RFEM Analysis of a Subway Station Considering Conditional Random Field

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Abstract. The concrete arch pre-support system (CAPS) is one of the construction methods of subway stations designed using deterministic soil strength parameters. A Finite Element Method (FEM) of analysis is used to calculate the design parameters, including the factor of safety and maximum displacement using deterministic soil strength values. In this study, the random and spatial variability of soil strength parameters for subway stations constructed using CAPS are considered for the first time to investigate the influence of the variability of these parameters on the design of subway stations. To generate spatially variable fields, first, selected artificial borehole data are employed in order to condition the spatially variable friction angle fields. Then, considering a typical value for the variability of the cohesion and the possible cross-correlation between the two, the spatially variable cohesion field is realized. Eventually, the design values are calculated using the Finite Element Method. Indeed, in this study, the importance of considering spatial variability for soil shear strength parameters is be addressed and its impact on the design of underground structures has been delineated. The results demonstrate that conditioning random fields and considering the cross-correlation between soil input parameters significantly reduce the level of uncertainty in the analysis and helps to render more reasonable and/or reliable results as input for design purposes.

1 Introduction

Subway station construction operations have increased significantly in recent years due to the rapid development of urbanization. The performance of such underground structures is significantly influenced by geological uncertainties. Thus, there has been a lot of interest in terms of investigating the effect(s) of spatial variability in soil parameters on this type of structure (Elkateb et al. 2003; Li et al. 2016a, b; Zhang et al. 2021b; Wang et al. 2016). It should be noted that research on tunnel stability analysis are mostly limited to considering the variability of soil parameters by applying the available probabilistic methods (e.g., random variable method (RVM)) (Hongzhan et al. 2019). The RVM disregards the spatial variability of soil parameters and model them as random variables. Thus, this presents an insufficient description of autocorrelation

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and the true uncertainty of the soil parameters. To solve this problem, random field theory was applied to characterize the soil spatial variability (Vanmarcke 2010). On this basis, numerous stochastic techniques have been well-documented, including the random finite element method (RFEM) (Griffiths and Fenton 2004), the stochastic response surface method (SRSM) (Li et al. 2011, 2015) and the random finite-difference method (RFDM) (Jamshidi Chenari and Alaie 2015; Jamshidi Chenari and Bathurst 2023). The RFEM typically uses an unconditional random field (URF) by random field theory in combination with the finite element method (FEM). Most researchers have deployed this approach to investigate the effect of spatial variability of soils and rocks on geotechnical reliability (Javankhoshdel et al. 2017; Mohammadi et al., 2019; Schweiger and Peschl 2005; Griffiths and Fenton 2004; Xiao et al. 2016; Luo and Bathurst 2018). However, little research has been reported on applying these probabilistic methods to the reliability of tunnel stability (Mohammadi et al. 2022). In contrast to URF, a novel approach known as the conditional random field (CRF) is employed, which accommodates the results of an in-situ site investigation, such as CPT data, to realize the inherent variability of the soil parameters. Yang et al. (2017) employed the Kriging method to generate CRFs with CPT data to study probabilistic stability of slopes. Li et al. (2016c) introduced a Markov Chain Monte Carlo (MCMC) method for generating CRFs from borehole data in order to represent the variability of geologic profiles. In addition, Gong et al. (2018) employed the Hoffman technique to generate CRFs of soil properties for a probabilistic evaluation of tunnel longitudinal performance. More recently, Sasanian et al. (2019) embarked on a selection of RFDM-based slope stability analyses by implementing a variety of existing site investigation data using the Kriging method. The most recent approach applied a sparse Bayesian learning method which was presented by Ching and Phoon (2017) and Ching et al. (2020). This method determines the statistical parameters (mean, standard deviation, and spatial correlation length), using limited site-specific geotechnical data. In this study, the same algorithm in combination with the RFEM analysis was used to investigate the influence of cross-correlated conditional random fields (CCRF). For the RFEM analyses, RS2 software (Rocscience 2022) was employed. Then, the effect of both RVM and CCRF on stability of a subway station were investigated.

