Stochastic Slope Stability Analysis: Exploring the Uncertainty of Input Motion

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Abstract. Slope stability issues are widely studied by geoengineers due to the significant risk they pose to human safety and the economy. Slope failures can be especially perilous, particularly in earthquake-prone regions, where even statically stable slopes can be triggered by dynamic loads. The pseudo-static (PS) approach is commonly employed in the initial stages of assessing seismic slope stability due to its effectiveness and efficiency. However, the variability of the PS coefficient is not commonly incorporated in the realm of stochastic slope stability analyses. In this study, the research focuses on simulating the spatial variability of soils in seismic slope stability analysis. The approach employed involves the integration of non-circular limit equilibrium method (LEM) of slices, Monte Carlo (MC) simulation, and random fields, termed as non-circular 2D-RLEM. A single random variable (SRV) approach is utilized for the pseudo-static (PS) load. The outcomes of parametric investigations are presented as design aids, providing valuable insights into the sensitivity of stochastic slope stability problems to various factors, including different levels of average PS loading and its uncertainty. It was observed that the impact of assigning different values to the coefficient of variation of seismic loading on the resulting slope failure probabilities was more significant for larger earthquakes. Meanwhile, a higher uncertainty level of the seismic coefficient was observed to be more critical for slopes with lower failure probabilities (i.e. less than about 40%).

Keywords: Stochastic Slope Stability Analysis · Pseudo-static Approach · Random Variability of Pseudo-static Loading · Soil Stochastic Modelling

1 Introduction

Over the past few decades, there has been a considerable amount of research focused on slopes, encompassing both natural and man-made formations, due to their widespread presence (Leshchinsky & San, 1994; Baker et al., 2006; Burgess et al., 2019). As slope failure is hazardous, especially in seismic-prone areas, in terms of life safety, environment or economy, the pseudo-static (PS) approach is commonly adopted to accelerate the seismic slope stability analysis compared to the more rigorous and computationally less efficient dynamic analysis, which requires detailed step-by-step numerical integration in time or frequency domain analysis. The stochastic modelling of soils improves

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the quality of simulations and helps reflect a more realistic picture of the soil structure in the models. Indeed, soil strength properties, primarily cohesion and friction angle, exhibit inherent stochastic characteristics arising from diverse deposition conditions and loading histories within a specific study area (Phoon and Kulhawy, 1999; Cho, 2010). The spatial variability of these properties can considerably influence the outcomes of geotechnical reliability analyses when accurately represented using mathematical models (Javankhoshdel et al., 2017). However, the effect of the uncertainty of the seismic load (Youssef Abdel Massih et al., 2008; Tsompanakis et al., 2010) has not been sufficiently explored in the realm of stochastic slope stability analysis yet. In effect, if the seismic load (i.e. PS coefficient) is treated as a random variable, then two slopes with nominally identical attributes can have different probabilities of failure because of the variability (i.e. level of uncertainty) of the seismic loading. More importantly, the assessment of the probability of slope failure is rather complicated because the seismic load can also have spatial variability, which will be investigated in later research.

2 Literature Review

Soil properties vary spatially due to various reasons including different depositional conditions and stress histories, as well as variations in the mineralogical composition in an area which is known as soil inherent variability. All in situ soil properties will vary in vertical and horizontal directions as a result of these natural processes. The investigation of this subject has been the primary focus of the research conducted by Phoon et al. (1995) and Phoon and Kulhawy (1999). Two statistical parameters represent the soil's inherent spatial variability: coefficient of variation (COV) of inherent variability and scale of fluctuation (SoF). The former can be obtained from detrending the soil data which can be considered as a combination of a deterministic trend and a homogeneous random function while the latter is an indication of the distance within which the soil property values are strongly correlated. Stationary Gaussian random fields are most used to model this stochastic behaviour due to the least number of inputs required (Fenton and Griffiths, 2008; Shah Malekpoor et al., 2022).

