An Analytical Method for Designing Laterally Loaded Piles

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Abstract. There is no doubt that the problem of a laterally loaded pile is an earth pressure analysis problem, involving the flexural rigidity of the pile. During the lateral loading, the earth pressures along the pile may take any value between the state at-rest and the active or the passive state. The current practice, as this is depicted by the various design standards, is the $p - y$ method, with the soil to be replaced by an array of parallel springs. This simplification is furthermore accompanied by rough assumptions for the Winkler spring constant. In the present paper, a new analytical method for designing laterally loaded piles is proposed. This method combines the earth pressure theory recently proposed by the author and the theory of elastic beams. The author's coefficient of earth pressures is a general expression for any soil state between the at-rest state and the active or the passive one, applicable to cohesive-frictional soils and both horizontal and vertical pseudo-static conditions. An analytical expression for the calculation of the required pile movement for the mobilization of the active or passive failure state of soil is also provided. The result of the proposed analysis is complete earth pressure and pile deflection diagrams.

Keywords: laterally loaded piles · static analysis of elastic beams · the Generalized Coefficient of Earth Pressure · intermediate earth pressures · method

1 Introduction

Pile foundations are commonly used in major projects (e.g., high-rise buildings, bridges, offshore structures) to withstand large axial and lateral loads. Designing piles subjected to lateral loads is, however, a difficult task. Already from the middle of the previous century, a rather great number of methods has been proposed. On the other hand, the methods included in a design standard are supposed to reflect the best current practice. In this respect, EN1997-1:2004 [1], prEN19973:2022 [2] (draft standard), American Petroleum Institute [3], FHWA [4] and AASHTO [5] largely rely on the $p - y$ method. A close look at the latter, however, would reveal major assumptions about the failure mechanism and the earth pressure distribution around the pile, significant simplifications and, surprisingly, analytical modelling of (short) rigid piles for studying the case of (long) flexible piles. For overcoming these drawbacks, these methods are supposed to have been calibrated against experimental data; however, not only these data are case-specific, but also, they are very limited in number.

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The problem of lateral loaded piles is, essentially, an earth pressure analysis problem, involving all soil states and especially the intermediate ones, both on the active and the passive "sides". Intermediate earth pressures act on the pile when the latter has not moved enough so that the active or the passive (failure) state to be reached.

The purpose of the present paper is to lay the basis for a new, analytical method for designing laterally loaded piles, respecting the actual mechanism. The new method will be combining the Generalized Coefficient of Earth Pressure, proposed recently by the author [6], and the theory of elastic beams.

2 The Generalized Coefficient of Earth Pressure

In 2019, the author [6] derived an earth pressure coefficient for any soil state between the at-rest state and the active or the passive state, applicable to cohesive-frictional soils and both horizontal and vertical pseudo-static conditions, through a continuum mechanics procedure. The basic earth pressure coefficient expression is:

$$K_{XE} = \frac{1-(2\lambda-1)\sin\varphi_m}{1+(2\lambda-1)sin\varphi_m} - (2\lambda-1)\frac{2c_m}{(1-a_v)(\sigma_v-u)}\tan\left(45^o-(2\lambda-1)\frac{\varphi_m}{2}\right) \tag{1}$$

where, c_m and φ_m are the mobilized shear strength parameters of soil, a_v is the vertical pseudo-static coefficient, u is the pore water pressure and σ_v is the vertical total stress. λ is a soil state coefficient being either 0 or 1, while $X = O, A, P, IA$ or IP denoting the at-rest, active, passive, intermediate active and intermediate passive state respectively. The readers should bear in their mind that the "Rankine's" form of Eq. 1, came naturally through the continuum mechanics procedure followed. The mobilized cohesion of soil, c_m, is calculated using Eq. 2, while the mobilized internal friction angle of soil, φ_m, derives from the solution of Eq. 3.

