An Analytical Method for the Calculation of the Shaft Resistance of Axially Loaded Piles in Cohesive-Frictional Soils

Lysandros Pantelidis(✉)

Cyprus University of Technology, Limassol, Cyprus
lysandros.pantelidis@cut.ac.cy

Abstract. According to the current practice, the unit shaft resistance of piles based on ground parameters is calculated usually with the a-method or the β-method. And while the physics behind these methods is simple and adequate for calculating the shaft resistance of piles in clays and sands for total and effective stress analysis respectively, the main difficulty in applying the effective stress approach in clays is to estimate the radial effective stress acting on the pile. In the present paper this is addressed by a proposed earth pressure at-rest coefficient, applicable to cohesive-frictional soils. The proposed coefficient is the product of a fully analytical, continuum mechanics approach, which, when applied to cohesionless soils, is simplified to the well-known $K_O = 1 - \sin\varphi'$, expression. Comparison examples show excellent agreement of the analytically derived shaft resistance capacities with the respective numerical ones.

Keywords: Shaft resistance of piles · Effective stress analysis · Earth pressure at-rest · Cohesive-frictional soils

1 Introduction

As known, the current practice for the calculation of the shaft resistance of pile based on ground parameters is the use of the following unit shaft resistances for total and effective stress conditions respectively:

$$q_s = \alpha c_u \tag{1}$$

$$q_s = \beta \sigma'_v \tag{2}$$

Equations 1 and 2 are best-known as the α-method [1] and the β-method [2] respectively. α is an adhesion factor for piles in (undrained) clays, $\beta = K_s \tan\delta$ is an empirical coefficient, K_s is an earth pressure coefficient, while δ is the friction angle of the pile – ground interface. K_s is usually Jaky's [3] earth pressure at-rest coefficient ($K_O = 1 - \sin\varphi'$), unless the soil is over-consolidated (requiring a higher value, most often empirical or an empirically corrected value of Jaky's K_O). And while the physics behind Eqs. 1 and 2 is adequate for calculating the shaft resistance of piles in clays

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R. E. Hammah et al. (Eds.): RIC 2023, AHE 19, pp. 209–216, 2023.
https://doi.org/10.2991/978-94-6463-258-3_22

(for total stress analysis) and sands (for effective stress analysis) respectively, the main difficulty in applying the effective stress approach (i.e., Eq. 2) in clays is the calculation of the radial effective stresses on the pile, that is, the coefficient K_s [4]. The purpose of the present paper is to suggest a new, reliable method for calculating the shaft resistance in cohesive-frictional soils (effective stress analysis of clays) focusing on the stress field around the pile; the empirical derivation of any adhesion factor a or friction angle δ for the pile-soil interface is out of the scopes of the present paper. The new method is based on the Generalized Coefficient of Earth Pressure proposed recently by the author [5]; the latter is described in brief in the next section. An application example for the calculation of the shaft resistance of axially loaded piles in cohesive-frictional soil with the proposed coefficient then follows.

2 The Generalized Coefficient of Earth Pressure

In 2019, the author [5] proposed a continuum mechanics method for deriving earth pressure coefficients for any soil state between the at-rest state and the active state or the passive state, applicable to cohesive-frictional soils and both horizontal and vertical pseudo-static conditions. The basic earth pressure coefficient expression is:

$$K_{XE} = \frac{1-(2\lambda-1)\sin\varphi_m}{1+(2\lambda-1)\sin\varphi_m} - (2\lambda-1)\frac{2c_m}{(1-a_v)(\sigma_v-u)}\tan\left(45^o-(2\lambda-1)\frac{\varphi_m}{2}\right) \tag{3}$$

where, c_m and φ_m are the mobilized shear strength parameters of soil, a_v is the vertical pseudo-static coefficient, u is the pore water pressure and σ_v is the vertical total stress. λ is a soil state dependent coefficient being either 0 or 1, while $X = O, A, P, IA$ or IP denoting the at-rest, active, passive, intermediate active and intermediate passive state respectively. The readers should bear in their mind that the "Rankine's" form of Eq. 1, came naturally through the continuum mechanics procedure followed.

