A Simplified Method of Incorporating Monitored Data for Settlement Prediction Using Bayesian Back Analysis Compared with Settle3

Merrick Jones[1,2](✉), Shan Huang[2], and Jinsong Huang[2]

[1] Science & Environment, University of Newcastle, Callaghan, NSW, Australia
Merrick.Jones@tetratech.com
[2] Tetra Tech Coffey Pty Ltd, Chatswood, NSW, Australia

Abstract. Simplification of the geotechnical model and soil parameters is common in engineering practice however review of the performance to verify and updated the prediction is seldom. However, oversimplification may not capture the appropriate conditions for reliable settlement prediction. Bayesian back analysis provides a way to update the adopted prior parameters using monitored data. Parameters such as the compression ratio, recompression ratio, creep strain rate and the coefficient of compressibility were treated as random variables. Prior predictions for a three layered model were analysed using two numerical analysis programs for comparison. Posterior predictions using a simplified model showed the surface settlement was well predicted utilising about 117 to 215 days of observed data. The settlement data was used to update the selected parameters through Bayesian back analysis to fit the time-settlement history.

Keywords: Bayesian updating · consolidation · embankment · settlement prediction

1 Introduction

Soft soils are typically described as a fine-grained material exhibiting low permeability and strength coupled with high potential for compressibility due to relatively large void ratios. It is this potential for relatively high compressibility under an increased effective stress or deformation under constant effective stress that typify the problems associated with infrastructure built upon them. The key issue in addressing these problems is to adequately characterise the ground conditions and provide reliable predictions so the associated risk or opportunity implications can be communicated with confidence to both technical and non-technical decision makers.

2 Bayesian Back Analysis

Bayesian back analysis is a stochastic method well suited to geotechnical engineering due to the scarcity of information available for a given subsurface problem. For surface settlement prediction the material properties, geometry and loading conditions are typical

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inputs and considered for Bayesian back analysis. Soil parameters are relatively uncertain compared to the geometry and loading conditions therefore the focus was on these soil parameters and their modelling as random variables. Using Bayes theorem, the posterior information is inferred by updating the prior probability distribution with the observed measurements and is expressed by:

$$P[x|d] = \frac{P[d|x]P[x]}{P[d]} \tag{1}$$

where $P[d]$ is the prior probability of d, $P[d|x]$ is the likelihood function and $P[x]$ is the prior distribution which reflects the prior knowledge about x obtained from the literature, engineering judgement or site investigations before field data is obtained. The posterior distribution $P[x|d]$ represents the updated knowledge obtained from our observations about x which incorporates the prior and updated information obtained from field measurements and testing data (Kelly and Huang 2015). The prior distribution can be obtained through assumption, measurement, or a combination of both. A measurement error e was introduced to describe the difference between the measured performance d and the modelled prediction $F(x)$ shown as:

$$d = F(x) + e \tag{2}$$

where x is the random variable, e is the 'error' difference between the measurement and model prediction. The likelihood function $L_i\ (x|d_i)$ presents the difference between the measurements d_i and the predictions F_i(x), which is caused by the measurement errors e_{i} which is assumed to follow a zero-mean Gaussian distribution and can be modelled explicitly through PDF f_e (•). The likelihood function is proportional to the probability of observing the behaviour for any given value of x (Vrugt 2016) and is expressed by:

$$L_i(x|d_i) = \frac{1}{(2\pi)^{\frac{N_d}{2}} det(R_i)^{\frac{1}{2}}} \times \exp\left\{-\frac{1}{2}[d_i - F_i(x)]^T R_i^{-1}[d_i - F_i(x)]\right\} \tag{3}$$

Where N_{d} is the number of points in a specific type of observation, i represents the number of types of monitored behaviour and R_{i} is the coefficient of variation (COV) of the measurement error corresponding to the monitored data and can represented by:

$$R_i = \begin{bmatrix} \sigma_{i,1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{i,1}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{i,N_d}^2 \end{bmatrix} \tag{4}$$

where $\sigma_{i,j} = COV_{i,err} x d_{i,j}$ and $COV_{i,err}$ is the coefficient of variation (COV) of the measurement error corresponding to the monitored data $d_{i,j}$ ($j = 1, 2, \ldots, N_d$). To undertake a simulation an estimate of the means, variances and probabilities associated must be input which are applied to the prior parameters in this case. Markov chain Monte Carlo (MCMC) simulations are used to assist in problems that defy analytical solutions. It can treat the input values as random variables with an assigned distribution to provide a probabilistic solution rather than a deterministic one. The MCMC algorithm is applied to generate the required samples from the posterior distribution of the model parameters.

