Searching for the 3D Critical Slip Surface in an Open Pit Mine Using Spline Surfaces

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Abstract. The use of software for 3D slope stability is becoming increasingly popular due to advances in computing technology. Though slip surfaces in slope stability are rarely circular in nature, many modern practitioners still rely on limit equilibrium results using spherical or ellipsoidal slip surfaces. Restricting slip surfaces to the spherical or ellipsoidal domain can lead to un-conservative results due to inflexibility of the slip surface during optimization. As the technology for identifying critical slips in slopes continues to improve, a new method which optimizes the shape of a slip surface using flexible splines in 3D was recently proposed. The method is applied in this paper towards the analysis of an open pit mine. Some simplification steps for complex models are presented which are recommended to be taken during the modelling procedure to increase the speed of searching for the critical slip surface and filter out unnecessary entities in the analysis, such as near-vertical weak layers and weak layers with reverse curvature.

Keywords: spline · search · limit equilibrium · slope stability · optimization · factor of safety · open pit · NURBS

1 Introduction

Limit equilibrium (LE) is a fast and powerful method for assessing the overall stability of slopes. Although it is available in both 2D and 3D modelling programs, the use of an appropriate searching algorithm to find the critical slip surface is imperative for safe design. The critical slip surface is the shape of the sliding surface which has the lowest factor of safety in a slope. Searching for the critical slip surface requires solving an optimization problem, whereby the shape of the slip surface is varied to minimize factor of safety.

Much of the early literature on LE understandingly assumes circular or spherical slip surfaces due to limitations in computing technology. In such formulations, the radius and location of the centroid of the circular slip surface is varied until a minimum factor of safety is found. However, the assumption of circular failure can be unconservative because the domain of shapes forming the slipping surface is greatly restricted. Sadly, even in modern times, grid search using circular surfaces remains one of the most popular methods of analysis by practitioners in this field. In many 3D limit equilibrium packages,

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the shapes of slip surfaces have often been limited to ellipsoids or other primitive shapes. However, the analysis of non-circular slip surfaces can be accomplished using a variety of methods, such as the algorithm proposed by Cheng and Yip [1] which is particularly useful as it can handle asymmetrical 3D slip surfaces and is used in Slide3.

Recently, a new class of slip surfaces in 3D was introduced by [6, 8]. The slip surfaces are based on 3D splines, which are more flexible than spheres and ellipsoids. In this context, a shape is flexible if it can easily be transformed into a wide range of forms to represent the diverse possibilities of slip surfaces by adjusting its parameters. As necessary, the spline surfaces can be locally optimized using the surface altering algorithm [7]. In this paper, the method of searching for the critical slip surface using spline surfaces is demonstrated using an example case study model. Steps are also taken to show how a geometrically complex 3D model such as the one presented can be simplified for the purposes of the analysis into a much more manageable problem which computes more quickly.

2 Spline Search

A review of the spline search method proposed in [6, 8], and the surface altering algorithm [7] is first presented.

In the simplest form, a spherical slip surface can be defined by a centroid point (x_O, y_O, z_O) and radius R, altogether comprising four parameters which can be varied in the search for the critical slip surface. Ellipsoids can be defined using up to nine parameters by extending the domain to three radii and three axis rotations about orthogonal directions. Ma et al. [6, 8] introduced an alternative set of eleven parameters which partially define a spline surface, illustrated in Fig. 1.

The spline surface parameters include four parameters (A_x, A_y, B_x, B_y) which define the reference points A and B for the surface located above the ground topography by

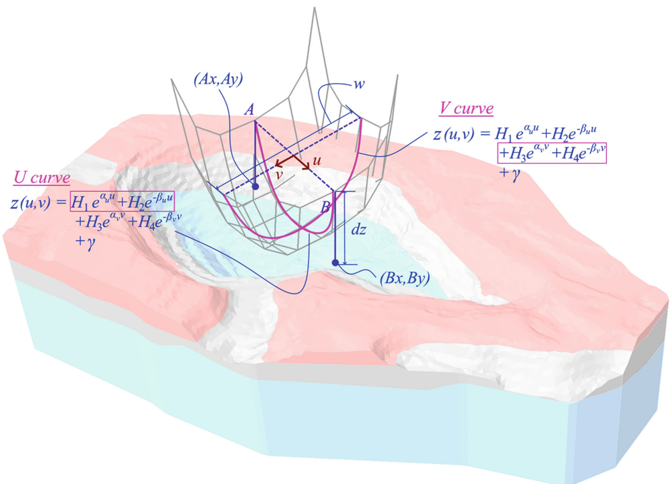

Fig. 1. Optimization parameters of the 3D spline surface defined in [6].