2 Methodology

The procedure used in this investigation to create CCRF is shown in Fig. 1. First, unconditional random fields were generated for friction angle. Next, 20 artificial boreholes were created for one of the fields to extract data at specific intervals, as shown in Fig. 2. Data intervals in the vertical direction were 0.2 m.

Table 1 displays the soil properties of the project site.

Using these artificial borehole data and sparse Bayesian learning approach, statistical characteristics of the friction angle random field were then determined (Table 2). Next, CRFs of friction angle were generated. The appropriate realizations for the cohesion field were then generated using the assumed standard deviation of 13.21 for cohesion, a cross-correlation of -0.5 between the cohesion and friction angle fields, and an algorithm proposed by Sasanian et al. (2019). The procedure of this method was illustrated in Fig. 1.

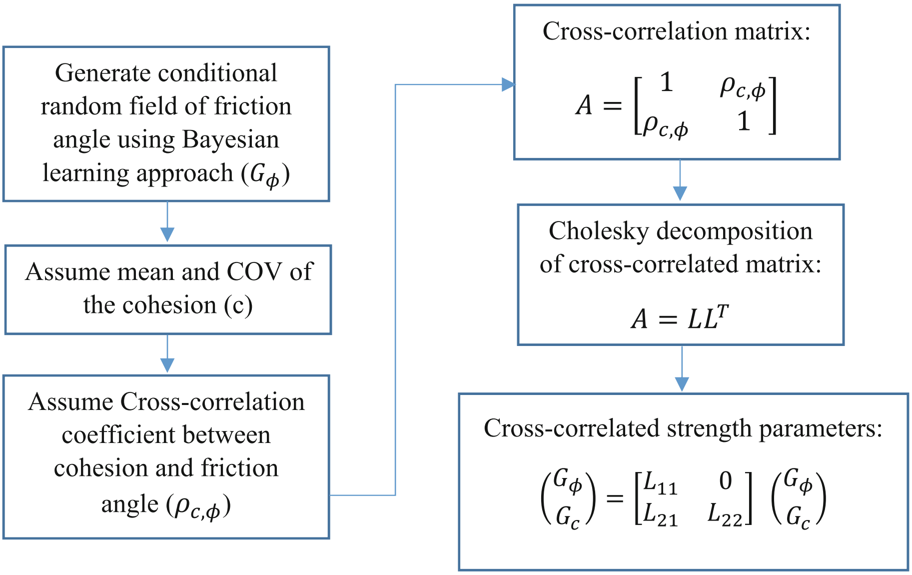


Fig. 1. Framework of the algorithm for conditional cross-correlated random field generation

Table 1. Soil properties of the project site

Parameter	Value
Unit weight $\gamma\,(kN/m^3)$	19.5
Elastic modulus E (MPa)	44.2
Poisson's ratio ν	0.3
Mean friction angle (°)	32.2
Mean cohesion (kPa)	26.4

Figure 3 illustrates the conditional and cross-correlated random fields of ϕ and c; since these fields are negatively cross-correlated, weak zones (blue) in Fig. 3a correspond to strong zones (green) in Fig. 3b.

3 Case Study

Figure 4 shows the geometry of the FEM model used in this study. The model's horizontal and vertical lengths were set to 100 m and 50 m, respectively, to reduce the boundary effect. In this study, the soil elements were modeled using the Mohr-Coulomb (MC) criterion, and supporting elements, such as piles, reinforced concrete arch beams, and shotcrete, were modeled as linear elastic materials.