A Markov chain Monte Carlo (MCMC) algorithm developed by Vrugt (2016) known as a Differential Evolution Adaptive Metropolis (DREAM) program which incorporates a likelihood function to estimate and update the posterior distribution function (PDF) of the model parameters was used. The posterior probability density function (PDF) was sampled by the algorithm DREAM. The posterior distribution represents the updated knowledge obtained from our observations and incorporates the prior information and the updated information obtained from field monitored behaviour (Kelly and Huang, 2015). The likelihood is assumed to follow a zero-mean Gaussian distribution and can be modelled explicitly through PDF. A more detailed description of the algorithm implemented is discussed in Zheng et al. (2018) and Zeng *et al.* (2019).

3 Embankment Prediction

3.1 Loading and Ground Model

An embankment approximately 16 m wide and 80 m in length along the crest with a final height of about 3 m with prefabricated vertical drains (PVD) was constructed on typical 'Ballina Clay' as described in Pineda et al. (2016). The soil stratigraphy comprised of a 1.5m topsoil crust, a 9m thick estuarine (Ballina) clay, a 4m clayey sand transition zone, followed by 5m of fine grained sands and a stiff to hard Pleistocene clay to depths greater than 40m. The lower sands and Pleistocene layers were combined for the three layered model as consolidation of these layers was expected to be comparatively small relative to the soft estuarine clay and low thickness of the fill embankment. The drainage is assumed to be free draining at the top and impermeable at the base of the soft to very stiff clay layer. The water table adopted was 1.5 m below ground with no assumed rise in ground water considered that may reduce the applied effective stress. Ground investigation data and interpreted laboratory results are outlined in Pineda et al. (2016).

The idealised three layered model is presented in Fig. 1. The construction time, loading stage time and vertical pressure from the low embankment are summarised in Table 1.

3.2 Geotechnical Parameters

The deformation parameters adopted for analysis were the compression ratio ($C_{c\varepsilon}$), the recompression ratio ($C_{s\varepsilon}$) and the creep strain rate ($C_{\alpha\varepsilon}$). The rate of consolidation parameter adopted was the coefficient of consolidation in the horizontal direction (C_h) due to prefabricated vertical drains (PVD). The vertical coefficient of consolidation (C_v) was taken as equal to the horizontal coefficient of consolidation (C_h). The values of equivalent radius (r_s), smear zone ratio (r_m/r_s) and permeability ratio (k_s/k_m) are shown in Table 3. The initial compression ratio was derived from laboratory testing, review of the literature and engineering judgement. The harmonic mean values were adopted from Zheng et al. (2018) and Gong and Chok (2018) to simplify the parameter values for the three layered models. Prior parameters for the three layered models are shown in Table 3.

The recompression ratio and creep strain rate were derived using a ratio from the compression ratio. The prior values for the recompression ratio were initially taken

