







Probabilistic Analysis of an Embankment Under Extreme Rainfall Events Considering Spatial Variability of Soil Strength Parameters

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Abstract. Studies have shown that the stability of embankment slopes are influenced significantly by extreme rainfall events. The results of previous studies have also indicated that embankments built with fine materials such as silt are more susceptible to extreme rainfall events than those built with coarse materials. In most of these studies, soil parameters are assumed to be random variables. However, soil properties can vary spatially within a soil profile. This has been shown by several researchers carrying out stochastic analyses. Therefore, as a preliminary study, this research investigates the influence of spatial variability of soil strength parameters including friction angle and unit weight on the probability of failure. A simplistic assumption of constant hydraulic conductivity throughout the soil profile is made. The results are compared with analyses in which assumption of random variable soil parameters is made. The results show a reduction in the value of the probability of failure for the case with spatially variable soil strength parameters compared to the case with random soil parameters. Therefore, it can be concluded that considering spatial variability in soil strength parameters can provide a better representation of the actual site condition.

Keywords: Slope stability · Spatial variability · Stochastic analysis

1 Introduction

Research has demonstrated that the stability of soil embankments can be significantly impacted by water infiltration, which often occurs during rainfall events. In addition, the initial moisture levels within the slope prior to the onset of rainfall can also play a role in the susceptibility of soil embankments to instability. Soil geotechnical parameters that control the stability of earth embankments can vary spatially since soils are inherently heterogeneous materials. In addition, measurement errors and insufficient testing contribute additional uncertainty to the soil properties used in the analyses. Therefore, all these uncertainties are expected to be considered in the evaluation of the slope stability of embankments [1].

The surface of soil embankments is exposed to the atmosphere. Water exchange at the soil-atmosphere boundary affects the soil water storage and pore water pressure distribution and eventually, the stability of soil embankments [2].

This study investigates the influence of spatial variability of soil strength parameters including friction angle and unit weight on the probability of failure of soil embankments. The probabilistic analysis considering spatially variability of soil parameters is carried out for a soil embankment under an extreme rainfall event with a variably saturated initial condition. The results are compared with the results of deterministic analyses and single random variability analysis (SRV).

1.1 Background

Changes in extreme rainfall patterns due to climate change have the potential to affect the stability of both natural and constructed slopes. Highway embankments, as an example, are likely to experience rainfall-induced slope failures because of their continuous exposure to changing atmospheric conditions.

Pk et al. [3] studied the effect of extreme precipitation events on the probable instability of a typical highway embankment in Southern Ontario, Canada. They conducted two-dimensional (2D) transient variably saturated seepage finite-element analyses to evaluate pore-water pressures and 2D limit equilibrium slope stability analyses for stability assessments of embankments. Their results indicated that the cumulative annual net infiltration in the future could significantly increase, which could result in a reduction in the embankments' factor of safety. They also reported that embankments constructed with low permeability fine materials such as silt showed a lower factor of safety (FOS) owing to their ability to retain water and conduct it slowly.

In 2020, Baninajarian et al. [4] developed the fragility curves for a slope of a soil embankment subjected to extreme rainfall in Niagara Falls, Ontario. Two different types of failure i.e., general and shallow failures, were considered. They used a coupled hydro-geotechnical model to assess the slope stability over time during extreme events. The fragility curves were developed using the First Order Second Moment (FOSM) method and the associated reliability analyses assumed various soil parameters to be random variables. The results indicated that the embankments built with fine materials such as silts, are susceptible to an increase in the probability of failures during prolonged rainfall events with a higher return period. It was also concluded that the occurrence of a shallow failure in this type of embankment is more probable than the occurrence of a general failure.

Cami et al. [5] conducted a study to provide a reference to present available methods to measure the scale of fluctuation parameter (spatial correlation length) which is a required parameter to best characterize and to simulate a spatially variable field. They also presented a database of horizontal and vertical scale of fluctuation values in different locations and for different materials, collected from published case studies. Javankhoshdel et al. [6, 7] and Cami et al. [8] presented the Random Limit Equilibrium Method (RLEM) approach to carry out probabilistic analysis using LEM considering spatial variability of soil properties.

