surface to the angle of the anisotropic plane, and depending on the proximity, determines whether the joint strength, bedding strength, or some interpolation of the two, is utilized. Although analysis in 3D is recommended, the computation of factor of safety over a series of simplified 2D sections is commonly adopted as an additional verification step. The conversion of a 3D anisotropic strength model, which can be obtained based on field data, into an equivalent 2D representation can be confusing because the section analysis in 2D may not always be aligned with the direction of anisotropy in 3D. In this paper, a recommendation is made to consider the 3D anisotropy fully in the 2D analysis. An alternative method is provided to convert 3D anisotropy into an approximated 2D model. The numerical results of an example with the proposed methods are compared.

1 Introduction

1.1 Anisotropy in Limit Equilibrium

In limit equilibrium (LE), the shape of a slip surface is varied to determine the critical slip surface corresponding to the lowest factor of safety (F). In the presence of anisotropic materials, especially those of rock masses with joints, the shear strength depends on the direction of the slip surface. If the direction of the slip surface is aligned with a joint plane, then the material strength should be reduced. Otherwise, the full bedding strength is assumed. A variety of methods have been adopted by practitioners to model the anisotropic strength [1–3]. A slip surface can have some sections aligned with the joints and other sections that are not aligned.

There are other reasons why practitioners may specify anisotropic material properties in a model, such as when faults are present. In any case, a good optimization routine will find the shape of the slip surface through the slope which minimizes the relative shear strength against the driving forces.

R. E. Hammah et al. (Eds.): RIC 2023, AHE 19, pp. 247–256, 2023.
https://doi.org/10.2991/978-94-6463-258-3_26

1.2 3D Anisotropy in 2D

Traditionally, slope stability has predominantly been performed via simplification of slopes into 2D sections, which corresponds to the plane strain assumption. Although the analysis of a 2D problem in LE is relatively fast compared to 3D LE, in the case where the plane of anisotropy is not aligned with the direction of 2D analysis, it can be difficult for practitioners to simulate the effects of 3D anisotropy in a 2D model. If the direction of plane strain is relatively aligned with the plane of anisotropy, then the analysis in 2D can assume the same angles of anisotropy. However, when the plane of anisotropy is not aligned with the direction of plane strain, there are situations where a slippage forced in the direction of the 2D section should not exhibit any reduction in strength due to anisotropic behavior at all.

In this paper, a recommendation is made for fully considering the 3D anisotropy in a 2D analysis. A method for converting 3D anisotropic models into approximated 2D anisotropic models is also presented.

2 Anisotropy in Limit Equilibrium

The specification of anisotropic shear strength in LE requires sets of material parameters corresponding to the joint direction(s), and a separate set of material parameters corresponding to the bedding strength. More generally, if the plane of the assumed slip surface at a particular slice or column in the LE discretization is closely aligned with a joint then the shear strength of that joint is adopted. Otherwise, a base material corresponding to all other directions is assumed. It is possible for multiple joints to exist; in which case one might wish to only consider the nearest-aligned joint, or take the minimum strength obtained by considering the alignment to multiple joints.

The joint and base material parameters can correspond to any material model (e.g. Mohr-Coulomb, Generalized Hoek-Brown, etc.). An example of a Generalized Anisotropic material model in Slide3 is shown in Fig. 1.

2.1 Tolerance Method

As the LE method requires searching of the critical slip surface shape, and thus altering the orientations of the slices or columns in the discretization, tolerance angles typically

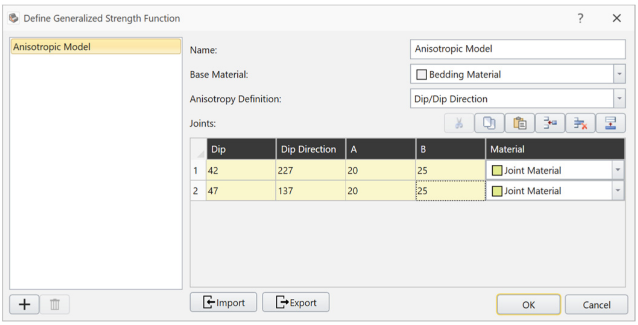

Fig. 1. An example of an anisotropic model in 3D

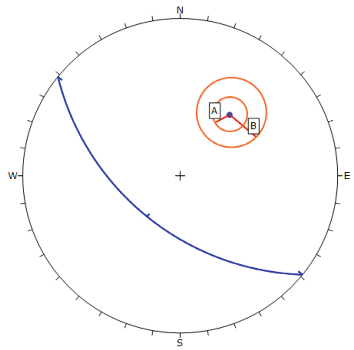

Fig. 2. Visualization of A and B parameters on a stereonet [4]