The mobilized cohesion of soil, c_m, and the mobilized internal friction angle of soil, φ_m, are calculated using Eqs. 4 and 5 respectively.

$$c_m = c'\frac{\tan\varphi_m}{\tan\varphi'} \tag{4}$$

$$\varphi_m = Re\left(\sin^{-1}\left(-(2\lambda-1)\frac{b_o+\frac{D_0}{C_0\zeta^\lambda}+C_0\zeta^\lambda}{3\alpha_o}\right)\right)\frac{180}{\pi}(^\circ) \tag{5}$$

where, $a_o = \left(1+e_2^2\tan^2\varphi'\right)$, $b_o = 1-(2(2\lambda-1)e_1e_2+e_2^2)\tan^2\varphi'$, $c_o = (e_1^2+2(2\lambda-1)e_1e_2)\tan^2\varphi'$, $d_o = -e_1^2\tan^2\varphi'$, $D_0 = b_o^2-3a_oc_o$, $D_1 = 2b_o^3-9a_ob_oc_o+27a_o^2d_o$, $C_0 = \sqrt[3]{\frac{1}{2}\left(D_1-\sqrt{D_1^2-4D_0^3}\right)}$, $e_1 = (1-A_0)/B_1$, $e_2 = (1+A_0)/[(2\lambda-1)B_1]+2c'/\left[(1-a_V)(\sigma_v-u)(2\lambda-1)B_1\tan\varphi'\right]$, and $\zeta = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$. The Re notation in Eq. 5 means that only the real part of the number is kept; the imaginary part (if any) is infinitesimally small, and thus, zero.

For the at-rest state (which is relevant to the problem considered herein), $\lambda = 1$, while for the static situation ($a_H = a_V = 0$; a_H is the horizontal pseudo-static coefficient), the A_0 and B_1 parameters are:

$$A_0 = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}\left(1 - \xi\sin\varphi'\right) \tag{6}$$

$$B_1 = \frac{2c'}{(\sigma_v - u)}\tan\left(\frac{\pi}{4} - \frac{\varphi'}{2}\right) \tag{7}$$

Generally, $\xi = (m - 1)/(m + 1) - 1$; m is a real positive number ranging from 1 to $+\infty$. The at-rest state is obtained for $m = 1$, giving $\xi = -1$ and thus, $A_0 = 1 - \sin\varphi'$ (recall Eq. 6).

Considering, now, the static situation, Eq. 3 can be rewritten in the following convenient forms:

$$K_O = \frac{1 - \sin\varphi_m}{1 - \sin\varphi_m} - \frac{2c_m}{\sigma_v - u}\tan\left(45^o - \frac{\varphi_m}{2}\right) \text{ or } 1 - sin\varphi' - \frac{2c_m}{\sigma_v - u}\tan\left(45^o - \frac{\varphi'}{2}\right) \tag{8}$$

Equation 8 (second form) has two parts, a frictional term, which is Jaky's "$1-\sin\varphi'$", and a cohesion term, which has the full form of the respective Rankine's term for the active state, but since the at-rest state is not a failure state, the mobilized cohesion, c_m, is used instead of c'.

Moreover, the total earth pressure at-rest is

$$\sigma_O = K_O(\sigma_v - u) + u \tag{9}$$

At this point the author would like to highlight that contrary to what people believe, Jaky's [3] "$1 - \sin\varphi'$" is, finally, neither empirical nor a simplification of Jaky's [6] $Ko = \left(1 - \sin\varphi'\right)\left(1 + 2/3\sin\varphi'\right)/\left(1 + \sin\varphi'\right)$. In this respect, the latter derives also from Eq. 3 but for $m = 2$ ($\xi = -2/3$; also: $a_h = 0$ and $c' = 0$) and corresponds to an intermediate state between the at-rest state and the active state.