The RLEM approach is used in this study to carry out probabilistic analysis using Slide2 software which is a 2D limit equilibrium slope stability program [9]. This software is utilized to perform groundwater finite element seepage analysis and assessment of factor of safety (FOS). In addition, probability of failure (PF) in soil embankments under rainfall event with variably saturated initial condition is estimated. Non-circular

Particle Swarm Search is a Metaheuristic approach that is used as the search method to locate the critical slip surface and the General Limit Equilibrium / Morgenstern-Price [10] method [9] is applied in this study.

The shear strength of unsaturated soil is estimated using Vanapalli et al. equation [11]:

$$\tau = c' + (\sigma - u_a) \tan \phi' + [(\theta - \theta_r)/(\theta_s - \theta_r)]\psi \tan \phi' \quad (1)$$

where c' is effective cohesion, ϕ' is the effective angle of internal friction, θ , θ_s and θ_r are soil volumetric water content, saturated water content and residual water content, respectively, and $(\sigma - u_a)$ and $(u_a - u_w)$ are the net normal stress and the soil suction, respectively.

Simulations are performed considering transient flow analysis and the factor of safety of the embankment is evaluated at different time steps. Deterministic analyses are conducted to establish a benchmark for comparing the results of probabilistic analyses to demonstrate the effect of the variability of input parameters on the probability of failure. The probabilistic analyses are carried out to evaluate the probability of slope failure considering the inevitable influence of uncertainties in some soil parameters such as friction angle and unit weight. Spatial variability analyses are carried out to quantify the effect of spatial variation of soil parameters within a soil mass on the probability of failure. When employing the RLEM approach for probabilistic analyses, it is necessary to establish random variables and their associated statistical parameters. This includes defining the probability distribution function (PDF) of the variables, as well as determining the standard deviation and scale of fluctuation in both horizontal and vertical directions.

1.2 Geometry and Materials

The embankment profile considered in the current study is representative of a typical highway embankment in Ontario, as shown in Fig. 1. The geometry of the embankment is symmetrical, so only one-half of the domain is simulated. The height of embankment is considered to be 8 m [12] and side slopes of the embankment are 2H:1V and a 3 m width unpaved shoulder was assumed at the top of the embankment. The distance between the slope toe and the right side of the model was set to more than three times the height of the slope to minimize the influence of the side boundary conditions [13]. Sandy silt material, from now on, referred as silt is considered in this study as the embankment fill material. The saturated unit weight (γ) of 19 kN/m³, the friction angle (ϕ) of 32° are considered for this material and the effective cohesion is assumed to be zero since the materials used in the construction of embankments are non-cementitious [1].

For the sake of simplicity, a constant hydraulic conductivity function is assumed throughout the soil profile in this study. The hydraulic behavior of unsaturated soils is defined by the soil-water Characteristic Curve (SWCC) and unsaturated hydraulic conductivity function (HCF) using the van Genuchten-Mualem approach [14]. Soil-water Characteristic Curves for silt is shown in Fig. 2 based on [1].

In this study, the variability in the values of two important soil parameters, the angle of internal friction and unit weight, are considered. To establish a rational range of values for these parameters, a simple method called Three-Sigma Rule is used in the

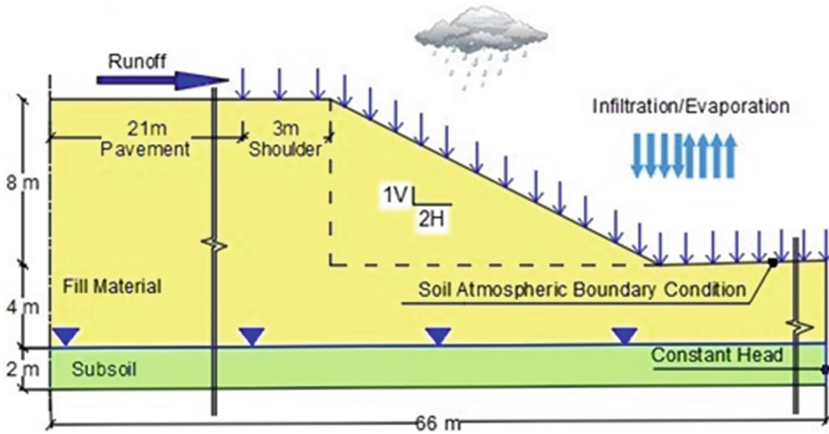


Fig. 1. Typical highway embankment in Ontario considered in this study (modified from [1])