distance dz. The width of the spline surface is w, and it has a reference elevation of γ. An additional four parameters define the skews of the spline surface (α_x, α_y, β_x, β_y) in each of the four local semi-positive lateral orthogonal directions. By altering the skew parameters, the fabric of the slip surface is pulled or pushed from the sides of the bounding rectangle. With these eleven parameters, the spline surface can be varied to produce shapes much more diverse and flexible than spheres and ellipsoids [6, 8]. However, it should be noted that despite their increased flexibility, the domain of spline surfaces which can be generated in this manner still does not include all possible slip surfaces.

The actual spline surface is defined as the Non-Uniform Rational Basis Spline (NURBS) spline [9] formed above Eq. 1 using control points discretized along a grid in localized (u, v) space sampled at the ordinates {–1.1, –1.0, –0.9, –0.8, –0.5, 0, 0.5, 0.8, 0.9, 1.0, 1.1} [6, 8].

$$z(u,v) = H_1 e^{\alpha_u u} + H_2 e^{-\beta_u u} + H_3 e^{\alpha_v v} + H_4 e^{-\beta_v v} + \gamma \tag{1}$$

The coefficients H_1 through H_4 are given in Eq. 2.

$$H_1 = \frac{-e^{\alpha_u}\left(C_1 + \gamma\left(e^{2\beta_u} - 1\right)\left(e^{\alpha_v} - 1\right)\left(e^{\beta_v} - 1\right)\right)}{D\left(e^{\alpha_u+\beta_u} - 1\right)} \tag{2a}$$

$$H_2 = \frac{-e^{\beta_u}\left(C_2 + \gamma\left(e^{2\alpha_u} - 1\right)\left(e^{\alpha_v} - 1\right)\left(e^{\beta_v} - 1\right)\right)}{D\left(e^{\alpha_u+\beta_u} - 1\right)} \tag{2b}$$

$$H_3 = \frac{-e^{\alpha_v}\left(C_3 + \gamma\left(e^{2\beta_v} - 1\right)\left(e^{\alpha_u} - 1\right)\left(e^{\beta_u} - 1\right)\right)}{D\left(e^{\alpha_v+\beta_v} - 1\right)} \tag{2c}$$

$$H_4 = \frac{-e^{\beta_v}\left(C_4 + \gamma\left(e^{2\alpha_v} - 1\right)\left(e^{\alpha_u} - 1\right)\left(e^{\beta_u} - 1\right)\right)}{D\left(e^{\alpha_v+\beta_v} - 1\right)} \tag{2d}$$

$$\begin{aligned} C_1 = &\left(A_z - B_z e^{2\beta_u}\right)\left(1 + e^{\alpha_v+\beta_v}\right) + \frac{A_z + B_z}{2}\left(e^{\alpha_v} + e^{\beta_v}\right)\left(e^{2\beta_u} - 1\right) \\ &- (A_z - B_z)e^{\beta_u}\left(e^{\alpha_v} + e^{\beta_v}\right) \end{aligned} \tag{2e}$$

$$\begin{aligned} C_2 = &\left(B_z - A_z e^{2\alpha_u}\right)\left(1 + e^{\beta_v+\alpha_v}\right) + \frac{B_z + A_z}{2}\left(e^{\beta_v} + e^{\alpha_v}\right)\left(e^{2\alpha_u} - 1\right) \\ &- (B_z - A_z)e^{\alpha_u}\left(e^{\beta_v} + e^{\alpha_v}\right) \end{aligned} \tag{2f}$$

$$C_3 = \left(e^{2\beta_v} - 1\right)\left(A_z e^{\alpha_u} + B_z e^{\beta_u} - \frac{A_z + B_z}{2}\left(e^{\alpha_u+\beta_u} + 1\right)\right) \tag{2g}$$

$$C_4 = \left(e^{2\alpha_v} - 1\right)\left(A_z e^{\alpha_u} + B_z e^{\beta_u} - \frac{A_z + B_z}{2}\left(e^{\alpha_u+\beta_u} + 1\right)\right) \tag{2h}$$

$$D = 1 + e^{\alpha_u+\alpha_v+\beta_u+\beta_v} + e^{\alpha_u+\beta_u} + e^{\alpha_v+\beta_v} - e^{\alpha_u+\alpha_v} - e^{\beta_u+\beta_v} - e^{\alpha_u+\beta_v} - e^{\alpha_v+\beta_u} \tag{2i}$$

