



Zoning of Soil Media by Raster Scan Algorithms for Two-Dimensional Steady State Flow

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Abstract. A method for representation of soil media and boundary conditions in a cross section of ground layers is presented. The method is based on algorithms used in computer graphics, called raster scan algorithms, RSA. The method may be applied to various soil mechanics solutions, especially slope stability problems. In this article it is applied to steady state two-dimensional groundwater flow. Seepage equations are modified to finite difference equations that can be used with the raster scan algorithms. The method is used to determine total head, pressure head and flow rates at any soil point (pixel). Most of the characteristic problems in groundwater flow are easily treated with RSA method such as, anisotropy, multi soil layers, pockets of special soils, different boundary conditions, confined and unconfined flow, and flow to wells.

Verifications included flow around a sheet pile, flow through earth dam and flow under a concrete dam. RSA method shows very promising to solve geotechnical problems efficiently.

Keywords: Seepage · Raster Scan Algorithm · Pixel Matrix · Zoning of Soil Media

1 Introduction

The movement of water through soil media is a basic part of soil mechanics. The elegance and logical structure of its theory renders it of interest to engineering scientists and applied mathematicians [1].

Nowadays, Seepage analysis is performed by computers and no longer by hand calculation. Several computer programs of seepage analysis are available for public and commercial use. Most of the computer programs of seepage analysis have good accuracy in the case of confined flow, while accuracy is decreased in the case of unconfined flow. The correct representation of soil media, boundary conditions, and better application of the groundwater flow equations will help getting a better accuracy. Therefore, the new method for representation of soil media adopting the raster scan algorithms RSA may be very useful in the future and can be extended to most soil mechanics theories.

The method of RSA representation was first applied by the author [2], in analyzing slope stability of Mosul Dam, north of Iraq. Then a research work of his student [3]. The method was used thereafter for a couple of projects successfully.

2 The Idea of Rasterization

Conversion of a picture to the form of pixels (small dots on the computer screen), which are saved and manipulated as sets of bits inside the computer, is called rasterization. Grouping the pixels in a matrix configures the raster scan device, as shown in Fig. 1. A group made up of horizontal or vertical pixels in the pixels' matrix are called horizontal and vertical scan lines, respectively. Usually, the idea is explained in textbooks of computer graphics, see for example reference [4].

On a screen of a raster scan device pictures are displayed depending on a stored memory of a frame buffer (or a buffer store). The matrix of pixels on the raster scan device may have one bit plane or more according to the light intensity and the color of pixels. In other words, for each pixel in the pixel matrix, there is one face or more in the frame buffer, as shown in Fig. 2. Each frame plane stores one bit for one of the pixel properties such as color or intensity. Therefore, each pixel should have a record, which is a field of memory that may be containing sub-records, to collect information or faces of the pixel from the bit planes at corresponding positions. Each pixel reads the information from its record using a horizontal or a vertical scanning technique to display information of pixels on the pixel matrix. The display technique is applied continuously and quickly for all pixels of matrix, see [4].

3 Application of Pixel Matrix to Represent Soil Media and Boundary Conditions

Soil media and boundary conditions may be represented as a matrix analogous to the matrix of pixels in which each pixel of this matrix is representing a square $b \times b$ in the ground, e.g., 100 mm by 100 mm squares. As the pixel size b is decreased the accuracy in representing the ground layers will increase. For each pixel, there is a main record and three sub records (faces). The main record includes the number of the pixel, its horizontal coordinate i , the vertical coordinate j which are original coordinates adjusted to integers to show location in the pixel matrix. Hence, the elevation head of the pixel is $j \times b$ with the datum taken at the bottom boundary. Meanwhile, the first face or sub record indicates the type of the pixel which may be one of the following,

1. An air pixel, where no properties for this type
2. A soil or rock pixel having two main properties k_h, k_v which are the coefficients of permeability in the horizontal and vertical directions, respectively.
3. An intermediate boundary pixel, where no properties are given for this type in the beginning. The solver can decide which soil this boundary should be, else, the properties will be the average of the surrounding pixels.
4. A water pixel having the main property as unit weight γ_w and for the sake of solution very high values for the coefficients of permeability. This can be omitted if replaced by head values at soil pixels. This type of pixels is important when you have a water pocket inside the soil media.