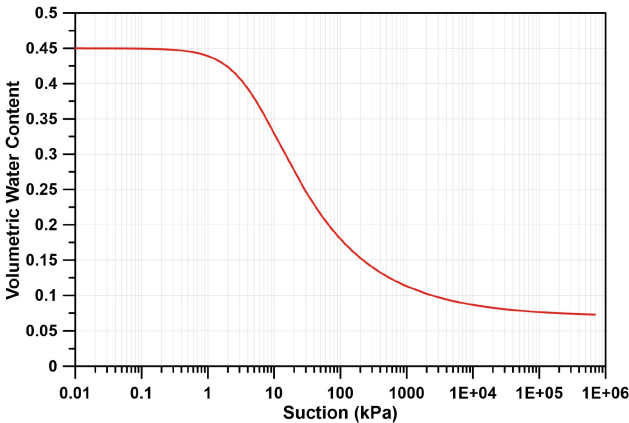


Fig. 2. Soil water characteristic curves for silt material, used in this study (modified from [1])

SRV Analysis. Based on this method suggested by Dai and Wang [15], an extremely low value would be three standard deviations below the average, and an extremely high value would be three standard deviations above the average.

For the angle of internal friction, the coefficient of variation (COV) between 2 to 13% has been suggested by several researchers (e.g., [16, 17], and [18]). In this study, COV of 12.5%, is chosen for the angle of internal friction. The suggested COV for the unit weight in the literature is 3% to 7%, a value of 5% is used in this research. Lognormal distribution is chosen as the probability distribution function for ϕ and γ . Soil parameters considered in probabilistic analyses are summarized in Tables 1 and for the probabilistic slope stability analyses, Latin-Hypercube method using 2000 samples is used.

Table 1. Mean values and standard deviations of soil parameters considered in this study

Parameters	Mean value	Standard Deviation	Rel. Min.	Rel. Max.
Friction Angle	32	4	12	12
Unit Weight	19	2	6	6

1.3 Initial and Boundary Conditions

The initial moisture condition dictates the pore water pressure in the embankment, which can have a significant influence on the stability of a slope. Baninajarian [1] calculated the average degree of saturation within the slope area to assess slope moisture conditions. The cumulative water storage in the slope area was estimated by carrying out variably saturated flow simulations using Hydrus 2D [19]. Statistical analyses were carried out to determine the average and maximum moisture conditions in the slope to identify critical times at which pore water pressure (PWP) distribution puts the slope at risk. Use of percentiles to represent initial condition, in terms of degree of saturation was proposed. In the current study, long-term variations of the spatial distribution of pore water pressure (PWP) within the embankments corresponding to the 90th percentile of saturation (P90%) is used.

The water table was conservatively assumed to be at the natural ground surface, which is 4m below the level of slope toe. A flux boundary comprising of an actual rainstorm pattern based on Keifer and Chu [20] known as the Chicago design storm is applied at the soil-atmosphere interface. This method has also been recommended by the Ministry of Transportation Ontario, for the assessment of the storm impacts on the drainage systems [21]. Figure 3 shows the design storms applied in this study. This Chicago curve was developed by Baninajarian [1] based on the future Intensity-Duration-Frequency (IDF) curves of 48-h rainfall with the 100 years return period for the city of Niagara Falls. However, the simulation is continued beyond the duration of the rainfall event to 60 h to ensure that the minimum FOS was obtained.

2 Analysis and Results

2.1 Deterministic Analysis

Figure 4(a) to 4(f) illustrate the PWP distribution from the seepage analyses through the slope at different stages including the (a) initial stage, and then (b) 15 h, (c) 18.5 h, (d) 22 h, (e) 48 h, and eventually, (f) 60 h after the rainfall starts. Figure 4(a) indicates the initial PWP condition in the slope. At this time, rainfall starts and the pore water pressure in the slope starts to increase by the infiltration. The change in the phreatic surface and pore water pressure due to the infiltration of the water into the slope are shown in the following figures.

The temporal variation of FOS of slope under the rainfall is presented in Fig. 5. The increase of pore water pressure shown in Fig. 4b through 4f results in reducing the soil shear strength and consequently, the factor of safety decreases during 48 h of rainfall.

