Application of Continuum-Based Voronoi Tessellated Models for Simulating Brittle Damage and Failure in Hard Rocks

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Abstract. Voronoi tessellations are commonly used in discontinuum numerical models to simulate the microstructure of brittle rocks. In this approach, the model domain is divided into several randomly generated polygonal blocks. Discontinuum-based Voronoi Tessellated Models (VTM) capture the brittle rock failure process more realistically than conventional continuum models. However, their higher computational costs often limit their practicality. This paper presents the applications of a continuum-based VTM for simulating brittle rock damage and failure at core and rock mass scales. The examples reviewed include: 1) laboratory behavior of a marble; 2) drilling-induced core damage; 3) failure mechanism of hard rock pillars; and 4) v-shaped notch formation around a circular tunnel. It is concluded that a properly calibrated continuum-based VTM can be used as a reliable tool for analyzing a wide range of geomechanical problems.

Keywords: Brittle Rock Failure · Hard Rock · Voronoi Tessellated Model (VTM) · Continuum Model

1 Introduction

Understanding, predicting and modeling brittle rock failure have been the focus of industries involved in the design of deep underground excavations. A common approach to simulate this phenomenon is to use continuum numerical methods utilizing the Cohesion Weakening Frictional Strengthening (CWFS) model. It is known that the CWFS model can replicate the depth and shape of failure around underground excavations in massive to moderately jointed hard rock masses [1]. However, this model does not capture the Excavation Damage Zone (EDZ), which is critical for the design of Deep Geological Repository (DGR) excavations. Alternatively, discontinuum numerical methods can be used, in which the heterogeneous nature of rock is modeled as an assembly of blocks. However, the disadvantage of this approach is its high computational costs.

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This paper presents the summary of research conducted at Dalhousie University on an alternative continuum-based modeling approach. In this approach, the 2D finite element program RS2 (by Rocscience) is utilized to construct heterogeneous rock models at different scales. The examples reviewed in this paper include: 1) laboratory properties and behavior of Wombeyan marble; 2) drilling-induced core damage in granite at Canada's Underground Research Laboratory (URL); 3) hard rock pillar failures under compressive and shear loading at the Quirke Mine, ON, Canada; and 4) v-shaped notch formation around a circular test tunnel at the URL.

2 Continuum-Based Voronoi Tessellated Model (VTM)

Recent advances in computational power and software programs have made it possible to incorporate various types of rock heterogeneity in numerical models. A numerical model with grain-scale geometric heterogeneity is called the Grain-Based Model (GBM). It is also referred to as the Voronoi Tessellated Model (VTM) in literature. The presence of heterogeneities (e.g., Voronoi blocks) in a numerical specimen results in the generation of localized tensile stresses even when the specimen is under an overall compressive stress field, allowing for capturing the brittle rock failure process.

In RS2, a VTM is generated using the built-in Voronoi joint network, which divides the homogeneous model domain into nonoverlapping convex polygons or Voronoi blocks (Fig. 1a). In the RS2-VTM, the contact between two blocks is a four-noded quadrilateral joint element (Fig. 1b). A total of 16 input parameters (micro-properties) are required in an inelastic RS2-VTM with the Mohr-Coulomb properties. The strength properties include the peak and residual cohesion (C), friction angle (φ) and tensile strength (σ_t) for blocks (mesh elements) and block boundaries (joint elements). The deformation properties include Young's modulus (E) and Poisson's ratio (υ) for blocks and normal stiffness (K_n) and shear stiffness (K_s) for block boundaries. These micro-properties are adjusted through an iterative process called 'calibration' until the macro-properties (peak strength) and macro-behavior (failure mode) are captured.

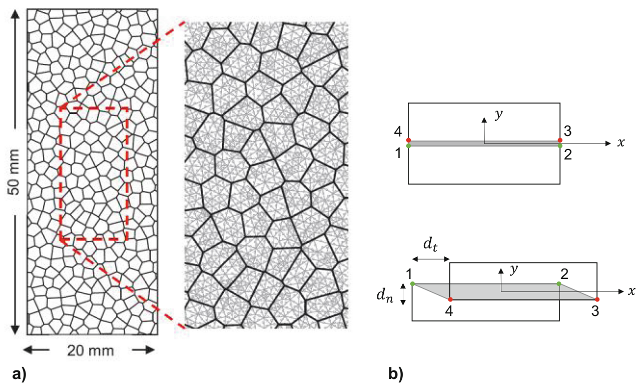

Fig. 1. a) Voronoi Tessellated Model in RS2 consisting of joint elements and polygonal blocks meshed with triangular elements; b) undeformed and deformed joint elements in RS2 [2]