The second face indicates the hydraulic status of the pixel which can be either of the following,

1. A hydrostatic pixel, where the total head is constant and does not participate in the flow zone.
2. An impervious pixel, where the flow cannot pass through.
3. A flow pixel, where the pixel may fall in the flow zone.
4. A phreatic line pixel, where the total head h_t equals the elevation head h_e . Phreatic line pixels appear only in the case of unconfined flow.

The third face indicates the value of the total head for the pixel. In other words, it is the latest value reached in the model at any stage of solution. To simplify how the soil media and boundary conditions are represented in the pixels' matrix, Fig. 3(A) shows a simple earth dam made of soil 1 laid over soil 2. The domain is discretized to a pixel's matrix where the geometrical records are given to the pixels as shown in Fig. 3(B), and the records for flow participation status are given to the pixels as in Fig. 3(C). The phreatic line shown is just a first choice and it will adjust itself during the solution iterations and hence, the pixel may get its flow status record flipped from one status to another through the iterations.

Pixels are numbered from 1 to the maximum number of pixels available based on the problem dimensions and pixel size chosen. For each pixel there is a set of two integer coordinates i, j for the horizontal and the vertical directions showing its location in the pixel matrix. They are assigned at the bottom left corner of the pixel. These integer coordinates are found using the real coordinates and the pixel size. Any point in the real domain is considered within the pixel i, j if its real coordinates fall within or on the boundaries of the pixel. As the points are lumped to pixels in rasterization of the soil media, there will be no need to think of real coordinates anymore.

4 Adopted Raster Scan Algorithms

If all boundaries, whether external boundaries of the problem or internal boundaries between different zones (soil, water, air), are modelled as single or multiple straight lines, the main two algorithms needed to rasterize the model are line drawing and polygon filling algorithms. The adopted raster scan conversion algorithms are called the Differential Digital Analyzer *DDA* algorithm for line drawing and Simple Seed Fill algorithm for area filling. There are other algorithms that can be evaluated as well such as Bresenham's algorithm for line drawing and the edge fill algorithm for polygon filling, cf. [4].

4.1 Digital Differential Analyzer (DDA) Algorithm

This algorithm is applied to locate the boundaries in the model. There are two types of boundaries. The first type is a geometric boundary, which separates soil types, water, air, or external boundaries, and the second type is a hydraulic state boundary, where the media is separated based on the flow status in the pixel distinguished to hydrostatic state or boundary, flow zone, impervious zone or boundary, or a phreatic line.

The slope m of a line starting at a left pixel number s with integer coordinates (i_s, j_s) and ending at a right pixel number t having the integer coordinates (i_t, j_t) , may be

calculated as,

$$m = \frac{j_t - j_s}{i_t - i_s} \tag{1}$$

With the start point and the slope of the line known, the line in pixels screen may be drawn. After determining the initial pixel on the line, the rest of the pixels forming the line are determined by taking unit steps with one coordinate and calculating the corresponding unit step for the other coordinate using the slope as realized [5]. The cases to plot a line in a pixel matrix from s to t may be summarized in the following cases:

Case 1: if $|m| < 1$,

$$i \text{ for the next pixel} = i \text{ for the previous pixel} + 1,$$

$$j \text{ for the next pixel} = j + \text{Int}(\Delta . m)$$

Case 2: if $|m| > 1$,

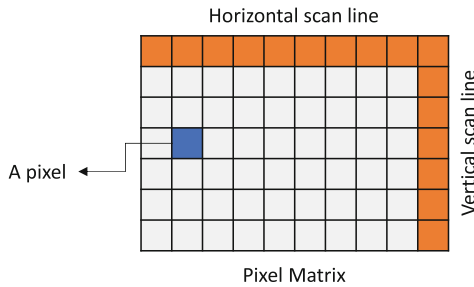


Fig. 1. The pixel matrix and the scan lines

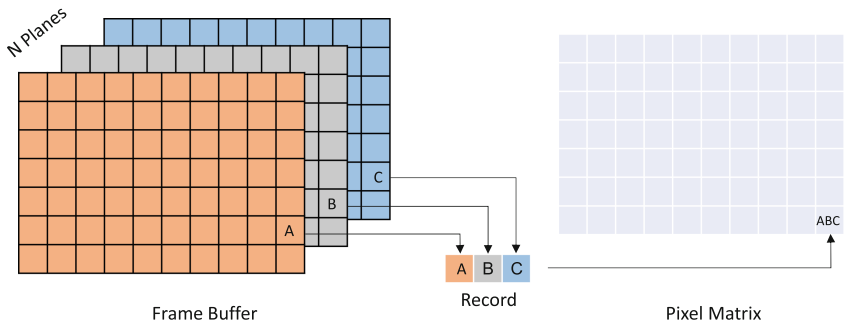


Fig. 2. Records made from bits on frame buffer planes and collected in the pixel matrix.