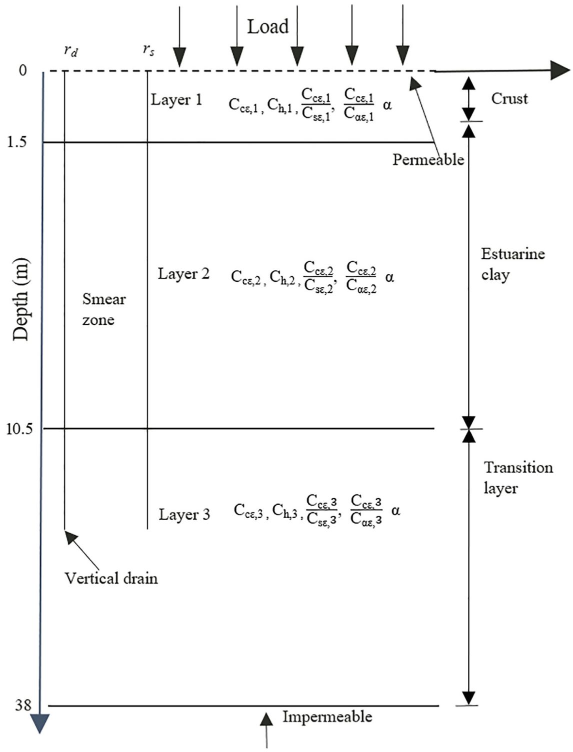


Fig. 1. Soil profile, boundary condition and random variables for the three layered model

Table 1. The stage time, description and embankment loading

Stage	Description	Time (days)	Total pressure (kPa)
1	Fill platform to 0.6m	1	0
2	Consolidate	12	12.6
3	Install drainage layer	20	12.6
4	Consolidate	50	21.6
5	Fill to 3.0m	60	21.6
6	Consolidate	974	61.5

as $C_{c\varepsilon}/5$ for and $C_{c\varepsilon}/15$ for the creep strain rate. The factor of compression ratio over recompression ratio ($C_{c\varepsilon}/C_{s\varepsilon}$) and compression ratio over the creep strain rate ($C_{c\varepsilon}/C_{\alpha\varepsilon}$) were deemed to be uniformly distributed with a range of 5 to 10 and 15 to 25, respectively. $C_{c\varepsilon}$ and C_h were deemed to be statistically lognormally distributed to ensure the random variable remained positive (Huang and Griffiths 2010). The preconsolidation pressure ($\sigma_p\prime$) across the whole profile was multiplied by a universal alpha (α) correction factor to

Table 2. The stochastic material parameters

Parameter	Properties	Distribution	COV $\left(\frac{\sigma}{\mu}\right)$
Compression ratio ($C_{c\varepsilon}$)	$C_c(1 + e_o)$	Lognormal	0.3
Coefficient of horizontal consolidation (c_h)	m^2/yr	Lognormal	3.0
Ratio of Recompression to compression ratio	$C_s(1 + e_o)$	Uniform	0.19 $\left(\frac{1.43}{7.5}\right)$
Ratio of Creep strain rate to compression ratio	$C_\alpha(1 + e_o)$	Uniform	0.14 $\left(\frac{2.88}{20}\right)$
alpha (α) for preconsolidation	-	Uniform	0.21 $\left(\frac{0.2}{0.95}\right)$

Note: COV's suggested by Duncan (2001) and Zheng et al. (2018)

account for the rate-effect and was taken as uniformly distributed with a range of 0.6 to 1.3. Therefore, each soil layer has four parameters $C_{c\varepsilon}$, C_h, $C_{c\varepsilon}/C_{s\varepsilon}$, and $C_{c\varepsilon}/C_{\alpha\varepsilon}$ that were treated as random variables and an alpha term α. The stochastic material parameters are summarised in Table 2.

The prior parameters were adopted from the Modified Cam-clay values used in Zheng et al. (2018) and converted to the conventional non-linear values for compression ratio, the recompression ratio, the creep strain rate and coefficient of consolidation in the horizontal direction equivalents. The over consolidation ratio (OCR) was also adopted from Zheng et al. (2018) by dividing the pre-consolidation pressure (σ_p') by the initial effective vertical stress (σ_{vo}') and adjusted based on the field in-situ data. Due to the strain rate effects the pre-consolidation pressure (σ_p') is multiplied by the correction factor (α). Therefore, there are four random variables per layer and one factor (α) per model. The prior predictions are shown in Fig. 2.