selected by practitioners to help the searching algorithm identify the weakness along the planes of anisotropy. If the orientation of the slice or column in a slipping mass above the sliding surface is satisfactorily aligned with a joint within the tolerance, the joint material strength is considered.

Define ϑ as the angle between the joint plane and the shearing plane, and tolerance angles are A and B. The material strength along the shearing plane is defined as follows:

- If $\vartheta \leq A$, then the joint material is assumed.
- If $A < \vartheta < B$, then an interpolation of the shear strength between those obtained via the joint material and base materials is assumed.
- If $\vartheta \geq B$, then the base material is assumed.

Note that the interpolation between the joint material and base material is assumed to be linearly varying, transitioning from fully adopting the joint material at A to fully adopting the base material at B. This interpolation scheme has become popular in practice and various iterations of it have been developed over the past few decades [2, 3].

Visually, the anisotropic model can be representing on a stereonet, shown in Fig. 2. The blue curve corresponds to the plane of a joint. ϑ can be obtained by simply taking the angle between the pole vectors of the shear and joint planes. As such, the circular boundaries A and B represent the range of pole vectors for the shear planes corresponding to their respective assumptions.

2.2 Anisotropy in 2D

For the plane strain analysis in 2D LE, the same tolerance method with angles A and B is used except that ϑ is the angle between the slip surface and the joint in the plane of the 2D analysis. As such, the joint plane is effectively assumed to be extruded orthogonally (out-of-plane) with respect to the 2D section.

2.3 Anisotropic Surfaces

The anisotropic model can be extended to apply towards anisotropic surfaces, whereby the joint direction is spatially varying. As an example, folded beddings such as the one illustrated in Fig. 3 require specification of anisotropic surfaces.

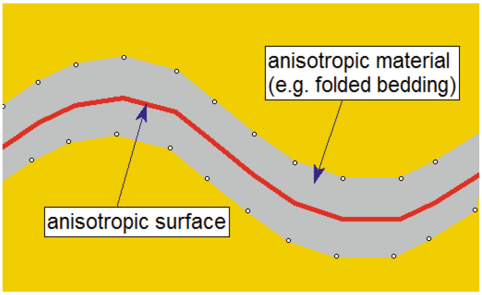

Fig. 3. An anisotropic surface representing folded bedding [4]

3 Conversion of 3D Anisotropy to 2D

The representation of 3D anisotropy in 2D space is understandably confusing and difficult to visualize for many practitioners. In this section, a method is presented which converts a 3D anisotropic model into an approximated 2D anisotropic model, complete with values for the apparent joint angle and effective values of A and B. A recommendation is also made where possible to simply consider the 3D orientation of the joint with respect to the 2D section when calculating the offset angle between the joint and the shear plane.

3.1 Equivalent 2D Values of A' and B'

Consider the following simple extruded slope where a single joint is represented by a 3D plane, inclined by some angle in the direction of the extrusion (Fig. 4).

The intersection of the joint plane with the 2D section produces the apparent direction of anisotropy in the 2D section. Recall that in a 2D LE analysis, the shear plane is assumed to be aligned with the direction of the 2D section (i.e., extruded in the orthogonal direction to the 2D section). This means that using the apparent dip of the anisotropic surface for this 2D LE analysis, as is traditionally done, would mean that the 2D analysis is considering an anisotropic orientation with an angle of 0° despite that the joint plane has an out-of-plane inclination.