Table 1. Micro-properties of RS2-VTMs calibrated to intact rock (core-scale) and rock mass (field-scale) properties

Micro-properties		Core-scale models		Field-scale models	
		Case 1	Case 2	Case 3	Case 4
		20 × 50 mm*	60 × 120 mm*	2 × 5 m*	0.6 × 1.2 m*
Blocks (mesh elements)					
Peak strength	C (MPa)	45	82	40	30
	φ (°)	37	59	50	27
	σ_t (MPa)	14	22	6	6.9
Residual strength	C (MPa)	15	0.1	40	10
	φ (°)	37	59	50	57
	σ_t (MPa)	0.1	0.1	0.1	0.1
Deformation properties	E (GPa)	80	78	68	65
	υ	0.3	0.22	0.2	0.25
Block boundaries (joint elements)					
Peak strength	C (MPa)	40	45	20	29
	φ (°)	50	59	50	11
	σ_t (MPa)	3	9.8	3.5	4
Residual strength	C (MPa)	0.1	0.1	0.1	0.1
	φ (°)	50	59	50	57
	σ_t (MPa)	0.1	0.1	0.1	0.1
Deformation properties	K_n (GPa/m)	240,000	145,000	5,000	10,500
	K_s (GPa/m)	24,000	59,000	500	4,100

*RS2-VTM size (width × height) used for laboratory test simulations and calibration

The objectives of the numerical studies presented in this paper were to investigate the capability of RS2-VTMs for capturing some of the characteristics of brittle rocks, such as laboratory peak strength and post-peak response, the transition in the failure mode as a function of confinement, unloading-induced brittle damage during core drilling, and the failure of hard rock masses within pillars and around tunnels. The micro-properties of the RS2-VTMs for these studies are given in Table 1.

3 Case 1: Laboratory Behavior of Wombeyan Marble

Granulated marble is a solid specimen in which the grains have been separated at their boundaries due to heating an intact marble specimen. Li and Bahrani [2] investigated the capability of the RS2-VTM in capturing the laboratory properties and behavior of intact and granulated Wombeyan marble reported by Gerogiannopoulos [3]. The stress-strain

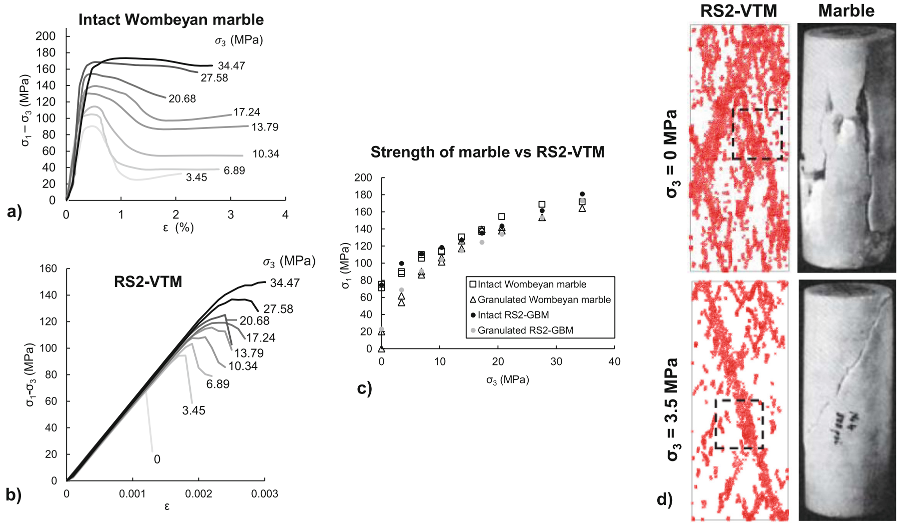

Fig. 2. Stress-strain curves of: a) intact Wombeyan marble [3]; and b) calibrated RS2-VTM [2]. c) Peak strengths of calibrated RS2-VTM [2] and intact and granulated Wombeyan marble [3]; d) failure modes of RS2-VTM [2] and intact Wombeyan marble [4]

curves of intact Wombeyan marble (Fig. 2a) indicate a transition from brittle behavior at low confinement ($\sigma_3 = 0 - 3.5$ MPa) to strain-softening with increasing confinement ($\sigma_3 = 10 - 21$ MPa), and to perfectly plastic at high confinement ($\sigma_3 > 27$ MPa).