The H coefficients are defined such that they map the localized (u,v) space into global space via Eq. 3.

$$A_z = z(-1,\ 0) \tag{3a}$$

$$B_z = z(1,\ 0) \tag{3b}$$

$$0.5(A_z + B_z) = z(0,\ -1) \tag{3c}$$

$$0.5(A_z + B_z) = z(0,\ 1) \tag{3d}$$

The purpose of the spline formulation is to improve the global search capability of metaheuristic algorithms such as Particle Swarm [5] towards identifying the region of the model containing critical slip surface. These metaheuristic methods simulate natural processes of optimization whereby the domain is searched for the location(s) of optimal fitness using an internally communicating and evolving swarm of particles, which sample their individual positions within the domain. They are used to optimize the values of the input parameters (i.e. the spherical, ellipsoidal, or spline parameters) to minimize the FS.

The Particle Swarm search is available in Slide3 and can be used to vary the parameters of the spline surfaces mentioned. Owing to its relatively simple implementation, Particle Swarm is among the most popular metaheuristic methods to search for the minimum FS in slopes [3, 4] and is also used for other geotechnical applications [2].

Metaheuristic search is useful for global optimization, which emphasizes the exploration of the global domain. As such, it is effective for finding the general locations of the global minimums in an optimization problem. While some localized searching is typically done towards the end of the Particle Swarm optimization, it is typically better to pass the minimum result obtained by the Particle Swarm to a more effective local optimizer. A further localized optimization routine discussed in the following subsection is recommended to further optimize and fine-tune the slip surface to produce the minimum FS.

2.1 Localized Surface Altering

Surface altering optimization (SA) [7] is a local optimization routine whereby the points on a NURB surface are manipulated using a series of simple transformations to minimize the FS. Since the spline surfaces in this study are already NURBS surfaces, they can be entered directly as inputs to the SA algorithm for optimization. SA is popular among practitioners and is available in Slide3 via the Surface Altering Optimization feature.

At the time of writing, practitioners commonly use ellipsoidal surfaces in the metaheuristic search. The minimum ellipsoid surfaces represent the general locations of potentially critical FS and are converted into spline surfaces for local refinement via SA. The new method of using spline surfaces during the metaheuristic stage obviates the need for conversion and may be more reliable at finding surfaces with lower factor of safety during the metaheuristic stage [8]. It should be noted, however, that it is plausible in some cases where a minimum ellipsoidal surface from the metaheuristic stage has a higher FS than the minimum spline surface from the metaheuristic stage for the same model, for it to occur that after SA both surfaces the ellipsoid surface which has been converted into a NURBS surface and locally altered may have a lower FS than the post-altered spline surface.

3 Case Study

The case study for this paper is a fictitious example adapted from the geometry of a typical undisclosed open pit mine and shown in Fig. 2. Five material models with either Mohr-Coulomb or Generalized Hoek Brown parameters were defined throughout the model and assigned based on typical values in practice, shown in Table 1.

For the faults, in the model, typical practice is to assume that sliding along the fault is represented using Mohr-Coulomb parameters (in this case, $c = 0$ kPa, $\varphi = 31°$).

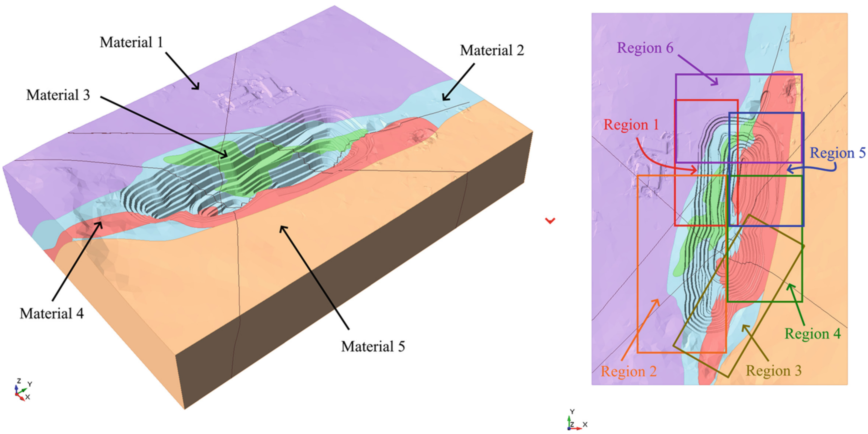

Fig. 2. Case study model of an open pit with six search regions defined.