Various algorithms for generating random fields are at disposal, such as the Turning-bands method (TBM), Covariance matrix decomposition (CMD), Fast Fourier Transform (FFT) method, and Local average subdivision (LAS) method (Fenton and Griffiths, 2008). If the specific problem demands a local average representation, such as in soil statistical modelling, then the most suitable option is the LAS method (Fenton and Vanmarcke, 1990). Soil stochastic attribute has been explored in various slope stability studies using different approaches such as RFEM (Random Finite Element Method), circular and non-circular RLEM (Random Limit Equilibrium Method) and RFDM (Random Finite Difference Method) (Griffiths and Fenton, 2004; Srivastava and Babu, 2009; Griffiths et al., 2009; Huang et al., 2010; Huang et al., 2013; Li et al., 2014; Javankhoshdel and Bathurst, 2014; Jamshidi Chenari and Alaie, 2015; Cami et al., 2017; Javankhoshdel et al., 2017; Burgess et al., 2019; Shah Malekpoor et al., 2020; Shah Malekpoor and Lopez-Querol, 2022).

Non-circular RLEM using Morgenstern-Price method was first employed by Cami et al. (2018) with the Monte Carlo technique or 'random walking' as the optimization technique in locating the low-safety-factor, noncircular surfaces. It was shown that their approach was able to find a similar failure path to the one using the RFEM approach, though being much more computationally efficient. Mafi et al. (2020) provided a comprehensive review of the literature on searching methods of critical noncircular slip surfaces in probabilistic slope stability analysis. They demonstrated that Surface Altering Optimization (SAO) is a computationally efficient and fairly accurate method for optimising non-circular slip surfaces.

On the other hand, in geoseismic engineering practice, slope stability is most frequently evaluated using the deterministic pseudo-static (PS) method, in which constant horizontal (and sometimes vertical) pseudo-static inertial forces are included in the safety factor calculations. The uncertainty of the seismic demand (i.e. pseudo-static horizontal acceleration) was considered in the development of fragility curves of a characteristic geostructure by Tsompanakis et al. (2010) using a lognormal distribution. Youssef Abdel Massih et al. (2008) employed the randomness of the horizontal seismic coefficient (K_h) in the reliability analysis of a strip footing subjected to a vertical load. It was shown that for higher values of the applied load, the effect of the random variability of the seismic load was significant. This aspect is further explored in this paper.

3 Methodology

The research employs simple, uniform slopes consisting of single material cohesive-frictional soils. The primary emphasis is placed on investigating the impact of the random variability of K_h. The soil property random fields are assumed to be isotropic stationary Gaussian, following a lognormal distribution due to their nonnegative nature, as supported by existing literature (Cho, 2007; Javankhoshdel et al., 2017). Similarly, the PS (pseudo-static) coefficient is also assumed to follow a lognormal distribution, reflecting the pattern of random variability (Tsompanakis et al., 2010). Additional assumptions encompass the absence of cross-correlation between the random variables and no consideration of pore pressure effects. Figure 1 illustrates the sample slope subjected to analysis within this study.

In brief, Table 1 displays the deterministic and statistical parameter values used in the simulations (Phoon and Kulhawy, 1999; Melo and Sharma, 2004; Jibson, 2011; Burgess et al., 2019; Cami et al., 2020).

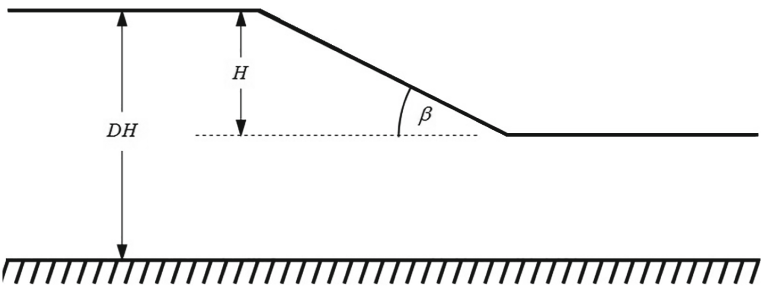

Fig. 1. Sample slope section

Table 1. List of simulation parameters (Shah Malekpoor et al., 2022)