The proposed method of considering anisotropy is explained as follows. Let the normal (pole) vector to the shear plane be $\boldsymbol{n_2}$, with magnitude 1. As shown in Fig. 5a,

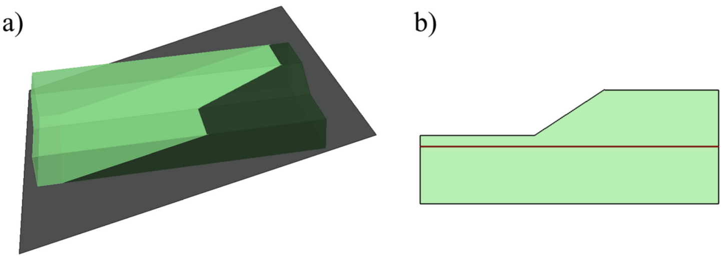

Fig. 4. Extruded slope with joint plane dipping towards the extruded direction (a) shown in 3D, and (b) its 2D section perpendicular to the extrusion

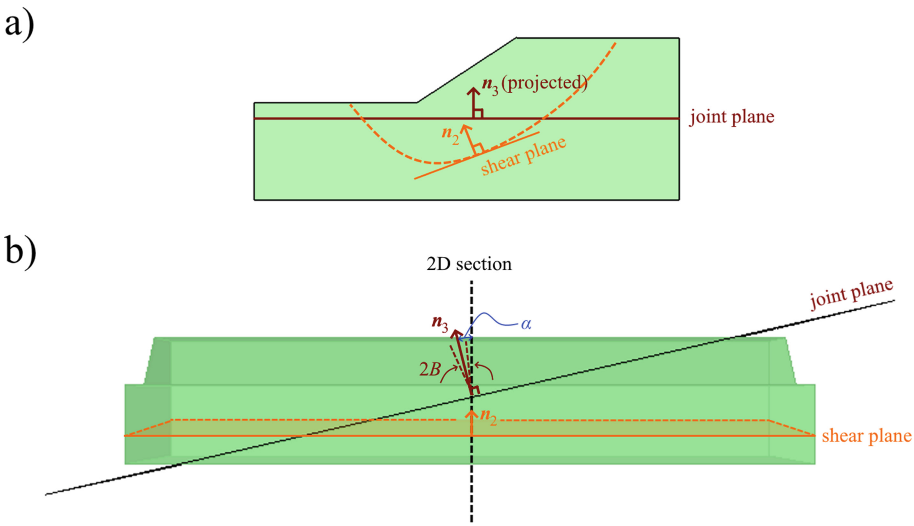

Fig. 5. A visualization of (a) the 2D section with pole vectors and a slip surface, and (b) the 3D model with pole vectors.

the orientation of $\boldsymbol{n_2}$ depends on the local inclination of the slip surface, but when viewed in 3D is limited within the plane of the 2D section (Fig. 5b).

Let $\boldsymbol{n_3}$ be the normal (pole) vector of the joint plane in 3D, with magnitude 1, and let α be the angle between $\boldsymbol{n_3}$ and the 2D section. Recall that ϑ is the angle between $\boldsymbol{n_2}$ and $\boldsymbol{n_3}$. Since $\boldsymbol{n_3}$ is not aligned with the 2D section, ϑ cannot possibly be zero in this example regardless of the orientation of the shear plane $\boldsymbol{n_2}$, which is restricted in the plane of the 2D section. The proposed method uses this value of ϑ, rather than the projected ϑ on the 2D section, to calculate the strength of the material. In essence, this means that the information about the orientation of the 3D plane is being considered in the 2D analysis, rather than its projection.

If the value of B is small enough (as illustrated in Fig. 5b), it is impossible for $\vartheta \leq B$ and as such the bedding material should apply for the entire slip surface.

In fact, the minimum possible value of ϑ is obtained when $\boldsymbol{n_2}$ points in the same direction as the projection of $\boldsymbol{n_3}$ onto the 2D section. This projection is always perpendicular to the intersection line between the 3D joint plane and the 2D section. Therefore, the intersection represents the closest possible orientation of the 2D shear plane relative to the actual joint plane in 3D. In other words, if the slip surface of the 2D analysis is parallel to the line formed by the intersection of the 2D section with the 3D joint plane, then ϑ is minimized but is not necessarily zero.