Li and Bahrani [2] captured the observed post-peak response of intact marble with increasing confinement using the RS2-VTM (Fig. 2b) with the micro-properties given in Table 1. Figure 2c shows how well the RS2-VTM captured the laboratory peak strengths of intact and granulated marble for the full range of confinement ($\sigma_3 = 0 - 34.47$ MPa). The laboratory failure modes of intact Wombeyan marble at confining pressures of 0 MPa and 3.45 MPa [4] and those of the calibrated RS2-VTM are compared in Fig. 2d. The patterns of yielding (left images) indicate that the simulated failure modes are consistent with laboratory test results: axial splitting under unconfined compression and shear failure under confined compression.

4 Case 2: Unloading-Induced Brittle Damage

It is recognized that drilling in high-stress environments induces damage in the form of micro-cracks to the cored samples, which may result in a reduced rock strength and elastic modulus measured in the laboratory [5]. Therefore, it is necessary to assess the severity of core damage and its influence on the laboratory-measured intact rock properties. Figure 3a shows an example of severe core damage, known as partial disking, in a core from a nearly vertical borehole at the 420 level of the URL. In this figure, micro-cracks are perpendicular to the long axis of the core (i.e., sub-parallel to σ_1 direction).

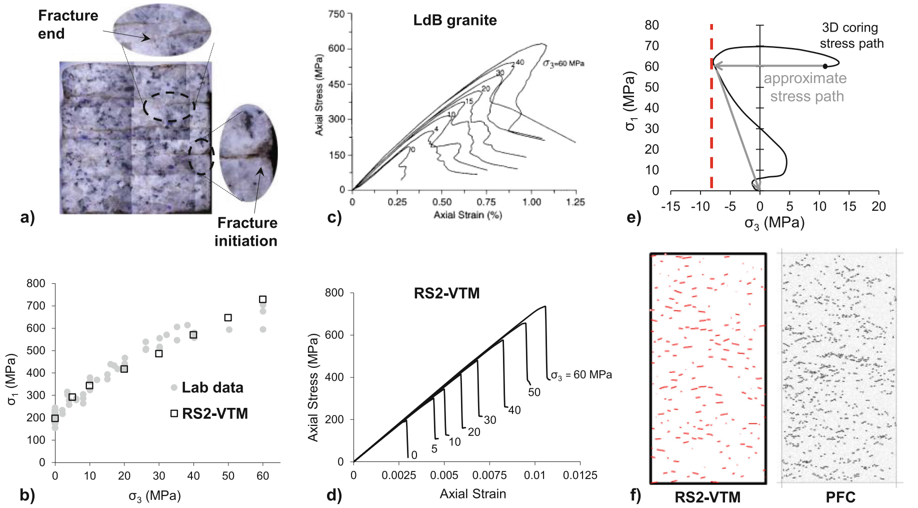

Fig. 3. a) Example of partial core disking at URL [8]; b) peak strengths of RS2-VTM and intact LdB granite [6]; stress-strain curves of c) intact LdB granite [9], and d) RS2-VTM [6]; e) 3D coring stress path and its approximate form applied to RS2-VTM [6]; f) simulated core damage in RS2-VTM [6] and PFC [7]

Amiri and Bahrani [6] used an RS2-VTM to simulate unloading-induced brittle damage during core drilling in high-stress environments. They first calibrated an RS2-VTM to the laboratory peak strength of undamaged LdB granite for the full range of confinement ($\sigma_3 = 0$ - 60 MPa), as shown in Fig. 3b (see micro-properties of Case 2 in Table 1). During the calibration process, they aimed to capture the post-peak response, which is brittle even at a confining pressure of 60 MPa (Fig. 3c). This behavior was replicated (Fig. 3d) by assigning a residual cohesion and tensile strength of 0.1 MPa to the blocks (mesh elements).