Parameter	Adopted values
β, Slope angle	20° to 85°
μ_ϕ, Mean of the soil friction angle	20°
Stability number, $\lambda = \mu_c/\gamma H tan\mu_\phi$	0.1, 0.3, 0.5, 0.7
μ_c, mean of soil cohesion	Determined based on λ
μ_{Kh}, mean of horizontal seismic coefficient	0.1, 0.3
COV_c, coefficient of variation of soil cohesion	0.3
COV_ϕ, coefficient of variation of soil friction angle	0.15
COV_{Kh}, coefficient of variation of horizontal seismic coefficient	0.1, 0.3, 0.5
Soil unit weight, γ	18 (kN/m^3)
Slope height, H	5 m
Depth factor, D[1]	2
$(\theta_{c,\phi})_{Ho}$[2]$/H$	40
$(\theta_{c,\phi})_{V}$[3]$/H$	0.3

Non-circular RLEM (Cami et al., 2018) using Janbu-simplified combined with the PS approach is employed in the current research. This approach reflects a more realistic picture of the failure surface compared to the conventional circular approaches and is called 2D- RPSLEM from now on. Before proceeding to include random variability of the PS coefficient in this novel methodology, the 2D- RPSLEM approach was validated with the non-circular Auto refine search method in Slide2 (Rocscience, 2023) for the cases where K_h is constant and the soil is spatially variable. To find the optimal number of Monte Carlo samples, a convergence sensitivity analysis was conducted first for different numbers of samples (Fig. 2).

4 Results

The effect of random variable seismic load on the probability of failure of a stochastic slope is investigated in the current research and the results have been presented as parametric studies. This includes the level of uncertainty of the seismic load and its magnitude. Burgess et al. (2019) explored the effect of a constant level of seismic load (i.e. PS loading) on the probability of slope failure and introduced the output of their charts as conditional probabilities of failure, the total value of which depends on the probability that a specific PS load would occur in a specific slope field. However, the current research gives out the total probability values considering a lognormal distribution for the existing PS loading.

[1] The depth factor is derived by dividing the depth to the hard layer by the slope height

[2] Horizontal scale of fluctuation of soil parameters.

[3] Vertical scale of fluctuation of soil parameters.

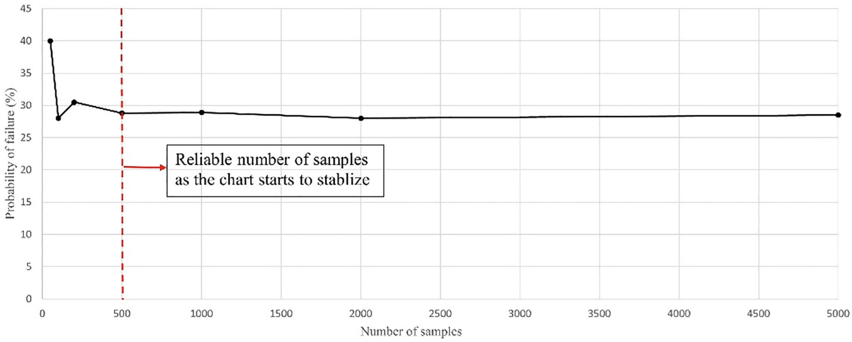

Fig. 2. Optimal number of MC samples (assuming $\beta = 30°$, $\mu_\phi = 20°$, $\lambda = 0.2$, $\mu_{Kh} = 0.1$, $H = 5\ m$, $\gamma = 18\ kN/m^3$, $COV_c = 0.3$, $COV_\phi = 0.15$, $COV_{Kh} = 0.5$, $(\theta_{c,\phi})_{Ho}/H = 40$, $(\theta_{c,\phi})_V/H = 0.3$)

As anticipated, an increase in the stability number (i.e. higher soil cohesion with constant slope height and friction angle) leads to a reduced probability of failure for a given slope angle under random variable PS loading. Conversely, a higher mean magnitude for the PS coefficient corresponds to an elevated risk of failure for a specific slope (Figs. 3 & 4).