$$c_m = c'\frac{\tan\varphi_m}{\tan\varphi'} \tag{2}$$

$$\sin\varphi_m\left(e_2 - \frac{\tan\varphi_m}{(2\lambda-1)\tan\varphi'}\right) = e_1 + \frac{\tan\varphi_m}{\tan\varphi'} \tag{3}$$

c' and φ' are the peak shear strength values. e_1 and e_2 are dimensionless parameters, the value of which depend on the shear strength, elastic parameters and unit weight of soil, the depth, the two pseudo-static coefficients, the pore water pressures and the horizontal displacement Δx. The calculation of Δx requires the knowledge of the flexural rigidity and the length of the pile. Avoiding repetition, the analytical solution of Eq. 3 can be found in Pantelidis [7]. An analytical expression for the calculation of the required retaining wall movement (at any depth) for the mobilization of the active or passive failure state is also given in [7]; the same is applicable to the problem examined herein. Finally, the total earth pressure (at any depth) is

$$\sigma_{XE} = K_{XE}(\sigma_v - u) + u \tag{4}$$

An exhaustive validation of Pantelidis' [6] continuum mechanics method against contemporary centrifuge tests and finite elements can be found in Pantelidis and Christodoulou [8]. The above formulae will be used herein for introducing a new, earth-pressure-based method for designing laterally loaded piles. The general procedure (with the calculation of the intermediate earth pressures along the structure and the iterations) will be similar to the one presented in [7]; however, this time, the unfavorable action on the structure is a horizontal point load at the top of the pile. Moreover, a laterally loaded pile is a three-dimensional problem, meaning that friction and/or adhesion contribute to the resistance to its lateral movement, while there is also a 3D-effect in the soil mass in front of the pile. Finally, it should also be mentioned that, in addition to bending, laterally loaded piles, may be rotated around a pivot point and/or be translated horizontally. In this short paper, only bending is considered.

3 The Proposed Method for Laterally Loaded Piles

The earth pressures acting on the circumference of a circular pile not subjected to any external horizontal loading is the earth pressures at-rest. When the pile is loaded laterally, the soil state around the pile may change from the state at rest to an intermediate active or passive state or even to an active or passive failure state.

Studying the earth pressures acting on the pile in the direction parallel to the direction of the lateral load (direction called hereinafter "tangential"), it can easily be shown that the resultant effective "tangential" force at depth z (per dz pile length) is:

$$(2R)(K_{IP} - K_{IA})(\sigma_V - u)dz \tag{5}$$

This will constitute the loading on the equivalent elastic beam for each segment along the beam of infinitesimal length dz. The values of K_{IP} and K_{IA} depend on the amount of deflection of the pile at each depth, which is caused by the lateral load. Of course, the same load may additionally cause to some extend rotation and/or translation to the pile, but these types of movement are not discussed here. Now, based on the elastic beam theory, the deflection of a cantilever beam with a point load P at the tip will be:

$$\Delta y_P(z') = (P - f_{ee}P_{ee})\frac{(3L - z')z'^2}{6E_pI_p} \tag{6}$$

E_pI_p is the flexural rigidity of the beam (in this respect, of the pile), L is the length of the beam (referring to the part of the pile between the point of (virtual) fixity, PVF, and the point of action of P) and z' is the $z-$ coordinate along the beam measured from its fixed support (i.e., from PVF); due to space limitations, the supporting figure has been added in Fig. 1, which appears later. P_{ee} is an equivalent load referring to the elastic energy stored in the pile due to bending and subtracted from P for avoiding the double inclusion of this elastic energy (augmented by a factor $f_{ee} = 1.5$ for calibration purposes against finite elements); it is found using the maximum calculated deflection value, $\Delta y_{P,max}$ (i.e., $P_{ee} = 3\Delta y_{P,max}E_pI_p/L^3$), while an iterative procedure is followed. The deflection values of Eq. 6 will be added to the deflection caused by the earth pressures acting on the two sides of the pile. A number of iterations is again needed until convergence, having as starting point the earth pressures at-rest (avoiding repetition, please see [7]). The lateral resistance of the pile, however, does not only come from the passive resistance of the ground in front of the pile and the flexural rigidity of the pile itself, but also from the friction at the pile – soil interface. For calculating the latter, the resultant effective "normal" force on the passive "side" of the pile per dz pile length must be known; as the pile moves away from the soil (active "side"), no frictional resistance is developed. Again, it can be easily shown that the resultant effective "normal" force corresponding to the pile semicircle on the passive "side" is:

$$(\pi R)K_O(\sigma_V - u)dz \tag{7}$$

Based on the above and treating adhesion in a similar manner, the resistance force (per dz pile length) due to interface friction and adhesion is:

$$(\pi R)\left[\left\{c'\right\}_{int} + K_O(\sigma_V - u)\left\{\tan\varphi'\right\}_{int}\right]dz \tag{8}$$

$\left\{c'\right\}_{int}$ and $\left\{\tan\varphi'\right\}_{int}$ are the adhesion parameter and the friction angle coefficient at the soil-pile interface respectively, with φ' being the peak friction angle of the soil at depth z. The interface shear resistance, however, is usually taken as a fraction of the shear strength of soil, i.e., $\left\{\tan\varphi'\right\}_{int} = \alpha \cdot \tan\varphi'$ and $\left\{c'\right\}_{int} = \alpha \cdot c'$, where α is a shear reduction coefficient (e.g., $a = 1$, 3/4, 2/2). A common shear reduction coefficient is usually used for both resistance components.

4 Application Example and Numerical Validation

The example presented herein has been solved both with the proposed analytical method (using KPileY v.1 educational software prepared by Dr Panagiotis Christodoulou and the author) and with Rocscience's RS2. Apparently, a laterally loaded pile is a three-dimensional problem. However, ignoring (at least for the moment) the lateral frictional resistance (assumption of perfectly smooth pile) and the fact that a three-dimensional soil wedge is actually affected in front of the pile (passive "side"), the problem is reduced to a plane-strain problem.

The geometry, mesh and boundary conditions are shown in Fig. 1. The pile is free to bend from $z = 0$ to 12 m under the influence of a $P = 200$ kN horizontal load. The latter is applied at the ground level ($z = 0$ m) in 11 steps with 20 kN interval. Below the $z = 12$ m depth, the pile is practically fixed, meaning that in the analytical procedure both the translational and rotational components of movement was assumed zero. In the numerical procedure, the anchorage of the pile was achieved setting a horizontal nodal displacement as low as 0.1 mm for $z = 12$ to 15 m. This very small nodal displacement was imposed in the lower three meters of the pile (instead of using a very stiff soil layer) for obtaining more stable results. The pile, which is solid and of circular cross-section (with one meter diameter, modulus of elasticity $E_P = 30$ GPa and thus, flexural rigidity $E_P I_P = 1,472,622\text{KPa} \cdot \text{m}^4$), functions in a homogenous, isotropic mass with cohesion $c' = 0$ kPa, internal friction angle $\varphi' = 35°$, unit weight $\gamma = 16$ kN/m^3, Young's modulus $E = 20$ MPa and Poisson's ratio $\nu = 0.3$. Pore-water pressures and seismic excitation was ignored for the sake of simplicity. Favoring reproduction of the example problem, all relevant information is given below (if something is not mentioned, the RS2 default value was used). The "*Gaussian elimination*" solver type was used. Regarding the "*stress analysis*" menu, the maximum number of iterations was 1000, the tolerance was set to 0.001, while the "*comprehensive*" convergence type was adopted. The "*mesh type*" was set to "*graded*", while 6-noded triangular elements were used (meaning 19.0 nodes/m^2 or 9.2 elements/m^2; see Fig. 1). The "*field stress type*" was "*gravity*" with "*stress ratio*" in- and out-of-plane equal to 0.4264 (use of Eq. 1 for zero displacement and static conditions). The "*initial element loading*" was "*field stress and body force*". The problem was solved statically ($a_H = a_V = 0$). The soil parameters were those given earlier (apparently, the "*plastic*" material type was chosen). The pile was modeled as "*structural interface*", with a "*standard beam*" element as liner and a "*joint*" element at both sides. The liner was considered to be "*elastic*" with Young's modulus $E_P = 30$ GPa and thickness 0.8383 m; this combination of values gives the same flexural rigidity $E_P I_P$ as in the analytical solution. The Poisson's ratio of the liner was 0.2, while the "*Timoshenko*" beam element formulation was adopted. Regarding "*joint*", the "*material dependent*" slip criterion was chosen with "*interface coefficient*" as low as 0.05 (besides, the proposed earth pressure analysis method is for smooth structures). Both the normal and shear stiffness of the joint element was set to 200 MPa/m.