Also, it is noted that, a neutral zone (zone where soil exerts no lateral pressure) is extended from soil's surface to depth z_{nz}. This depth is different from, but corresponding to, the tension crack depth of cohesive soils being in the active state, and it is given by the following equation:

$$z_{nz} = \frac{c'}{\gamma}\tan\varphi' \tag{10}$$

An exhaustive validation of Pantelidis' [5] continuum mechanics method against contemporary centrifuge tests and finite elements can be found in Pantelidis and Christodoulou [7].

The above formulae will be used herein for introducing a new method for calculating the shaft resistance capacity of piles in cohesive-frictional soils based on ground parameters. The method is illustrated though the following application example (the basic procedure does not differ from the α-method [1] or the β-method [2].

3 Application Example of the Proposed Method

Let a 12-m-long pile of one meter diameter in a homogenous soil medium with c'=40 kPa, φ'=10° and $\gamma = 20$ kN/m^3. The pile is infinitively stiff, while the interface shear strength is defined by Eqs. 11 and 12. The shear reduction coefficient α_i is assumed equal to 1 (rough pile), while pore pressures are ignored for simplicity.

$$c_{int} = \alpha_i \cdot c^{'} \tag{11}$$

$$\tan\varphi_{int} = \alpha_i \cdot \tan\varphi' \tag{12}$$

The K_O coefficient, as this varies with depth for the specific example is shown in Fig. 1. More specifically, K_O is negative up to depth equal to 0.353 m where soil exerts no lateral pressure (negative values in the analysis are, apparently, replaced with zero, assuming that soil receives no tension).

For facilitating hand-calculations, the soil mass is divided into six (sub)layers, just for representing the $K_O - z$ relationship with an equal number of straight lines (such a straight line has been drawn for layer 3; indicated as L3 in Fig. 1). The shaft resistance capacity force $Q_{s,i}$ corresponding to each sublayer is calculated using Eq. 13.

$$Q_{s,i} = \pi \cdot 2R \cdot \Delta z_i \cdot \alpha_i \left(c' + \overline{K_O}_i \cdot \gamma \cdot \bar{z}_i \cdot tan\varphi'\right) \tag{13}$$

The procedure is summarized in Table 1. The maximum unfactored axial loading corresponding to the total shaft resistance of pile derives, finally, from Eq. 14, and for the given example, it is equal to 2518.5 kPa.

$$q_{s,tot} = \sum_{i=1}^{6} \left[Q_{s,i}/\pi R^2\right] \tag{14}$$

4 Numerical Validation

The analytical example presented above is used herein for the necessary numerical validation of the proposed method. The numerical results were obtained using Rocscience's RS2; RS2 is a commercial program for 2D finite element analysis of geotechnical structures for civil and mining applications. The geometry and boundary conditions, as well as the mesh and the basic settings used are all shown in Fig. 2. More specific data for the pile material and the surrounding soil follows. For the pile, Material Type = Elastic/Isotropic, Unit Weight = 0.0001 kN/m^3, Poisson's ratio = 0.2 and Young Modulus = 10 GPa. For the soil: Material Type = Plastic/Isotropic, Unit Weight = 20 kN/m^3, Poisson's ratio = 0.3 and Young Modulus = 0.02 GPa, Peak and Residual Cohesion = 40 kPa and Peak and Residual Friction Angle = 10°.