By extension, effective values of A and B in the 2D sectional analysis relating to the true 3D orientation of anisotropy can be derived. Let ϑ_2 be the offset angle between $\boldsymbol{n_2}$ and the projection of $\boldsymbol{n_3}$ onto the 2D section, $\mathrm{proj}_2\boldsymbol{n_3}$. In physical terms, ϑ_2 is simply the offset between the slip surface inclination angle and the joint angle at some location, within the plane of the 2D analysis. The joint angle can vary depending on location along

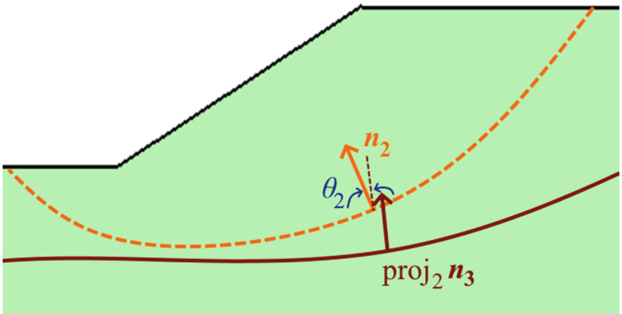

Fig. 6. Definition of the 2D offset angle ϑ_2.

the slip surface (i.e. where an anisotropic surface with spatially-varying inclination is present) (Fig. 6).

Define effective angles A' and B' to be the values of the offset angle ϑ_2 corresponding to $\vartheta = A$ and $\vartheta = B$. That is, for the 2D analysis,

- If $\vartheta_2 \leq A'$, then the joint material is assumed.
- If $A' < \vartheta_2 < B'$, then an interpolation of the shear strength between those obtained via the joint material and base materials is assumed.
- If $\vartheta_2 \geq B'$, then the base material is assumed.

To derive these values, the geometrical relationships in Eq. (1) and Eq. (2) are noted:

$$\cos\vartheta = \boldsymbol{n_2} \cdot \boldsymbol{n_3} \tag{1}$$

$$\boldsymbol{n_2} \cdot \mathrm{proj}_2\boldsymbol{n_3} = \left\|\mathrm{proj}_2\boldsymbol{n_3}\right\| \ \cos\vartheta_2 \tag{2}$$

$$\mathrm{proj}_2\boldsymbol{n_3} = \boldsymbol{n_3} - \boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{n_3}) \tag{3}$$

where $\|\cdot\|$ denotes the magnitude of the vector in the argument, and $\boldsymbol{v}$ is a vector perpendicular to the 2D section. The following relations are also recognized to aid with the solution:

$$\boldsymbol{n_2} \cdot \boldsymbol{v} = 0 \tag{4}$$

$$\left\|\mathrm{proj}_2\boldsymbol{n_3}\right\| = \cos\alpha \tag{5}$$

By expanding Eq. (2) and combining the result with Eq. (1), the following solution in Eq. (3) can be obtained.

$$\cos\vartheta_2 = \cos\vartheta \ \left\|\mathrm{proj}_2\boldsymbol{n_3}\right\| \tag{6}$$

Therefore, for the scenarios $\vartheta = A$ and $\vartheta = B$, the corresponding values of $\vartheta_2 = A'$ and $\vartheta_2 = B'$ are given in Eq. (7a and 7b).

$$A' = cos^{-1}\left(\frac{\mathrm{cos}A}{\mathrm{cos}\alpha}\right); 0 \leq \frac{\mathrm{cos}\mathrm{A}}{cos\alpha} \leq 1 \tag{7a}$$

$$B' = cos^{-1}\left(\frac{\cos B}{\cos\alpha}\right); 0 \leq \frac{\cos B}{\cos\alpha} \leq 1 \tag{7b}$$

If the term in the restriction in Eq. (7b) is outside of the range [0,1], then it is not possible for ϑ to be less than B, and the base material should be adopted. This should be taken into consideration when comparing 3D results to 2D results. If the 3D A and B are extremely narrow, or very far from the 2D plane, or both, then the 2D analysis will automatically take the base material. For best results comparison, looser 3D values of A and B should be used.