First, the stability of the station was investigated under conditions where both parameters c and ϕ have spatial variability and were cross-correlated. For this purpose, one of

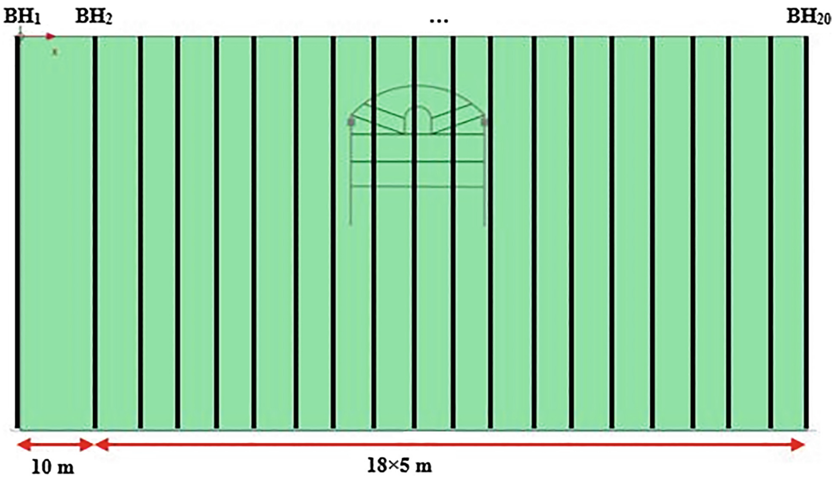


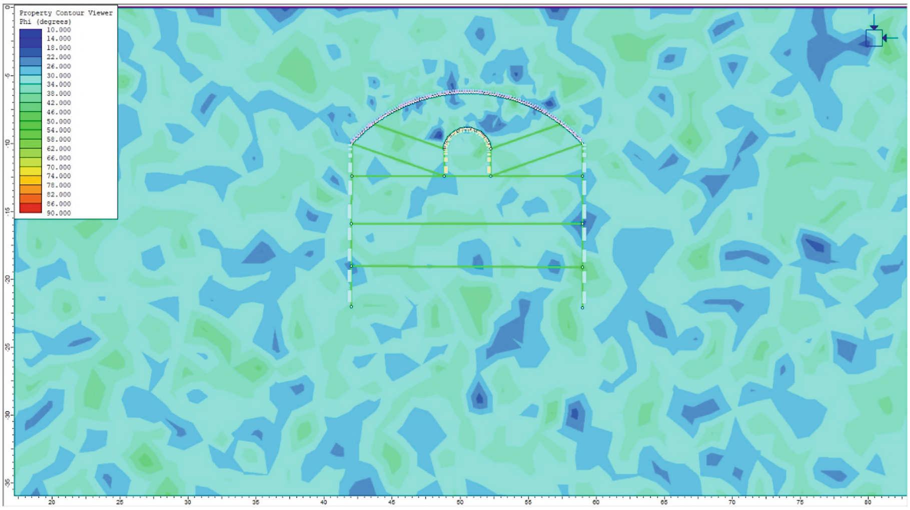
Fig. 2. Boreholes arrangement and subway station excavation location in cross-section (grey outline) for the problem under study

Table 2. Measured statistical parameters of friction angle

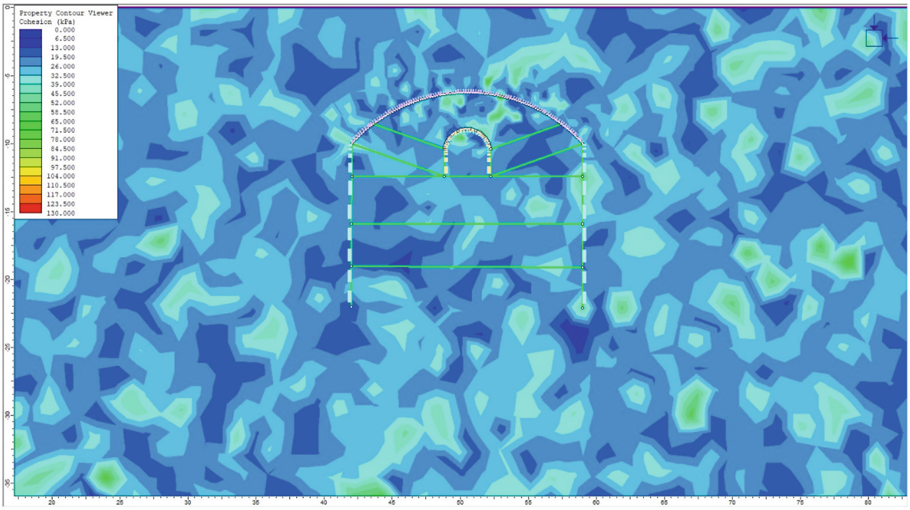
Parameter	Mean	COV (%)	Horizontal correlation length (m)	Vertical correlation length (m)
Friction angle (°)	32.2	17.5	1.65	0.97