Table 1. Material properties of case study model.

Material	Unit Weight (kN/m^3)	c (kPa)	φ (°)	UCS (MPa)	GSI	mi	D
Material 1	27	-	-	22	39	12	0.7
Material 2	26	85	34	-	-	-	-
Material 3	28	-	-	95	55	24	0.7
Material 4	29	-	-	85	45	18	0.7
Material 5	27	-	-	45	40	11	0.7
Faults	-	0	31	-	-	-	-

3.1 Model Simplification

Due to recent advances in technology, large volumes of geometrical data can be obtained from geotechnical models and translated into computer models via software. Triangulations of large and complicated models with high resolution geometry can reach data sizes in the order of millions of triangles and several gigabytes of memory. It is important to discuss some simplification strategies that can be used by practitioners to simplify their models for the purpose of the analysis that they wish to run. As an example of concept, models with triangulations resolving at distances of less than one meter may be unnecessarily detailed if the slope stability analysis evaluates sliding surfaces on the scale of hundreds of meters. Based on the practicing experience of the authors, the LEM analysis of a model with millions of triangles can be sped up by orders of magnitudes if its geometry is simplified accordingly.

Additionally, the geometrical models need to be partitioned into separate soil volumes with differing material properties. In 3D, if the geometrical data is very thorough, there can be many small volumes which do not contribute significantly to the analysis or may be located too deep beneath the ground surface to affect the critical slip surface. During computations, the presence of many small volumes of material can adversely impact the speed of the analysis. For the purposes of simplification, these material volumes can be collapsed and treated as part of their surrounding volumes.

The following options in Slide3 were used to simplify the geometry of a model:

1. Simplify Triangulation
2. Collapse Material Boundaries
3. Collapse Small Volumes

Four faults were within the vicinity of the pit for the model and highlighted in Fig. 3. Faults are typically modelled as weak layers with reduced material parameters. Where slip surfaces intersect weak layers, they can be subject to clipping to the weak layer surface in Slide3, but this can result in an unreasonable shape of the slip surface in many cases, especially when the weak layers have irregular geometry or near-vertical portions. The analysis of the slip surface with such weak layers can take up significant extra computational time which can be avoided if the weak layer is deemed not to significantly affect the results of the analysis.

To simplify the analysis for faster processing without compromising the results, three of these faults were suppressed and excluded from the limit equilibrium analysis for (a) being too steep and pointed away from the logically opposite direction of sliding, and/or (b) exhibiting a high degree of reverse curvature. Reverse curvature occurs when a vertical line can be drawn at some location on a weak layer which intersects it more than once. Furthermore, limit equilibrium was not formulated with the intention of handling near-vertical or vertical slips due to the numerical issues associated with steep angles of discretized column bases above a slip surface. The following general recommendations are made with regards to weak layers in a complicated model:

1. Near-vertical weak layers should be excluded from the analysis unless they are oriented downwards in the same direction as the slope. Alternatively, a tension crack can be defined in the model to simulate the vertical truncation of slip surfaces in the vicinity of the weak layer.

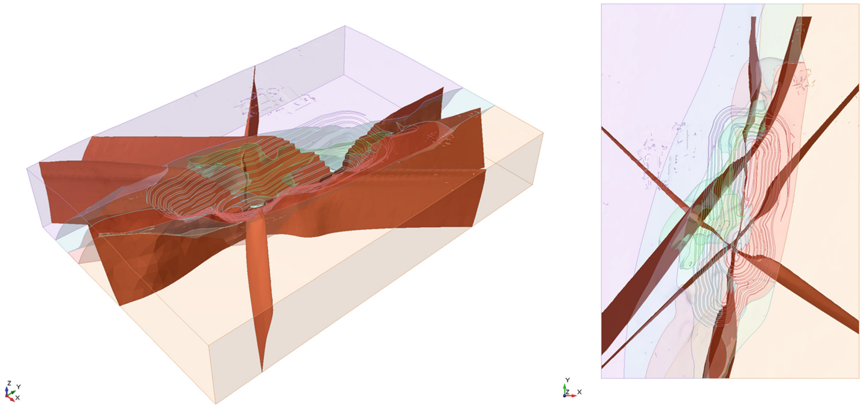

Fig. 3. Locations of faults in the case study model.