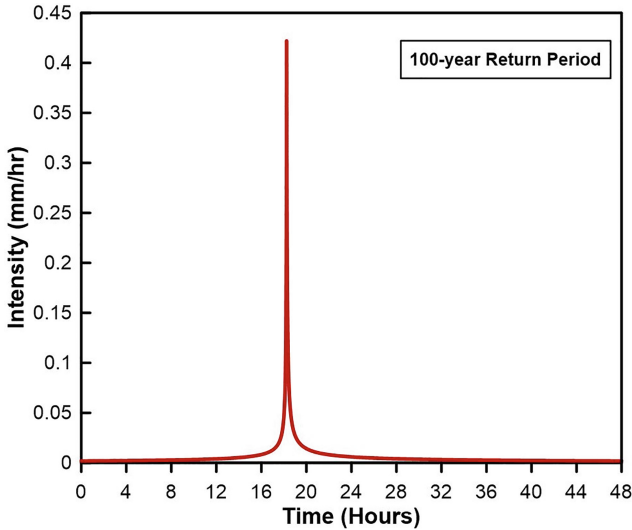


Fig. 3. Chicago curve for 48-h extreme rainfall – City of Niagara Falls (modified from Baninajarian [1])

However, the variation of FOS is rather insignificant, under the rainfall, at different stages due to the higher retention and lower conduction capacity of the silt. It can be observed from Fig. 5 that by the time that the rainfall finishes, the FOS starts increasing. To investigate the influence of the variation of soil input parameters on the probability of failure, probabilistic analyses are carried and are discussed in the following sections.

2.2 Single Random Variability Analysis (SRV)

For the probabilistic analyses, soil unit weight and friction angle are considered as random variables and their values are as presented in Table 1 earlier. The slope probability of failure and values of mean FOS at different stages obtained from SRV analysis are shown in Fig. 6. This figure shows an insignificant change in the mean FOS over time. The mean FOS changes from 1.28 at initial hours of rainfall to 1.27 at the end of rainfall and 12 h thereafter. In addition, the probability of failure varies from around 6% to 6.5%.

It is noticeable that values of probability of failure obtained from SRV analysis are relatively high considering an acceptable mean FOS of about 1.3 and based on target PF = 0.01% recommended by Silva et al. for permanent well-engineered and constructed unreinforced soil slopes and embankments [22]. This indicates that ignoring the spatial variability of soil properties results in an unrealistically high probability of failure. The following section shows the influence of considering spatial variability of soil properties.

2.3 Stochastic Analysis

To consider spatial variability of soil parameters, horizontal correlation of 33.2, and vertical correlation of 2.08 reported by Cami et al. [8] are defined in the analyses along with the cross-correlation coefficient of 0.5 between unit weight and friction [6].