the generated random fields for c and φ was considered, and the analysis results were presented in Figs. 5, 6 and 7. These figures show that the left pile exhibits greater deformation in the heterogeneous cases, a reflection of weaker soil in that very region. The main plastic strain around the station can also be seen in Fig. 8. However, it is important to note that the behavior of the piles can vary in different random fields, and a more accurate estimate of the station's behavior can be obtained by examining them.

As can be seen, at the subway station investigated in this study, under homogeneous soil conditions, deformations are close to symmetrical and both right and left piles experience almost similar deformations (Figs. 5a-7a) (the difference in deformations is due to the slight variation in the piles' geometry); however, in the cases with spatial variability of the c and ϕ fields (for selected random fields), the maximum deformation was observed in the left pile (Figs. 5b-7b). This asymmetry in deformation is not unexpected, because in the cases where the spatial variability of soil shear strength parameters are incorporated in the analysis, the resistance conditions of different points around the station will not be the same, and this will lead to different deformations in various places. Moreover, under homogeneous soil conditions, yielded elements are close to symmetrical, but in the cases with spatial variability of the c and ϕ fields, the area where yielding does not occur is more extensive. This is quite reasonable considering the resistance conditions of the soil parameters (c & ϕ) in this area (Figs. 8a-8b). It is crucially important to pay



(a)



(b)

Fig. 3. Sample conditional and cross-correlated random fields of a) friction angle; b) cohesion

attention to this issue while designing such geotechnical structures leading to the more realistic design of the structures.

Thus, incorporating spatial variability in the analysis can lead to asymmetrical deformations and yielded elements, which can have significant implications for the safety and performance of the structures. Therefore, it is crucial to pay attention to this issue while designing geotechnical structures to ensure a more realistic and accurate design that considers the variability of soil parameters in the analysis. This can help improve the safety and reliability of geotechnical structures and prevent potential failures.

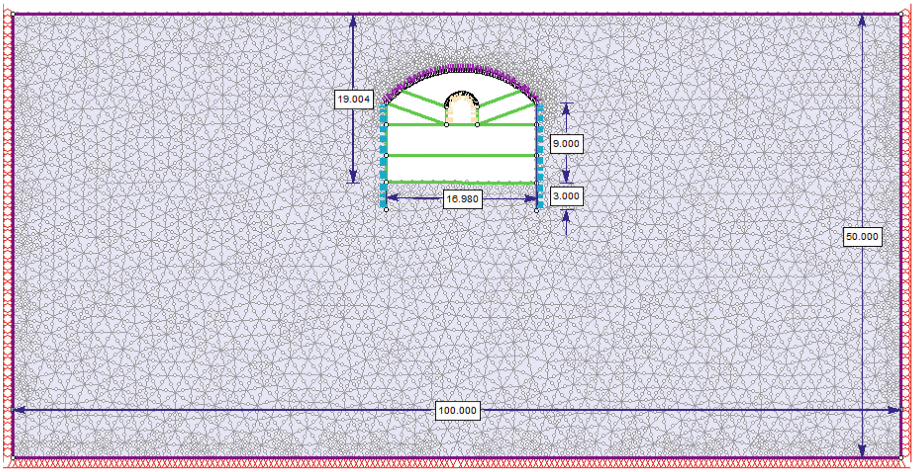
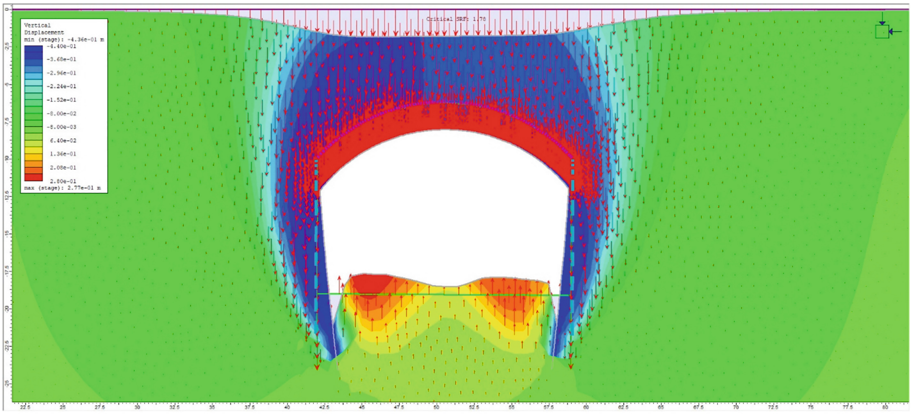