2. Weak layers with reverse curvature should be removed from the analysis or trimmed and/or split into separate entities.
3. Weak layers which reach the ground surface should rise above and clear the ground topography completely, to prevent geometrical issues during clipping of the slip surface to the weak layer.
4. If there are weak layers in the model which are too small or do not appear to be aligned towards the expected direction of sliding, consider suppressing them from the analysis to improve performance and ease the search process.
5. Use more advanced techniques such as finite element analysis or toppling analysis where the assumptions of limit equilibrium are inadequate to consider the true mechanics of the faults.

The automatic weak layer handling feature in Slide3 was used to consider whether the remaining weak layer would produce a lower FS or not when clipping each slip surface generated during the search.

Finally, for models covering large geographical regions, it is best to restrict the search area within a smaller region, and to run the analysis in multiple regions. Separating the overall problem into smaller regions is recommended due to the tendency of the metaheuristic search algorithms to intensify towards regions of the domain containing local minima. To ensure adequate coverage of searching on the various pit walls, six search regions were defined for the model, labelled Regions 1 through 6 in Fig. 2.

As a result of the simplification of the model using these procedures, the required time to analyze the model, including the surface altering procedure can be completed within an hour on a laptop computer.

3.2 Comparison of Results

The analysis of the open pit in each of the search regions was performed first by metaheuristic Particle Swarm search using ellipsoid surfaces, and then with spline surfaces. The FS based on the General Limit Equilibrium (GLE) assumption was computed in

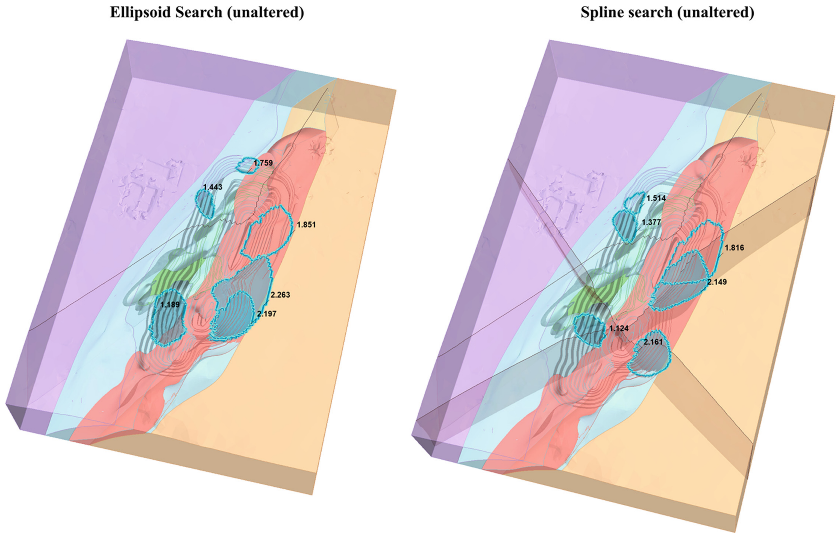

Fig. 4. Minimum ellipsoid and spline surfaces in each search region from Particle Swarm before surface altering.

each case using Slide3. The results for pre-altered ellipsoidal and spline surfaces are shown in Fig. 4.

With both results obtained using Particle Swarm, when compared to the ellipsoid search, the spline search found lower FS in each region during the metaheuristic search. In Regions 2, 4 and 6, minimum FS surfaces were found in different regions between the two searches. This can occur due to any of the following reasons:

1. When considering all possible ellipsoids and splines that can be drawn in the topography, the true minimum FS ellipsoidal surface is in a different location than true minimum FS spline surface.
2. It is more difficult to fit an ellipsoidal surface through localized regions of weak materials when compared to the more flexible spline surfaces.
3. The random nature of the metaheuristic searching has caused the search to converge to a local minimum. This problem is exacerbated if the searching parameters are inadequate to provide sufficient search coverage.