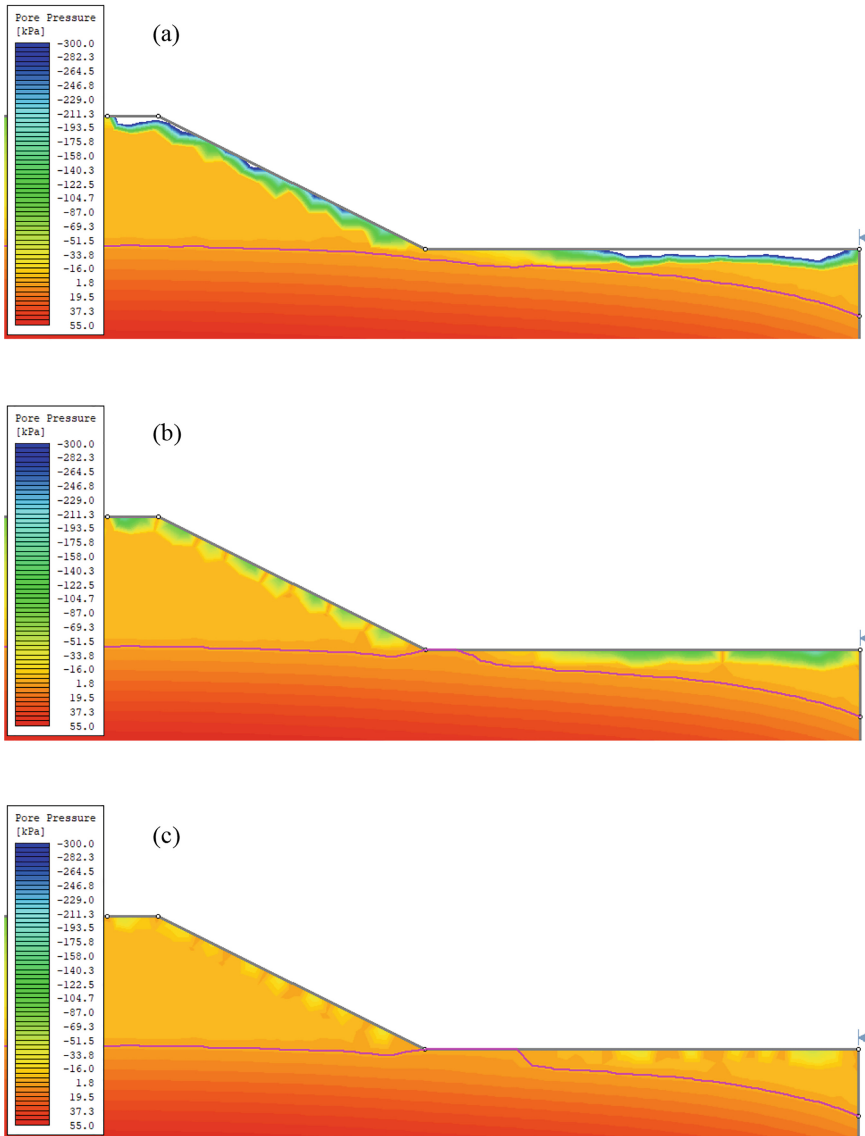


Fig. 4. Spatial distribution of PWP in the slope at the (a) initial stage, after (b) 15 h, (c) 18.5 h, (d) 22 h, (e) 48 h, and (f) after 60 h

The variation of PF and FOS over time obtained from Stochastic analysis is presented in Fig. 6. It can be observed that the stochastic analyses result in the PF of about 1% for FOS of around 1.20. This value of PF is more acceptable considering the design values of PF and FOS [23].