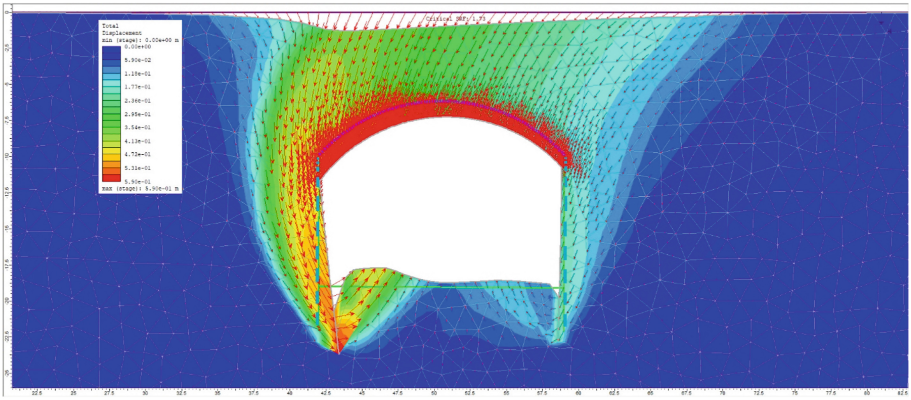
Fig. 4. An example model used in this research

In the next step, to assess the possible advantages of the CRF over the RVM, using the coefficient of variation of soil parameters derived from friction angle and cohesion conditional random fields, an alternative single random variable analysis is performed. For this purpose, the log-normal distribution for c, ϕ, and E parameters, as well as $COV_c = 0.5$, $COV_\phi = 0.2$, and $COV_E = 0.2$ are considered, and 500 random samples were generated using the Latin hypercube sampling method for these parameters. Then, the probability density functions of the safety factor and the maximum displacement around the cavity facing are investigated (Figs. 9 and 10).

As can be seen in Figs. 9 and 10, the two output variables (factor of safety and maximum displacement), represent a normal probability density function. On the other hand, it can be seen that the use of the CCRF not only affects the standard deviation of the outputs, but also causes a change in the mean value. In comparison to the random variable approach, CCRFs showed a decrease in the coefficient of variation and mean factor of safety at approximately 84% and 1.7%, respectively. The corresponding values for the maximum displacement are 61% decrease, and 76% increase. Additionally, for the factor of safety and maximum displacement, there has been a reduction of 83.5% and 15% in terms of data distribution (see Table 3). Therefore, it can be concluded that the application of CCRF has increased the certainty of the analysis and expected to a more realistic design.