Amiri and Bahrani [6] used an indirect modeling approach to simulate unloading-induced brittle damage using the calibrated RS2-VTM. In this approach, the coring stress path from a 3D elastic continuum model (black curve in Fig. 3e) was first approximated and then applied to the calibrated RS2-VTM to introduce damage. The grey arrow in Fig. 3e is the approximate coring stress path for a vertical borehole at the 420 level of the URL. As shown in Fig. 3f (image on the left), the simulated micro-cracks (yielded block boundaries) are nearly horizontal (i.e., subparallel to σ_1 direction) and randomly located within the specimen. This figure shows that the unloading-induced brittle damage captured by the RS2-VTM is comparable with PFC simulation results (image on the left) by Bahrani et al. [7].

5 Case 3: Failure of Hard Rock Pillars

The Quirke Mine is an abandoned uranium mine located in the Elliot Lake district, ON, Canada. The rib pillars in this mine were initially laid out with their long axes parallel to the dip direction of an orebody with a dip angle of 20°. In the central part of the mine, the pillars were re-oriented at 45° to the orebody dip direction. This realignment resulted in adverse shear loading within the pillars that led their failure [10]. An example of a failed pillar in the Elliot Lake district is shown in Fig. 4a.

Since the intact rock (quartzite) at the Quirke Mine has a peak strength of 230 MPa, and the rock mass is massive to moderately jointed (GSI > 65), the s-shaped or tri-linear failure criterion [11, 12] was used to estimate the rock mass strength. Hamediazad and Bahrani [13] calibrated an RS2-VTM to the peak rock mass strength estimated using the tri-linear and its equivalent Hoek-Brown rock mass strength envelopes proposed by Bewick et al. [12]. Figure 4b illustrates the peak strength of the calibrated RS2-VTM. Figure 4c shows the geometry of the RS2 model of a pillar with a width of 3 m and a W/H of 0.75. The models of pillars in compression and shear were created as a heterogeneous material using the block geometry and micro-properties of the calibrated RS2-VTM (see micro-properties of Case 3 in Table 1).

The pillar loading was simulated in two stages. First, two parallel drifts with a diameter of 4.5 m were excavated. The modeled pillars were further loaded by gradually excavating the stopes at both sides. It is evident in the stress-strain curves of both pillars shown in Fig. 4d that the strength of the pillar in shear is lower than that in compression. The simulation results presented in Fig. 4e indicate that the mode of yielding in both pillars is mostly tension. In the modeled pillar in compression, the yielding pattern is relatively symmetrical. The pillar in shear, however, begins to yield from the opposite corners. The yielding is then propagated diagonally to the core. The simulated failure modes are consistent with field observations at the Elliot Lake district (Fig. 4a).

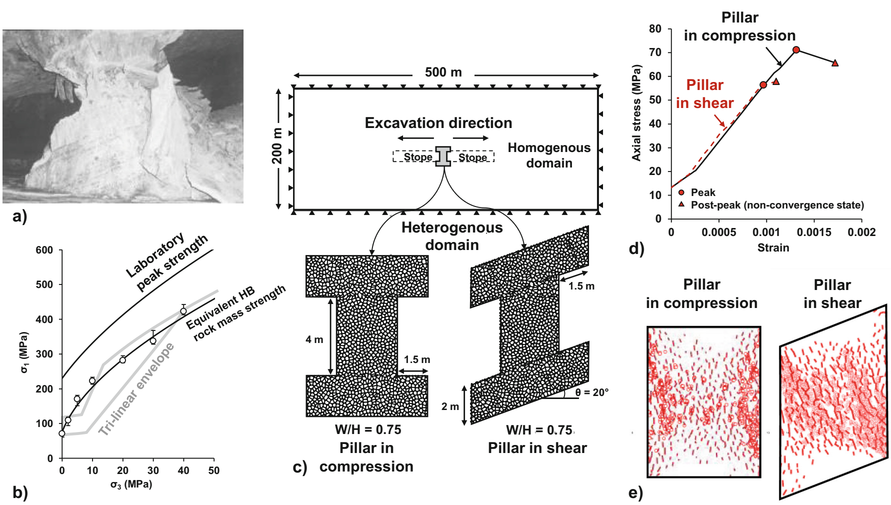

Fig. 4. a) Failure of hard rock pillar in Elliot Lake district [14]; b) peak strength of RS2-VTM compared to estimated rock mass strength [13]; c) RS2 model geometry [13]; d) simulated stress-strain curves, and e) failure modes of pillars in compression and shear [13]