Regarding the proposed analytical procedure, the deflection Δy (at any depth z) is calculated combining the proposed earth pressure method with the elastic beam theory, where the pile is treated as a cantilever beam; the earth pressures on the two sides of the pile along with the lateral loading P constitute the loading acting on the beam. An iterative procedure is needed. As a starting point (1st iteration), it is logical and convenient to assume that the soil on both sides of the pile is at the state at-rest. However, since a horizontal load acts, this state will change; the pile will have the tendency to bend towards the passive "side". The deflection (Δy_{tot}) of the pile caused by the combined action of the earth pressures on the two sides of the pile and the horizontal load is then calculated. For the 2nd iteration these Δy_{tot} values are the new Δy data values for calculating the

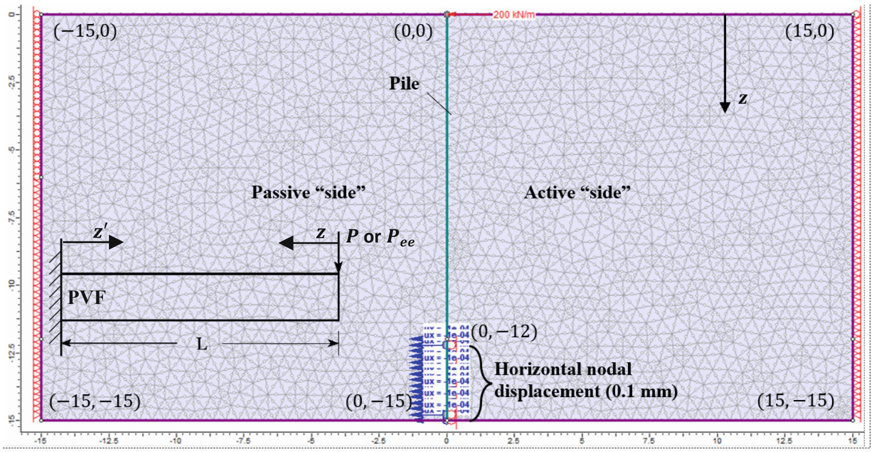

Fig. 1. Geometry, mesh and boundary conditions of the RS2 model.