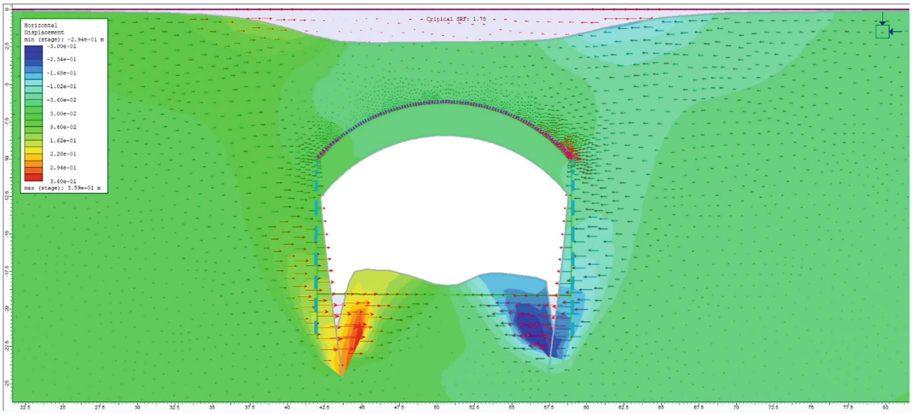


(a)



(b)

Fig. 5. Vertical displacement in a) homogeneous soil deposit; and b) conditioned cross-correlated random fields soil deposit



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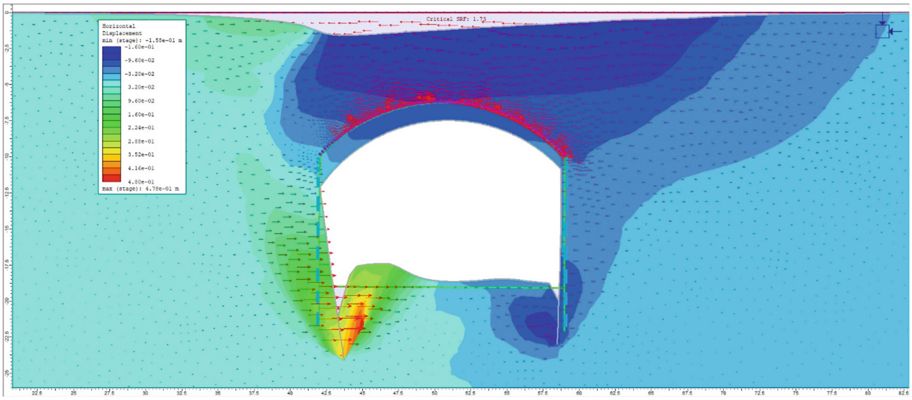
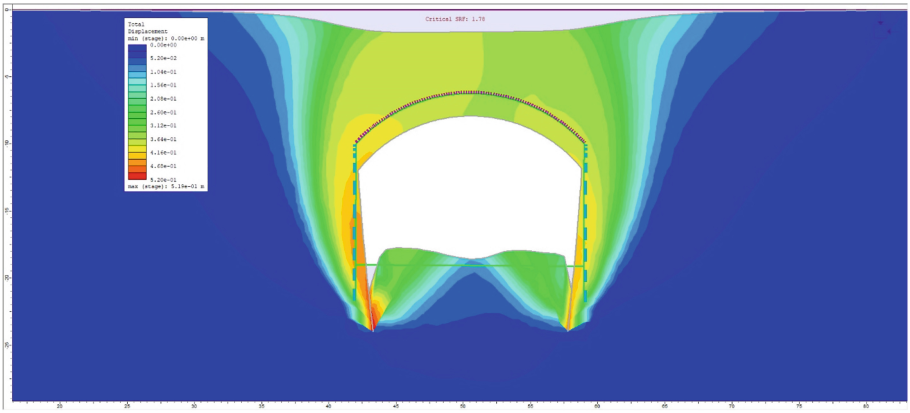
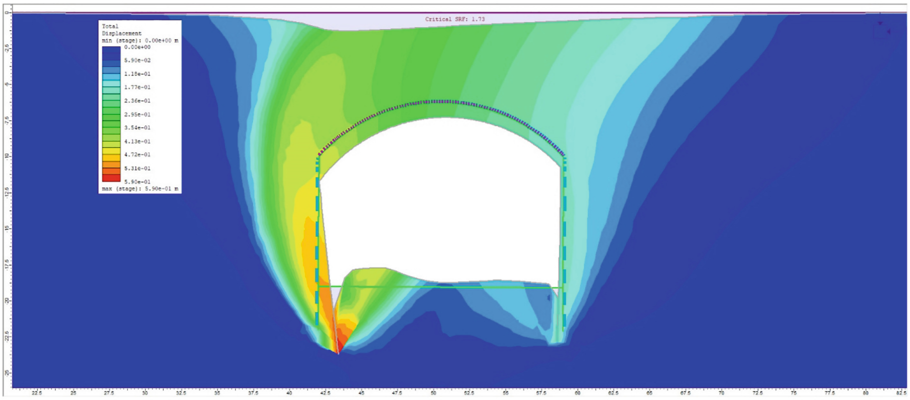