A uniform distributed load 10% greater than the respective analytically derived pile shaft resistance (see Table 1) is applied on the top of the pile in 55 loading stages with constant increment (Stage 1 is a zero-load stage). The zero-tip resistance was achieved by

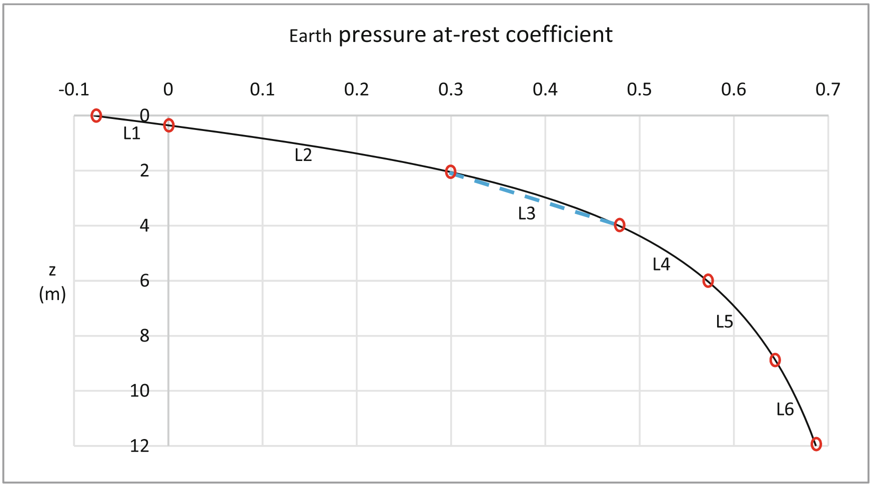

Fig. 1. Earth pressure at-rest coefficient, K_O, against depth, z, chart ($c' = 40$ kPa, $\varphi' = 10°$ and $\gamma = 20$ kN/m^3)

Table 1. Table for the calculation of the total resistance of pile shaft based on the proposed method

z_{up}	$K_{OE,up}$*	z_{down}	$K_{OE,down}$	$\overline{K_{OE}}$	Δz	$\overline{z}$	$Q_{s,i}$
(m)		(m)			(m)	(m)	(kN)
0	(−0.079)	0.353	0	0	0.343	0.18	43.1
0.353	0	2	0.293	0.1465	1.647	1.18	210.1
2	0.293	4	0.478	0.3855	2	3	277.0
4	0.478	6	0.572	0.525	2	5	309.5
6	0.572	9	0.646	0.609	3	7.5	528.8
9	0.646	12	0.687	0.6665	3	10.5	609.6
			Total shaft resistance force, Q_s (kN)				1978.1
			Total resistance of pile shaft, $q_{s,tot}$ (kPa)				2518.5

*Negative K_{OE} values in the calculations of the shaft resistance of pile are replaced with zero.

allowing the nodes at the lower end (tip) of the pile to freely move downwards in the void. The fixed horizontal but free vertical components of displacement at the tip of the pile, in essence, create an opening in the lower external boundary. The diameter of the opening is a few millimeters greater than the diameter of the pile itself, allowing, therefore, a thin soil shear zone to be created around the pile. Shearing the soil mass itself (instead of the pile-soil interface) corresponds to the α_i=1 case and, in practice, to a bored, cast in-situ, concrete pile. The pile is weightless; thus, each staged load on pile corresponds to the respective mobilized shaft resistance. The numerical ‘staged loading on pile’ against ‘pile settlement’ curve has been drawn in Fig. 3. A second ‘$\kappa = \Delta q/\Delta \rho$’ versus