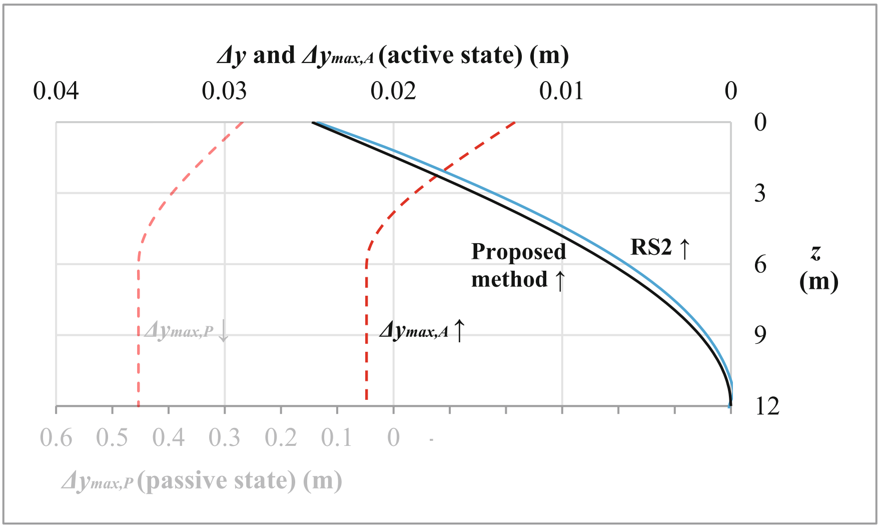

Fig. 2. Deflection of pile versus depth chart. Note: $\Delta y_{max,P}/\Delta y_{max,A}$=20.998.

updated earth pressure coefficients and so on, until convergence to be achieved. The results obtained in the 10th iteration (already stable from the 5th iteration) are compared against the respective numerical ones in Figs. 2 and 3.

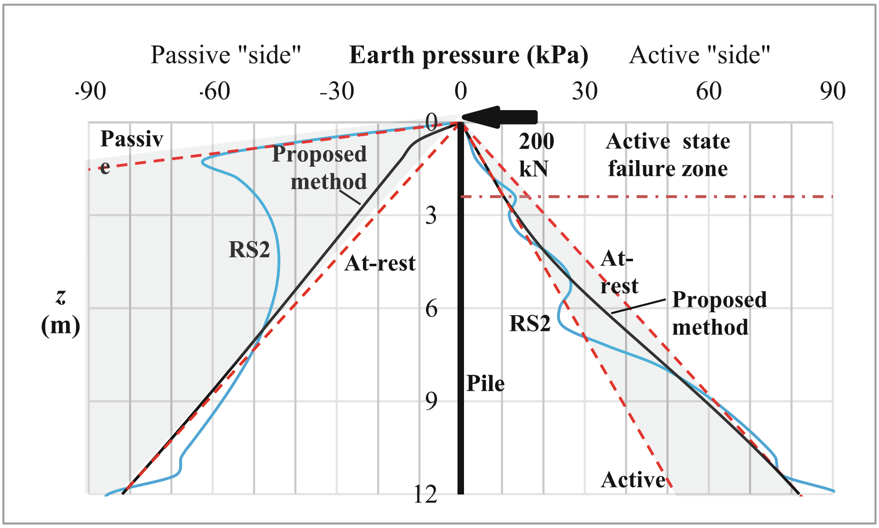

Fig. 3. Earth pressures versus depth chart. The red dashed lines indicate the "active", "passive" and "at-rest" state. Shaded areas indicate the allowable earth pressure values.

It is noted that for f_{ee}= 1 (no calibration), the maximum relative difference in the deflection of pile, which is referred to the highest point of the pile, is about 20% (on the conservative side), while negligible differences are observed in the earth pressures.

5 Conclusions and Future Work

The knowledge of intermediate earth pressures (i.e., earth pressures between the at-rest state of soil and the active or the passive one) is very important in designing laterally loaded piles. However, in the absence of a reliable method for calculating these pressures (which depend on the amount of movement and bending of the pile), the various design standards worldwide rely on the simplistic $p - y$ method. In this paper a new method for designing laterally loaded pile, which combines the earth pressure theory recently proposed by the author and the theory of elastic beams, is proposed. Indeed, following the proposed method, complete earth pressure and pile deflection diagrams can be drawn. Comparing the analytical results with the respective numerical ones, one can observe excellent prediction for the deflection and the earth pressures on the active "side" of the pile. Indeed, more stable earth pressure values are obtained. For the passive "side" of the problem, smaller earth pressure values are obtained near the surface, which, however, can be justified by the small deflection values.

The material presented herein was just an introduction of the proposed method. Since the problem of a laterally loaded pile is a three-dimensional problem, future work will be focused on the validation and if necessary, the calibration of the proposed method against three-dimensional finite element analysis and field test data.

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