(b)

Fig. 6. Horizontal displacement in a) homogeneous soil deposit; and b) conditioned cross-correlated random fields soil deposit

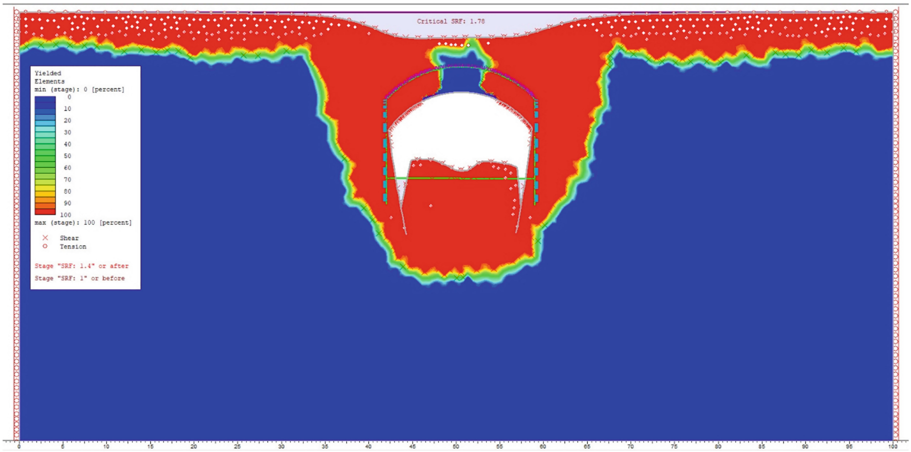


(a)

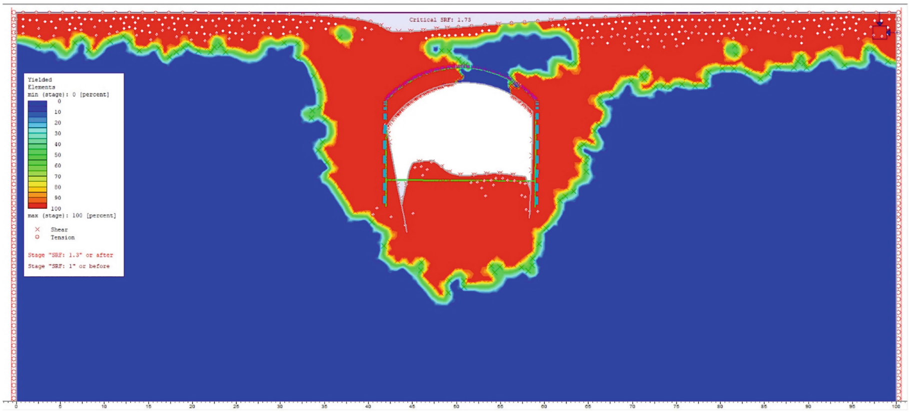


(b)

Fig. 7. Total displacement in a) homogeneous soil deposit; and b) conditioned cross-correlated random fields soil deposit for SRF of 1.78



(a)



(b)

Fig. 8. Plastic zones formation around the subway station in a) homogeneous soil deposit; and b) conditioned cross-correlated random fields soil deposits