4 Numerical Prediction

4.1 Prior Prediction

Prior numerical consolidation analyses for surface settlement prediction were conducted using Settle3 and a finite difference numerical solution of one-dimensional equations for consolidation. The finite difference program Consolidation Analysis Of Soils (CAOS) was developed by Prof. Harry Poulos and is discussed in Kelly (2008), implements a forward marching finite difference procedure to solve the uncoupled consolidation equations for one dimensional consolidation, radial consolidation with wick drains and combined vertical and radial consolidation. Creep settlements are implemented by Bjerrum's concept (Bjerrum 1967) modified by the creep transition equation (Wong 2006). Both prior predictions for Settle3 and CAOS differed from the measured settlement both in overall settlement prediction and the shape of the settlement curve as shown in Fig. 2 below. The results from the Settle3 prior prediction are shown in Fig. 3.

Table 3. The prior soil parameters and vertical drain values adopted for the model

Parameter	Properties	Crust	Soft clay	Stiff clay
		1	2	3
Layer thickness	H (m)	1.5	9.0	27.5
Over-consolidation ratio	OCR	4.8	1.7	1.0
Unit weight of soil	γ (kN/m^3)	18.75	15.2	19.0
Initial void ratio	e_0	0.81	2.47	0.54
Compression index	C_c	0.36	1.56	0.61
Recompression index	C_r	0.07	0.31	0.12
Compression ratio ($C_{c\varepsilon}$)	$C_c(1 + e_o)$	0.2	0.45	0.40
Recompression to compression ratio ($C_{c\varepsilon}/5$)	$C_r(1 + e_o)$	0.04	0.09	0.08
Creep strain to compression ratio ($C_{c\varepsilon}/15$)	$C_\alpha(1 + e_o)$	0.013	0.03	0.027
Coefficient of horizontal consolidation (C_h)	m^2/yr	50	3	100
Horizontal to vertical consolidation ratio	C_h/C_v	1	1	1
Preconsolidation pressure	$\sigma_p{\prime}$	68	89	237
Coefficient of Variation	$[C_{c\varepsilon},C_{s\varepsilon}]$	0.3	0.3	0.3
Coefficient of Variation	$[C_{\alpha\varepsilon}]$	3.0	3.0	3.0
Drain spacing	S_p (m)	1.2	1.2	1.2
Drain pattern	-	Square	Square	Square
Drain radius	r_s (m)	0.025	0.025	0.025

4.2 Posterior Prediction

Bayesian back analysis was used to incorporate measured settlement data to update prior soil parameters. When the monitored data from 0 to 47 days was used for prediction on the 974th day the mean settlement was 720mm which is about half of the measured settlement of 1,427mm and shows a clear deviation between the predicted and measured results. Two more iterations were incorporated one from 0 to 76 days and 0 to 117 days before the prediction converged with the measured results. 0 to 76 days showed an over prediction of 1630mm however 0 to 117 days showed a prediction of 1,460mm which compares well with the measured result of 1,427mm. The accuracy of the prediction based on the soil parameters increased with an increased amount of monitored data. The number of parameter dimensions (d) for the three layered model was nine and 12,000 simulations were used. The surface settlement results shown in Fig. 4 and mean values of posterior distributions are presented in Table 4.