Figure 3 shows that different levels of uncertainty of the horizontal seismic coefficient do not considerably affect the vulnerability of a stochastic slope for low mean levels of input load ($\mu_{Kh} = 0.1$) while this matter becomes significantly important for a higher load for all slope angles, see Fig. 4.

For both mean seismic coefficient values (i.e. $\mu_{Kh} = 0.1$ & 0.3), the critical or worst-case COV_{Kh} value, which results in a higher slope probability of failure compared to other COV_{Kh} values, changes from 0.1 to 0.5 for slopes with risk of failure less than about 40%. For example, with respect to a high stability number slope ($\lambda = 0.7$) when the PS coefficient mean value is increased to 0.3, a higher uncertainty level of this yields much higher probabilities of failure compared to a constant K_h approach (Fig. 4).

It is worthy to note that the constant K_h approach (i.e. no uncertainty in the input load) shows compatible trend with the trend in the curves of different uncertainty levels (from $COV_{Kh} = 0.1$ to $COV_{Kh} = 0.5$) (Figs. 3 & 4).

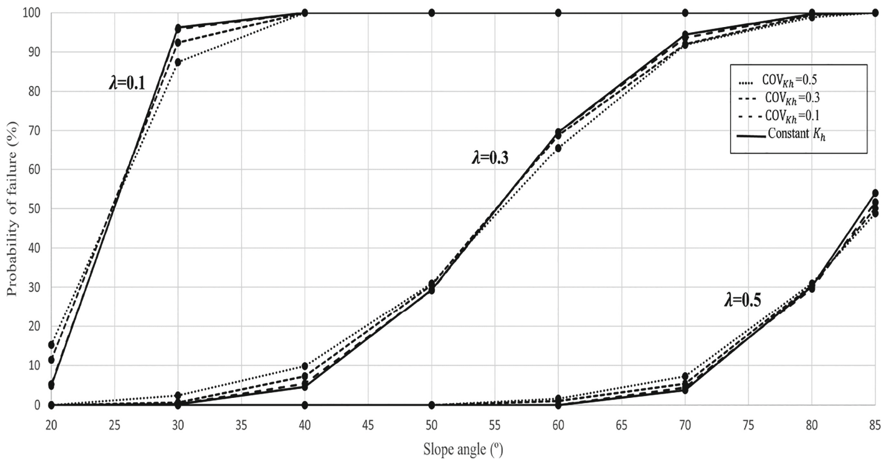

Fig. 3. The impact of different levels of COV_{Kh} and λ assuming $\mu_{Kh} = 0.1$, $\mu_\phi = 20°$, $COV_c = 0.3$, $COV_\phi = 0.15$, $(\theta_{c,\phi})_{Ho}/H = 40$, $(\theta_{c,\phi})_V/H = 0.3$ with a Markovian ACF

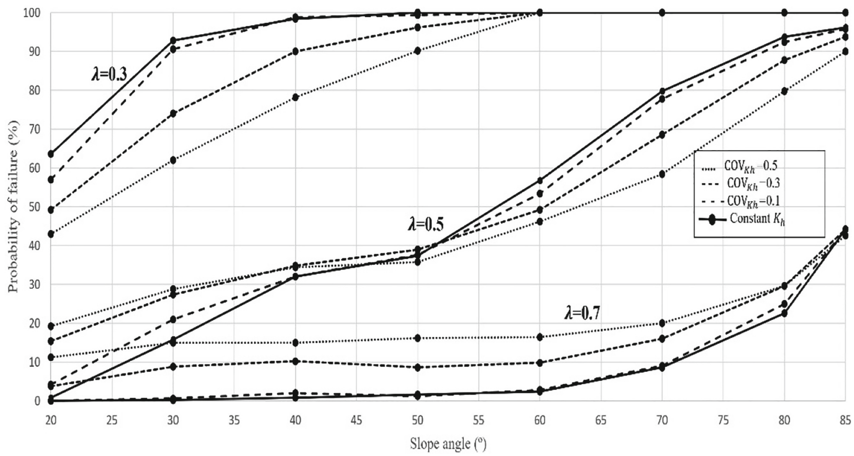

Fig. 4. The impact of different levels of COV_{Kh} and λ assuming $\mu_{Kh} = 0.3$, $\mu_\phi = 20°$, $COV_c = 0.3$, $COV_\phi = 0.15$, $(\theta_{c,\phi})_{Ho}/H = 40$, $(\theta_{c,\phi})_V/H = 0.3$ with a Markovian ACF