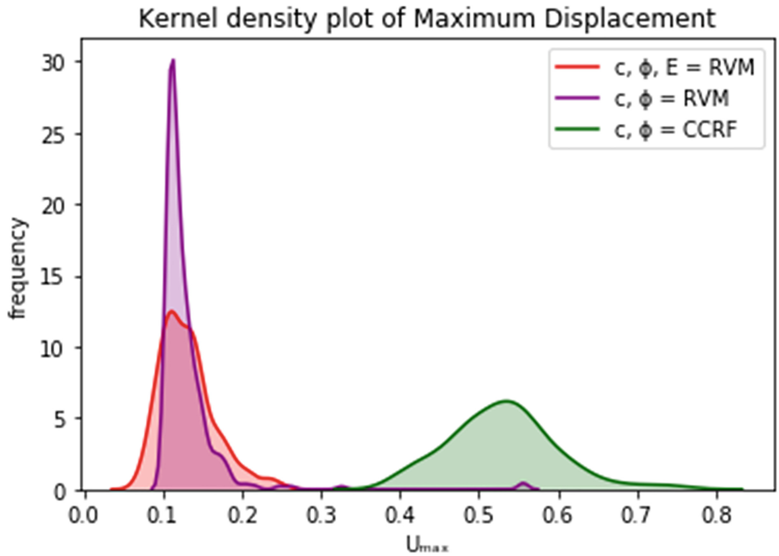


Fig. 9. Probability density functions of the maximum displacement

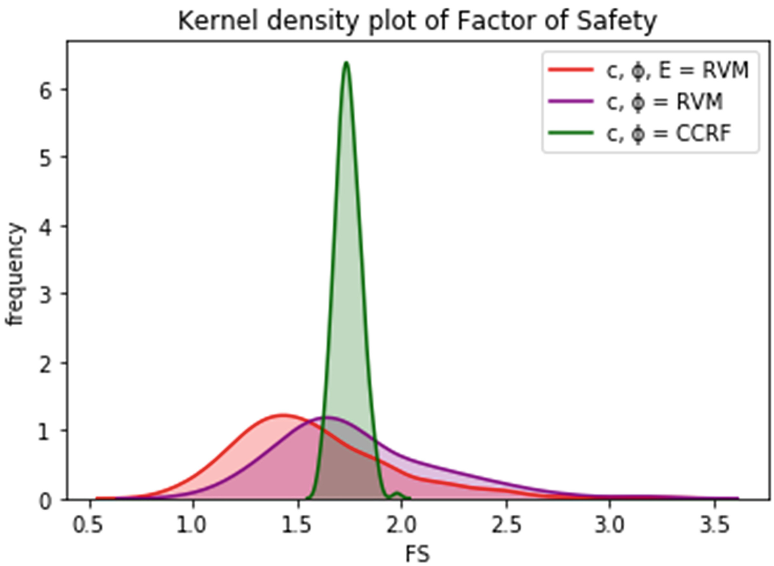


Fig. 10. Probability density functions of the safety factor

Table 3. Measured lower and upper bound and coefficient of variation for the factor of safety and maximum displacement obtained from RVM and CCRF methods

Parameter	Random variable method (RVM)			Cross-correlated conditional random fields (CCRF)		
	Min	Max	COV	Min	Max	COV
FS	1.0	3.25	0.22	1.61	1.98	0.035
U_{max}	0.1	0.56	0.33	0.38	0.77	0.130

Min: minimum; **Max**: maximum; **COV**: coefficient of variation

4 Conclusion

In this study, conditional cross-correlated random fields for soil shear strength parameters (c and ϕ) were generated, and the influence of these assumptions was investigated through the stability analyses of a subway station using RFEM. In order to generate correlated conditional random fields, Bayesian concepts and Cholesky decomposition method were used. The results confirm that the output random variables (factor of safety and maximum vertical and horizontal displacement) follow a normal distribution. Also, distinct plastic zones formation patterns have been reported compared to a homogeneous soil condition. Therefore, it is possible to identify high-risk areas in the conditioned non-homogeneous case more precisely. It is also worth mentioning that conditioning the random fields corresponding to the shear strength parameters, based on select in-situ loggings, notably increases the level of certainty and thus, significantly reduces the probability of failure.

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