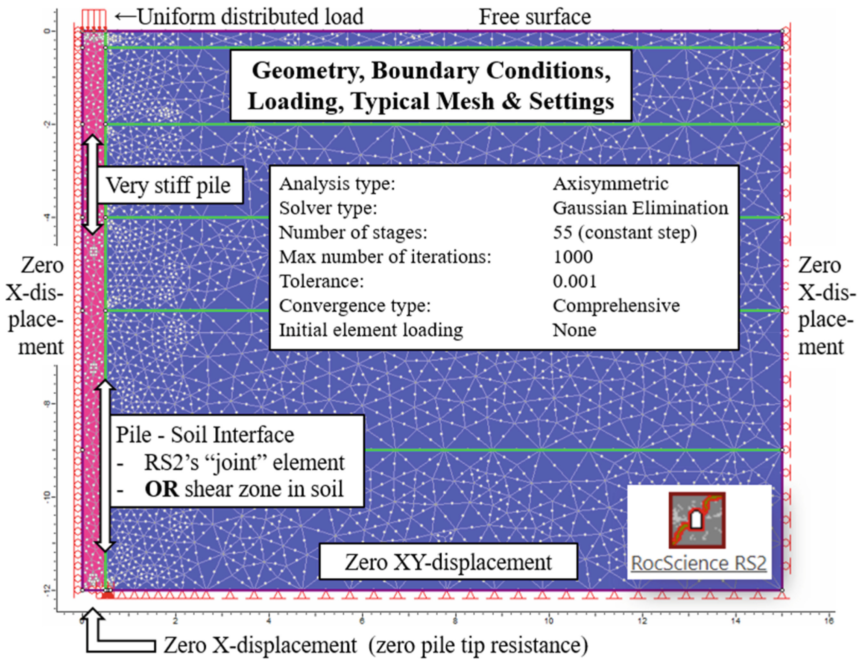

Fig. 2. Geometry, boundary conditions, mesh and basic settings for the example used

'settlement' curve has also been drawn in the same chart, facilitating in identifying the failure load; the latter corresponds to the shaft resistance capacity of pile. The RS2 failure point (red point on the chart) was found by interpreting the two curves (simultaneous and significant drop in both curves).

The shaft resistance as this derived by the proposed method is also indicated on the chart. The comparison revealed a relative difference just 2.2%; please compare the numerically derived 2462.6 kPa value with the respective analytical 2518.5 kPa (proposed method).

5 Conclusions

According to the current practice, the shaft resistance of axially loaded piles based on ground parameters is calculated using the a-method or the β-method, for total or effective stress conditions respectively. And while the physics behind the a-method and the β-method is adequate for calculating the shaft resistance of piles in clays and sands for total and effective stress analysis respectively, the main difficulty in applying the effective stress approach in clays is to calculate the radial effective stress acting on the pile. In the present paper, this is addressed by the suggested coefficient of earth pressure at-rest derived from a continuum mechanics approach. The effectiveness of the proposed method is illustrated through a comparison example, where the analytically derived shaft resistance capacity of the pile considered deviate by only a few percentage points from the respective numerical one. The proposed method could be used as a general $c - \varphi$

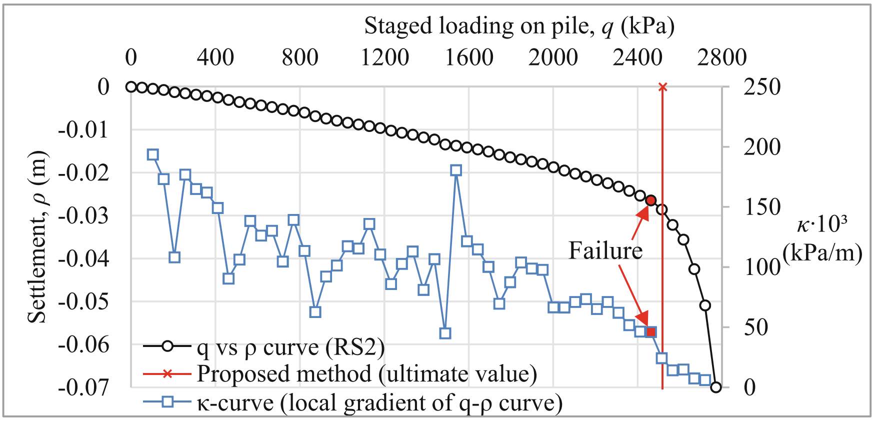

Fig. 3. Staged loading on pile against pile settlement curve

procedure, applicable also to the cases covered by the a-method and the β-method. Finally, given that cohesion reduces the effective stresses acting radially on the pile (unfavorable factor), the proposed method can lead to more efficient design.

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