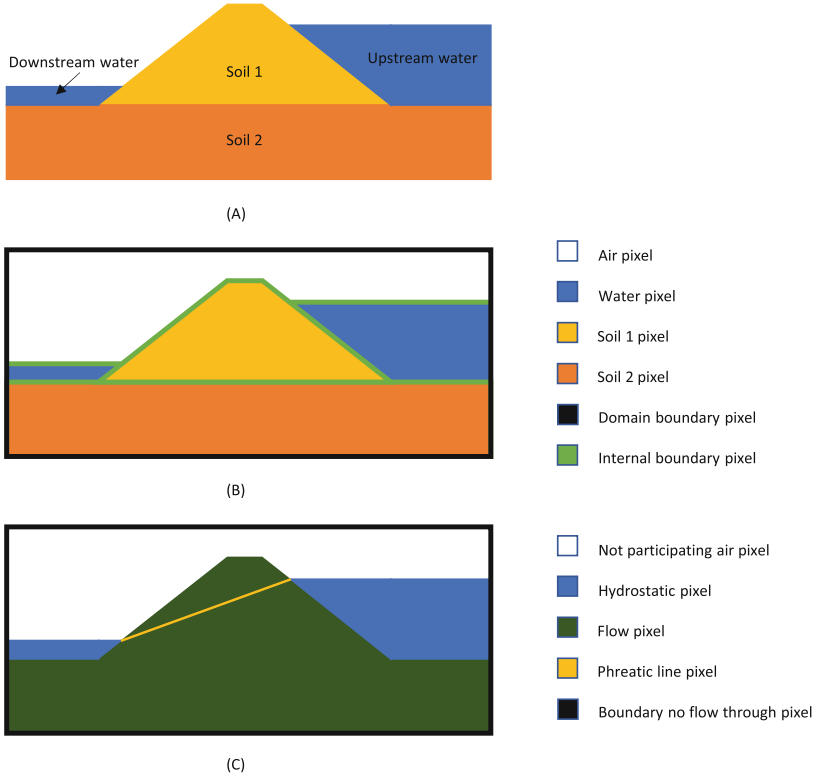


Fig. 3. Modeling a flow problem in soil media (A), with geometry pixels (B) and for participation status in the groundwater flow (C)

$$j \text{ for the next pixel} = j \text{ for the previous pixel} + 1,$$

$$i \text{ for the next pixel} = i + \text{Int}(\Delta / m)$$

Case 3: if $|m| = 1$,

$$i \text{ for the next pixel} = i \text{ for the previous pixel} + 1,$$

$$j \text{ for the next pixel} = j + m$$

where Δ is the number of steps moved. Hence, the calculated step using the slope will not get realized to full pixel unless the step size exceeds unity. Refer to [5] for further explanation.

The *DDA* algorithm ends when i reaches i_t or j reaches j_t . The input data for this procedure shall be,

1. The coordinates of end points of boundary (i_s, j_s) and (i_t, j_t)
2. The type of geometrical boundary (intermediate boundary or flow boundary)
3. The type of hydraulic status of the boundary
4. The total head at the pixel as a first trial or if it is fixed value.

The DDA here will not only draw the line of pixels but also assigns the geometric and the flow boundary types to the pixel and the initial total head.

4.2 Simple Seed Fill Algorithm

This algorithm is applied to give the pixels in the areas, (polygonal zones), that are bounded by the pixel lines drawn by the DDA, their property records. These zones have their geometric and hydraulic status records. The geometric zone shall include pixels of the same type such as air, water, or specific soil type. Similarly, the hydraulic zones have pixels of the same hydraulic flow status such as impervious pixels, constant head, phreatic line, or flow pixels.