5 Conclusion

The effect of uncertain seismic load on the vulnerability of a stochastic slope is explored in this study. The most important conclusion is the significance of this variability for the higher mean value of the seismic input load, meaning that different levels of variability of the seismic loading impose different vulnerabilities as opposed to the low levels of mean PS coefficient. This matter asserts the importance of considering the uncertainty levels of the seismic loading in large earthquakes for which their equivalent PS coefficient is accordingly high. Meanwhile, a higher uncertainty level of the seismic coefficient showed a more critical effect for slopes with probabilities of failure less than about 40%.

References

Baker, R., Shukha, R., Operstein, V., Frydman, S.: Stability charts for pseudo-static slope stability analysis. Soil Dyn Earthq Eng 26(9), 813–823 (2006). https://doi.org/10.1016/j.soildyn.2006.01.023

Burgess, J., Fenton, G. A., Griffiths, D. V.: Probabilistic seismic slope stability analysis and design. Can Geotech J 56(12), 1979–1998 (2019). https://doi.org/10.1139/cgj-2017-0544

Cami, B., Javankhoshdel, S., Bathurst, R.J., and Yacoub, T. 2018.: Influence of mesh size, number of slices, and number of simulations in probabilistic analysis of slopes considering 2D spatial variability of soil properties. In Proceedings of IFCEE 2018. pp. 186–196. https://doi.org/10.1061/9780784481585.019

Cami, B., Javankhoshdel, S., Lam, J., Bathurst, R. J., Yacoub, T.: Probabilistic analysis of a tailings dam using 2D composite circular and non-circular deterministic analysis, SRV approach, and RLEM. In: 70th Canadian Geotechnical Conference, 7 p. Ottawa, Canada (2017).

Cami, B., Javankhoshdel, S., Phoon, K. K., Ching, J.: Scale of fluctuation for spatially varying soils: Estimation methods and values. ASCE-ASME J Risk Uncertain Eng Syst A: Civ Eng 6(4), 03120002 (2020). https://doi.org/10.1061/ajrua6.0001083

Cho, S. E. 2007.: Effects of Spatial Variability of Soil Properties on Slope Stability. Engineering Geology 92: 97–109.

Cho, S.E. 2010.: Probabilistic assessment of slope stability that considers the spatial variability of soil properties. J Geotech Geoenviron Eng, 136(7): 975–984. https://doi.org/10.1061/(ASCE)GT.1943-5606.0000309

Griffiths, D. V., Fenton, G. A.: Probabilistic slope stability analysis by finite elements. J Geotech Geoenviron Eng 130(5), 507-518 (2004). https://doi.org/10.1061/ASCE1090-02412004130:5507

Srivastava, A., Babu, G. L. S.: Effect of soil variability on the bearing capacity of clay and in slope stability problems. Eng Geol 108(1), 142–152 (2009). https://doi.org/10.1016/j.enggeo.2009.06.023

Griffiths, D. V., Huang, J., Fenton, G. A.: Influence of Spatial Variability on Slope Reliability Using 2-D Random Fields. J Geotech Geoenviron Eng 135(10), 1367–1378 (2009). https://doi.org/10.1061/(asce)gt.1943-5606.0000099

Huang J. S., Griffiths D. V., Fenton G. A.: System reliability of slopes by RFEM. Soils Found 50(3), 343–353 (2010). https://doi.org/10.3208/sandf.50.343

Huang, J. S., Lyamin, A. V., Griffiths, D. V., Krabbenhoft, K., Sloan, S. W.: Quantitative risk assessment of landslide by limit analysis and random fields. Comput Geotech *53*, 60–67 (2013). https://doi.org/10.1016/j.compgeo.2013.04.009