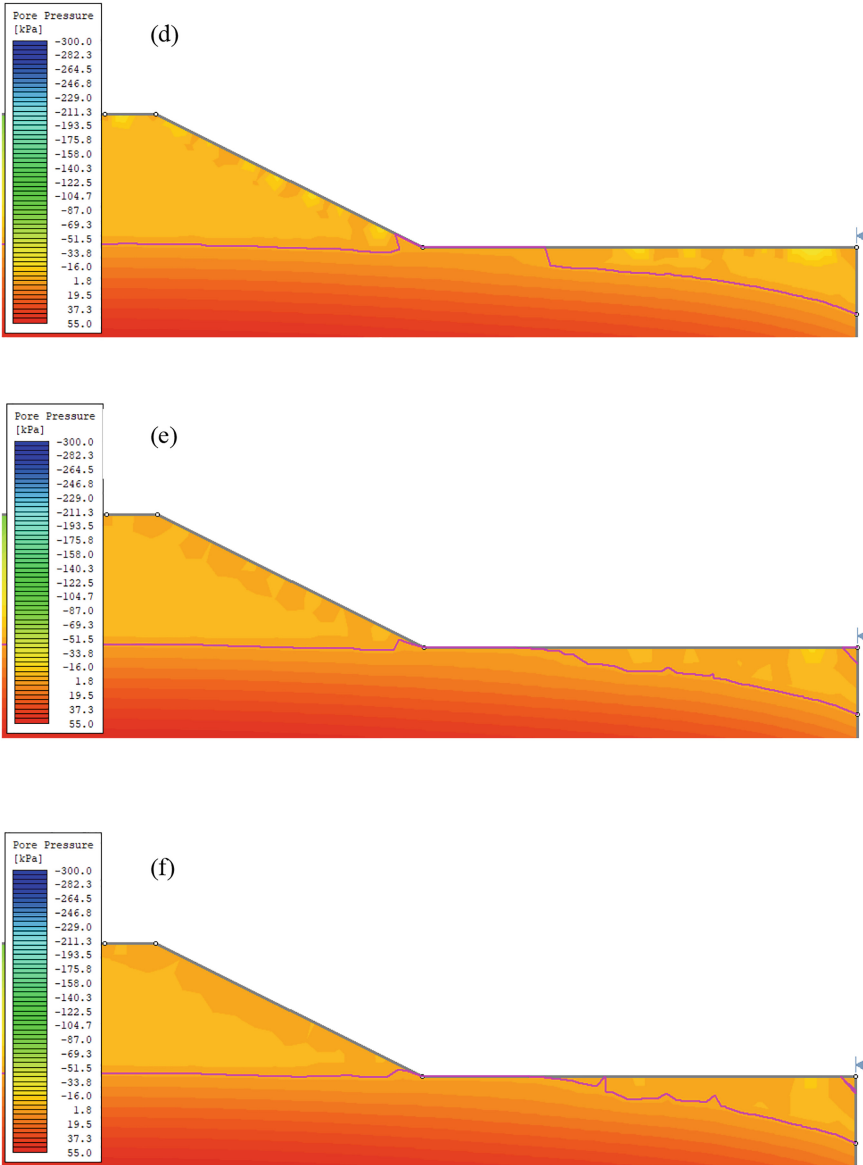


Fig. 4. (continued)

3 Concluding Remarks

This study investigates the influence of spatial variability of soil strength parameters, friction angle and unit weight on the probability of failure for an embankment under an extreme rainfall event. A Finite Element Seepage analysis is carried out to calculate the

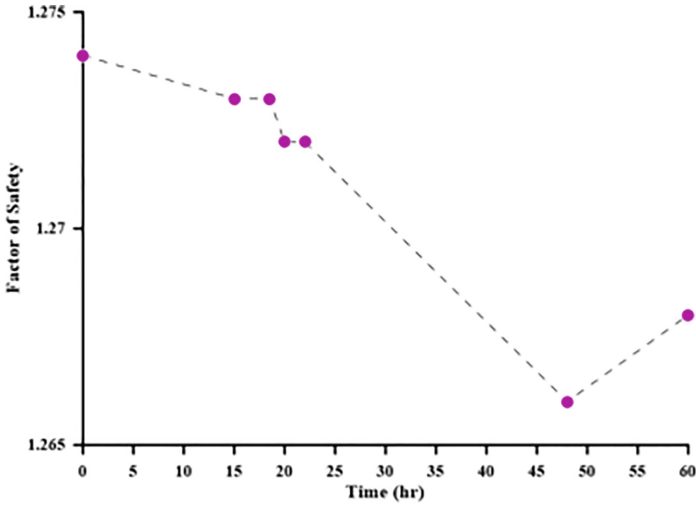


Fig. 5. Temporal variation of FOS obtained from the deterministic analysis

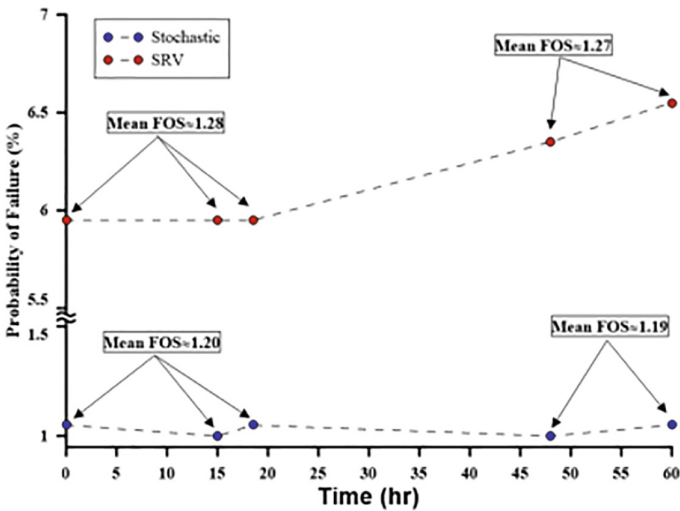


Fig. 6. Variation of FOS and PF over time - SRV Analysis and Stochastic Analysis

pore water pressure at different times during the rainfall event using a transient boundary condition. The results are compared with analysis in which assumption of random variable soil parameters is made. The results show a reduction in the value of the probability of failure for the case with spatially variable soil strength parameters compared to the case with random soil parameters. It should be noted that the spatial variability analysis can provide PF values closer to the design PF for slopes. However, in order to gain more accurate results and reduce the level of uncertainties, it is recommended to consider conditional random field.

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