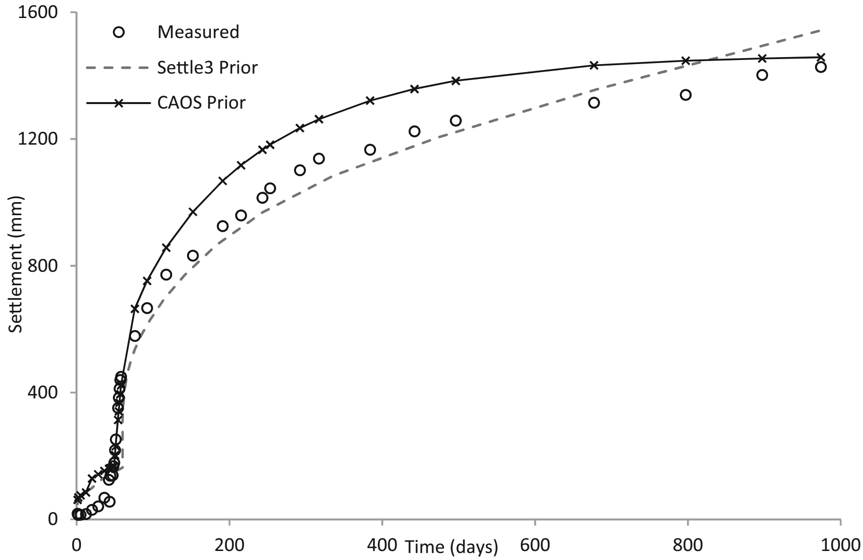
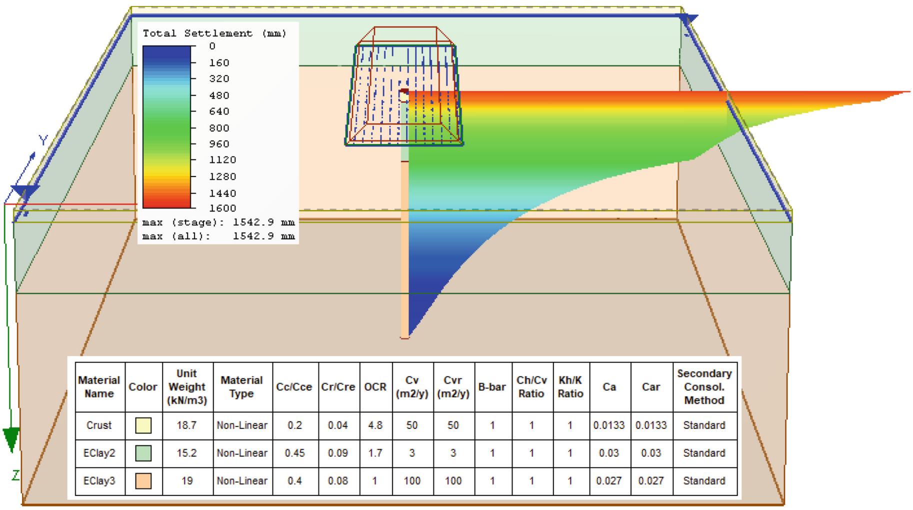


Fig. 2. Prior predictions using Settle3 and finite difference program CAOS



Material Name	Color	Unit Weight (kN/m3)	Material Type	Cc/Cce	Cr/Cre	OCR	Cv (m2/y)	Cvr (m2/y)	B-bar	Ch/Cv Ratio	Kh/K Ratio	Ca	Car	Secondary Consol. Method
Crust		18.7	Non-Linear	0.2	0.04	4.8	50	50	1	1	1	0.0133	0.0133	Standard
EClay2		15.2	Non-Linear	0.45	0.09	1.7	3	3	1	1	1	0.03	0.03	Standard
EClay3		19	Non-Linear	0.4	0.08	1	100	100	1	1	1	0.027	0.027	Standard

Fig. 3. Prior prediction results from Settle3 showing the material parameters and total settlement

5 Discussion

The prior prediction from both numerical models differed from the measured surface settlements. Simplified geotechnical models were updated using observed data incorporating Bayesian back analysis to verify the parameters and surface settlement over time. It demonstrated the use of an advanced method such as Bayesian back analysis