The results from both methods were further optimized using the local surface altering optimization. The results for post-altered ellipsoid surfaces are shown in Fig. 5.

The post-altered spline surfaces mostly had lower FS in part because the pre-altered surfaces were already lower in FS, but also because the ellipsoid surfaces need to be approximated into spline surfaces. During the conversion of an ellipsoid into a NURBS surface for SA, a loss of accuracy and a jump of the FS usually occurs [7]. Note that the local optimization processes for the ellipsoidal surfaces in Regions 2, 3, 5 and 6 were unable to find slip surfaces with lower FS than the pre-altered ellipsoidal surfaces due to the jump in FS during the conversion process. A summary of the results for the case study in each of the regions is shown in Table 2.

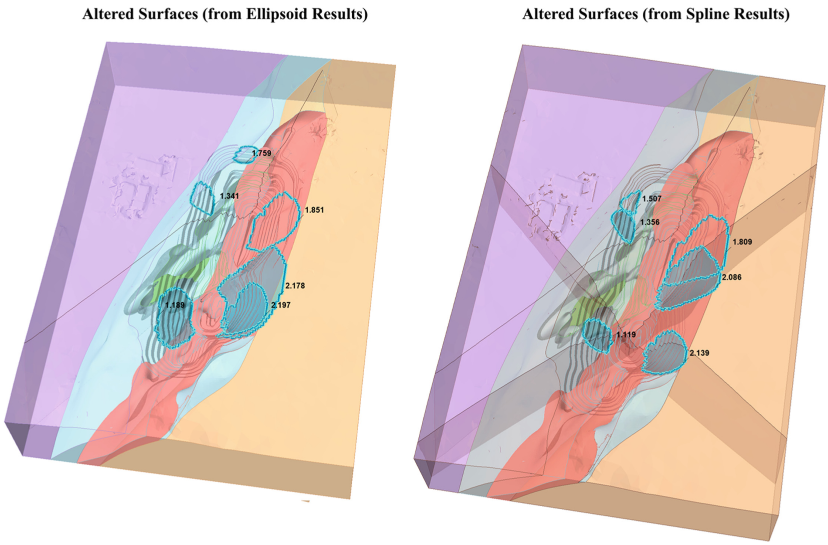

Fig. 5. Minimum post-altered ellipsoid and spline surfaces in each search region.

Table 2. Summary of factor of safety analysis results in each region (minimum values bolded)

Region	Ellipsoid (unaltered)	Spline (unaltered)	Ellipsoid (altered)	Spline (altered)
Region 1	1.443	1.377	1.341	1.356
Region 2	**1.189**	**1.124**	**1.189**	**1.119**
Region 3	2.197	2.161	2.197	2.139
Region 4	2.263	2.149	2.178	2.086
Region 5	1.851	1.816	1.851	1.809
Region 6	1.759	1.514	1.759	1.507

Note that for Region 1, the post-altered ellipsoid result had a lower FS than the post-altered spline result even as the pre-altered ellipsoid result was higher. This can happen due to the tendency of the surface altering procedure to become stuck in local minimum solutions

In general, the spline search was able to find lower factors of safety than the ellipsoid for this example. After surface altering, the spline search still performed better for all search regions. However, this conclusion cannot be generalized for all models. Neither does the search guarantee to always find the true critical minimum surface. Models that are extremely complex or spatially large may not be adequately searched and can converge to local minima. Due to the high degree of pseudo-randomness employed in the search process either method can find a lower FS than the other. In general, to increase the likelihood of running a successful search, it is recommended to employ the simplification steps mentioned previously while ensuring adequate search coverage.

4 Conclusion

The spline search method is a powerful metaheuristic searching method which helps to identify the critical slip surface in a 3D slope with relative geometrical flexibility. Because they are non-uniform rational basis spline (NURBS) surfaces, the spline surfaces can be further optimized locally using the surface altering method, without the need for conversion. Specifically in the case studied by this paper, the spline search was observed to be more effective versus ellipsoid surfaces at finding locations of low factor of safety along multiple regions of the open pit mine. Recommendations are made with regards to the analysis of complex models, which include simplifying the triangulation, subdividing the model into multiple search regions, and thinking critically about whether certain weak layers can be excluded from an analysis.

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