If the term in the restriction in Eq. (7a) is outside of the range [0,1], then it is not possible for the slip surface to align at an angle which causes the joint material to govern fully. In such a case, even if the slip surface is aligned with the projection of the anisotropic plane onto the 2D section ($\vartheta_2 = 0°$), an interpolation between the joint and base material is required, as follows:

- If $\vartheta_2 \geq B'$, then the base material is used.
- If $\vartheta_2 = 0°$, then the shear strength τ is taken as per Eq. (8).
- If $0° \leq \vartheta_2 \leq B'$ then an interpolation between the above two values is taken.

$$\tau = \tau_{joint} + \frac{\alpha - A}{B - A}\left(\tau_{bedding} - \tau_{joint}\right) \tag{8}$$

An equivalent model to use in the case where A' in Eq. (7a) does not satisfy the restriction would be to set $A' = 0°$ and the shear strength corresponding to $\vartheta = A'$ (that is, an effective joint strength) to the value obtained from Eq. (8).

Note that the proposed method is approximation of 3D anisotropy in 2D. It is approximated because mapping of strength from ϑ_2 between A' and B' does not scale in the same manner as the mapping of ϑ between A and B. If the interpolation in the transition zones between A' and B' (and A and B) are linear, then there will be some error induced in the approximated model. For example, when ϑ_2 is halfway between A' and B', the true value of ϑ may not necessarily be halfway between A and B. Further research is recommended in this regard to develop a more accurate mapping relation between 2D and 3D.

3.2 Fully Considering the 3D Anisotropy

Finally, where the information of the 3D anisotropy can be carried into a 2D analysis with respect to the section, it is always best to compute the true value of ϑ against the 3D orientation of the joint(s). Programs such as Slide2 and Slide3 are beginning to implement this functionality to prevent the need for conversion. The value of ϑ can be evaluated by inversing the cosine on Eq. (1) via Eq. (9).

$$\vartheta = \cos^{-1}(\boldsymbol{n_2} \cdot \boldsymbol{n_3}) \tag{9}$$

Ensure that the vectors in the dot product are normalized (i.e. scaled such that their magnitude equals unity).

4 Numerical Example

Consider the simple extruded slope shown in Fig. 7. The plane of anisotropy is at a 20° angle in the direction of the slope extrusion. The material properties of the base and joint materials are listed in Table 1. The values of A and B are 20° and 30°. The Spencer FS in 3D is 0.69. The bottom of the sliding surface is somewhat planar, aligning with the direction of the joint.

A 2D section is taken perpendicular to the slope as shown in Fig. 7 and computed in Slide2. Three methods of computation are used: (a) by simply adopting the same values of A and B from 3D into anisotropy defined along the 2D section plane (hereafter referred to as the "apparent dip" method), (b) by the proposed approximation method presented in the previous section, (c) by considering the fully 3D anisotropy. Via Eq. (7a and 7b), the equivalent values of A' and B' are computed to be 0° and 22.84° and entered as parameters for 2D anisotropy in the 2D analysis. The results are shown in Fig. 8.

It can be seen in Fig. 8(a), that considering the apparent dip results in a global minimum slip surface which tries to conform to the anisotropic direction with inclination between $A = 20°$ and $B = 30°$. This results in a very low FS and a 2D slip surface. In Fig. 8(b), with using the approximated method the computed FS is much closer to the 3D value. Fully considering the 3D orientation of the anisotropy in Fig. 8(c) results in an even closer result (Spencer FS = 0.71).

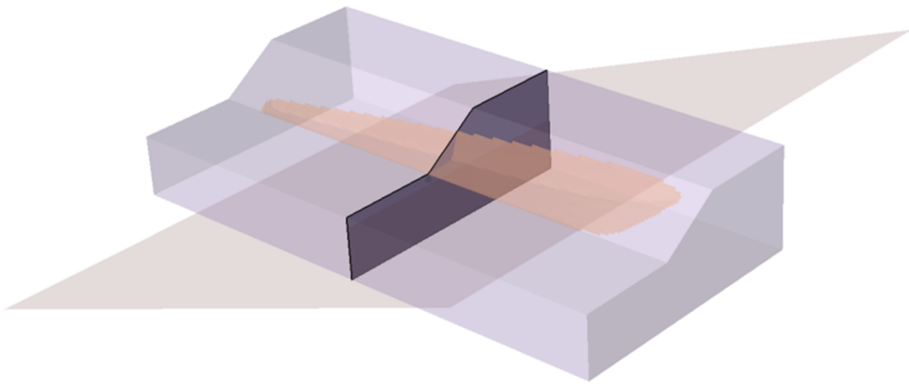

Fig. 7. Extruded slope with plane of anisotropy. FS = 0.69 in 3D. 2D section through model also shown