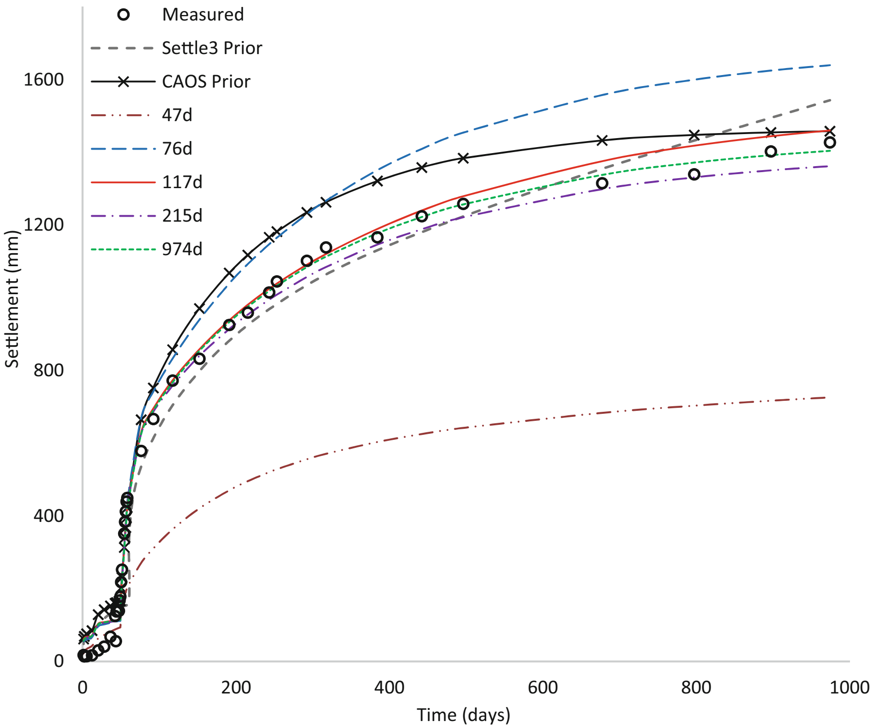


Fig. 4. Posterior (mean) settlement predictions using monitored data from 0 to 974 days.

Table 4. Summary of the mean posterior values

ID	Parameter	Properties	Prior (mean)	Posterior (mean)				
				47d	76d	117d	215d	974d
1			0.20	0.087	0.236	0.247	0.253	0.25
2	$C_{c\varepsilon}$	$C_c(1 + e_o)$	0.45	0.668	0.325	0.257	0.206	0.228
3			0.40	0.430	0.704	0.727	0.728	0.738
4			0.137	0.188	0.047	0.057	0.054	0.074
5	C_h	m^2/day	0.008	0.011	0.012	0.014	0.014	0.013
6			0.274	0.337	0.467	0.450	0.483	0.484
7	$C_{c\varepsilon}/C_{s\varepsilon}$	-	5	6.800	5.141	5.281	5.117	5.262
8	$C_{c\varepsilon}/C_{\alpha\varepsilon}$	-	15	19.32	21.15	20.90	20.78	19.95
9	α	-	1	0.846	1.086	1.094	1.080	1.073

in combination with simplified geotechnical models can produce reasonably reliable predictions. It also showed that approximately 117 to 215 days of monitored data for the simplified models was required for a prediction to converged to the field data which is similar to the results obtained in Zheng et al. (2018). Predictions prior to the construction phase being completed, in this case up to about 60 days, did not produce reliable results for settlement prediction as shown by the '47d' prediction results likely due to the embankment construction phase. This method demonstrates the use of random variables for $C_{c\varepsilon}$, C_h, $C_{s\varepsilon}$, $C_{\alpha\varepsilon}$ and α. The method used the ratio of the compression index to recompression index and creep strain rate as a means of reducing the parameters and therefore computational effort required for analysis.

6 Conclusion

Settle3 and finite difference program CAOS were used to predict surface settlement based on the prior soil parameters and results compared. Bayesian back analysis was then incorporated using CAOS based on the measurement data to progressively update soil parameter to predict the long-term surface settlements. A function for incorporating Bayesian back analysis in Settle3 to the authors knowledge does not yet exist. The following conclusions can be drawn:

- Prior predictions tend to deviate from the measured settlement data based on the parameters selected and numerical analysis adopted.
- Both Settle3 and COAS prior predictions showed a deviation from the measured settlement both in the overall prediction and shape of the settlement curve.
- Surface settlement predictions using Bayesian back analysis incorporating about 117 to 215 days of measured data was required to produce a reliable updated prediction.
- Settlement prediction can be improved by incorporating monitoring data progressively to update the soil parameters.
- Bayesian back analysis is a useful tool to enable soil parameters to be progressively updated using monitored data.

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