6 Case 4: Brittle Failure Around a Circular Test Tunnel in Hard Rock

The Mine-By Experiment (MBE) was conducted in massive LdB granite at the 420 level of the URL to investigate the response induced in the rock mass by excavating a 3.5-m diameter circular tunnel using a non-explosive technique. A v-shaped notch failure was observed during the excavation of this tunnel. Two known methods for simulating the brittle failure around the MBE tunnel using continuum methods are: 1) the CWFS by Hajiabdolmajid et al. [1]; 2) and the Damage Initiation and Spalling Limit (DISL) by Diederichs [11]. Due to their fundamental limitations, these methods do not predict the extent of the EDZ. Sanipour et al. [15] developed an RS2-VTM (Fig. 5a) and calibrated the model to the tri-linear strength envelope to resolve this limitation. The emergent strength envelope (Fig. 5b) was found to have an s-shape; it follows the crack initiation stress level of LdB granite at low confinement ($\sigma_3 < 5$ MPa) and the spalling limit up to a confinement of about 30 MPa.

The core softening approach was used to simulate the 3D tunnel advance in the 2D model (Fig. 5c) to capture the progressive excavation-induced damage and failure. The simulation results (Fig. 5d) suggest that the RS2-VTM successfully captures the depth and shape of failure around the MBE tunnel (Fig. 5e). Additionally, the extent of block boundary yielding (damage) is consistent with the recorded micro-seismic events, suggesting that the RS2-VTM also replicates the EDZ.

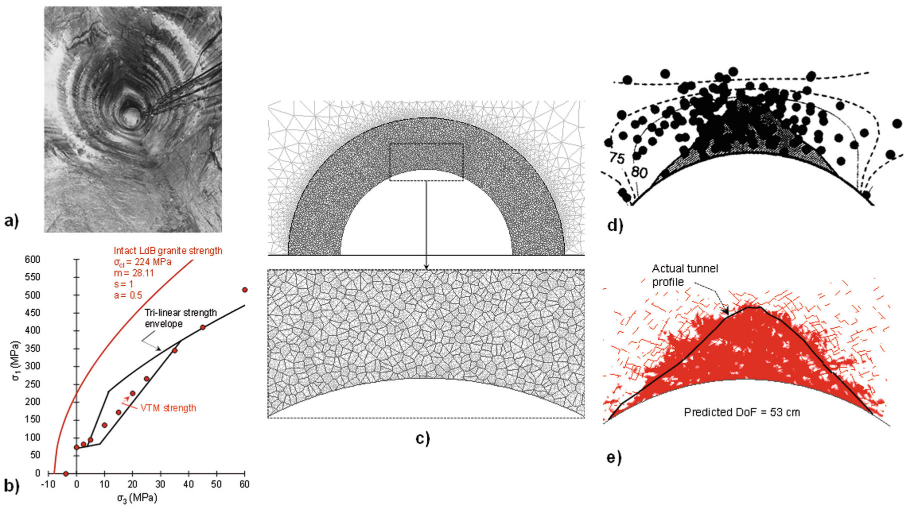

Fig. 5. a) V-shaped notch failure around MBE tunnel [16]; b) strengths of RS2-VTM calibrated to tri-linear strength envelope [15]; c) RS2 model geometry [15]; d) recorded micro-seismic events [9]; e) simulated failure (yielded mesh elements) and EDZ (yielded joint elements) [15]

7 Summary and Conclusions

In this paper, the application of a continuum-based heterogeneous model for simulating brittle damage and failure at different scales was reviewed. The Voronoi joint network in RS2 was used to develop a Voronoi Tessellated Model (VTM). The RS2-VTM consisted of randomly generated polygonal blocks separated by block boundaries (i.e., joint elements). The core-scale examples reviewed in this paper included the simulations of the laboratory behavior of Wombeyan marble and unloading-induced brittle damage in LdB granite. The field-scale examples comprised the failure of hard rock pillars at the Quirke Mine in Ontario, Canada and the v-shaped notch breakout around the Mine-By Experiment tunnel at Canada's Underground Research Laboratory. The linear Mohr-Coulomb constitutive model was used for the blocks and block boundaries in all these models. By calibrating the models, linear (intact rock in Case 1), non-linear (intact rock in Case 2 and rock mass in Case 3) and s-shaped (rock mass in Case 4) strength envelopes and realistic failure modes were captured. It is concluded that a properly calibrated RS2-VTM can be used as a reliable tool for analyzing a wide range of geomechanical problems.

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