Jamshidi Chenari, R., Alaie, R.: Effects of anisotropy in correlation structure on the stability of an undrained clay slope. Georisk 9(2), 109–123 (2015). https://doi.org/10.1080/17499518.2015.1037844

Javankhoshdel, S., Bathurst, R. J.: Simplified probabilistic slope stability design charts for cohesive and cohesive-frictional (c-ϕ) soils. Can Geotech J 51(9), 1033–1045 (2014). https://doi.org/10.1139/cgj-2013-0385

Jibson, R. W.: Methods for assessing the stability of slopes during earthquakes- A retrospective. Eng Geol 122(1), 43–50 (2011). https://doi.org/10.1016/j.enggeo.2010.09.017

Li, D. Q., Qi, X. H., Phoon, K. K., Zhang, L. M., Zhou, C. B.: Effect of spatially variable shear strength parameters with linearly increasing mean trend on reliability of infinite slopes. Struct Saf 49, 45–55 (2014). https://doi.org/10.1016/j.strusafe.2013.08.005

Mafi, R., Javankhoshdel, S., Cami, B., Jamshidi Chenari, R. and Gandomi, A.H., 2020.: Surface altering optimisation in slope stability analysis with non-circular failure for random limit equilibrium method. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, pp.1–27.

Melo, C., Sharma, S.: Seismic coefficients for pseudostatic slope analysis. In: 13th World Conference on Earthquake Engineering. Vancouver, Canada (2004).

Fenton, G. A., Griffiths, D. V.: Risk assessment in geotechnical engineering. John Wiley & Sons (2008).

Fenton, G. A., Vanmarcke, E. H.: Simulation of random fields via local average subdivision. J Eng Mech 116(8), 1733-1749 (1990). https://doi.org/10.1061/(ASCE)0733-9399(1990)116:8(1733)

Javankhoshdel, S., Luo, N., Bathurst, R. J.: Probabilistic analysis of simple slopes with cohesive soil strength using RLEM and RFEM. Georisk 11(3), 231–246 (2017). https://doi.org/10.1080/17499518.2016.1235712

Leshchinsky, D., San, K.: Pseudostatic seismic stability of slopes: Design charts. J Geotech Eng 120(9):1514-1532 (1994). https://doi.org/10.1061/(ASCE)0733-9410(1994)120:9(1514)

Phoon, K. K., Kulhawy, F. H.: Characterization of geotechnical variability. Can Geotech J 36(4), 612–624 (1999a). doi:https://doi.org/10.1139/t99-038

Phoon, K. K., Kulhawy, F. H., Grigoriu, M. D.: Reliability-based design of foundations for transmission line structures. Research project (1995).

Rocscience Inc. 2023. Slide2, 2D Limit Equilibrium Slope Stability software. Rocscience, Toronto, Ontario, Canada.

Shah Malekpoor, P., R. Jamshidi Chenari, S. Javankhoshdel.: Discussion of 'Probabilistic seismic slope stability analysis and design'. Can Geotech J 57 (7), (2020). https://doi.org/10.1139/cgj-2019-0386

Shah Malekpoor, P., Lopez-Querol, S.: A novel stochastic pseudo-static approach for slopes. In: 16th Young Geotechnical Engineers Symposium edited by Stirling, R. and Nadimi, S., Newcastle University (2022). https://doi.org/10.57711/kz8q-q554

Shah Malekpoor, P., Lopez-Querol, S., Javankhoshdel, S.: Spatial variability of input motion in stochastic slope stability. TMIC2022 conference.

Tsompanakis, Y., Lagaros, N. D., Psarropoulos, P. N., Georgopoulos, E. C.: Probabilistic seismic slope stability assessment of geostructures. Struct Infrastruct Eng 6(1–2), 179–191 (2010). https://doi.org/10.1080/15732470802664001

Youssef Abdel Massih, D., Soubra, A. H., Low, B. K.: Reliability-based analysis and design of strip footings against bearing capacity failure. J Geotech Geoenviron Eng 134(7), (2008). https://doi.org/10.1061/ASCE1090-02412008134:7917