Table 1. Material properties of base and joint material for the example in Fig. 7. Failure criterion is Mohr-Coulomb and unit weight is 20 kN/m^3

	Cohesion (kPa)	**Phi (°)**
Base Material	5.0	35
Joint Material	0.5	5

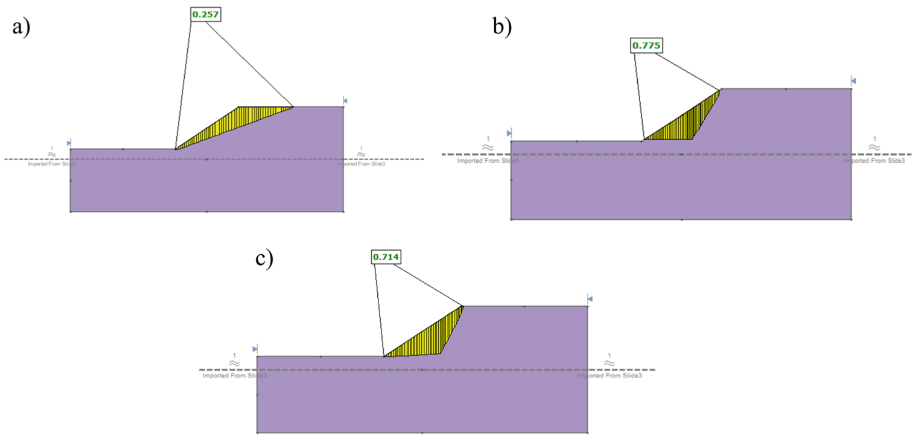

Fig. 8. a) Apparent Dip method, FS = 0.26; b) Approximate 2D anisotropic method, FS = 0.77; c) Full 3D Anisotropy method, FS = 0.71

The resulting FS and slip surfaces from the approximated and full 3D methods are very much in agreement with the Slide3 results. However, note that the approximate method using equivalent A' and B' still differs by around 10% from the true 3D value because the linear mapping of ϑ_2 is not exact. For example, if the slip surface is inclined $\vartheta_2 = 10°$ in the 2D section then the interpolation between $A' = 0°$ and $B' = 22.84°$ implies a shear strength nearly halfway between the joint and base strengths when interpolated linearly. However, the true value of ϑ in this case is 22.3°, which would imply a shear strength interpolated only a quarter of the way from the joint strength ($A = 20°$) to the base strength ($B = 30°$) when considering the full 3D anisotropy. Perhaps a different interpolation scheme (e.g. cosine variation rather than linear) could be used to bring the result closer.

5 Conclusions

The consideration of anisotropic strength in limit equilibrium has become popular among practitioners in the field of slope stability. Although analysis in 3D is recommended, the computation of factor of safety over a series of simplified 2D sections is commonly adopted as an additional verification step. Traditionally the apparent dip method has been adopted by many practitioners, which simply cuts the anisotropic surface at the location of the 2D section and assumes the same tolerance angles in 2D as in 3D. This often leads to wildly different results between the 3D analysis and the 2D analysis and may not accurately represent the true anisotropic behavior in the orientation of the 2D section. A new method is proposed in this paper which allows the 2D analysis to be informed by the 3D orientation of the anisotropic surface. This leads to results that are much more in agreement between 3D and 2D LE. A method to calculate the equivalent projected values of A' and B' for the 2D section is also proposed. It was shown that narrow tolerance angles in 3D often become even narrower in 2D (per the included derivation), which can result in cases in 2D where only the base material is considered.

Overall, it is demonstrated that it is best for the 2D analysis to be fully informed of the true 3D orientation of the anisotropy where possible.

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