To fill an area enclosed by polygons with a certain type of pixels using the algorithm needs only one random pixel to be picked from the zone. This is called the “seed pixel”. Let this pixel be (i_s, j_s) , the four adjacent pixels are examined as shown in Fig. 4. Each tested pixel, which is found not like the seed pixel will be pushed to a stack. The stack is simply an array into which pixels may be sequentially placed or removed. As new pixels are added to the stack, all previously stored pixels are pushed down one level. After testing the four adjacent pixels a new seed pixel is chosen by taking out the last stored pixel in the stack. The new seed pixel shall pop out and get the same properties (records of the zone. Such a stack is referred to as a first in last out stack (FILO) with a push-pop procedure. Again, new four adjacent pixels are tested and if any of them is not a boundary pixel nor a pixel that has correctly defined record, the pixel is pushed to the stack. This process continues until the stack gets empty. Further details may be found in [4].

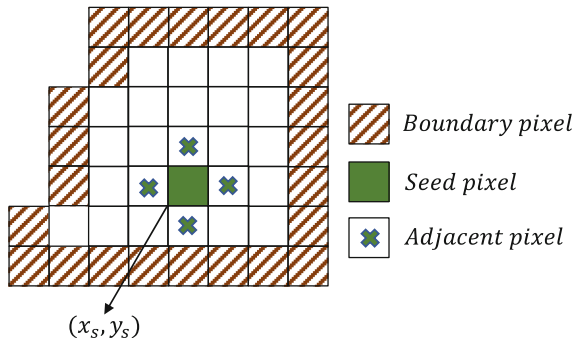


Fig. 4. Area filling by the seed fill algorithm

5 Representation of Soil Media and Boundary Conditions in Seepage Analysis

To prepare the pixel matrix with all required records for geometry and boundaries, and hydraulic properties and status, the following steps are followed:

A. Geometry:

1. Give the lower left corner and upper right corner vertices of the rectangle containing all the model.
2. Number all the pixels in that rectangle with their locations in the pixel matrix based on the pixel size chosen.
3. Start giving the numbers of the vertices (or the coordinates) for the external boundaries for the soil media. Assign the pixels and the type of each boundary line specifying the two vertices of each line. Draw the line through the pixels using the simple DDA.
4. Specify internal boundaries between soil zones by the vertices of lines between soils and drawing the lines with DDA. Assign one of the two separated soils to the internal boundary line.
5. Start filling the zones of pixels based on the soil type by picking one point inside the zone and filling the rest of the zone using Simple Seed Fill algorithm.

The record for the geometry of the pixels will hold the following, number of the pixel, its location in the pixel matrix, its type of material (air, water, soil type, external boundary).

B. Hydraulic properties:

1. Go back to each pixel and give its hydraulic status by assigning first the hydraulic boundary lines. Re-divide the external boundaries of the soil media based on their hydraulic status, either no flow impervious status or constant head. Use DDA and assign the head for the constant head pixels.
2. If you have unconfined flow you need to assign one more line or polyline for the initial estimate of phreatic line. Draw by DDA and assign the pixels initial total head as their elevation, the vertical coordinate in length units.
3. Assign the hydraulic properties (coefficients of permeability in both directions) for all soil pixels. Use Simple Seed Fill algorithm for that. For impervious soil zones (pockets or intermediate layers) use very low permeabilities or assign them as impervious boundaries.

Accordingly, the hydraulic record of each pixel will be, pixel number, hydraulic status, (pervious, constant head or impervious), k_h , k_v , and the total head h_t .

6 The Flow Pixel Matrix

6.1 Equation of Seepage Analysis

Let the pixel P_0 corresponding to the coordinates position i, j in the pixel matrix, be the pixel under scanning. Scanning usually is done row by row, left to right, from the top to bottom of the pixel matrix. Considering first the case where the P_0 is in a flow zone, the four pixels around it will be designated as P_1, P_2, P_3 , and P_4 as shown in Fig. 5.

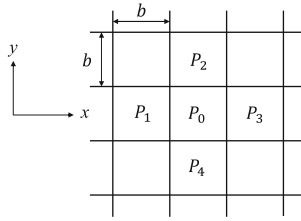


Fig. 5. Pixel notation for flow analysis at pixel P_0

The horizontal flow from the pixel P_1 to P_0 pixel may be expressed by Darcy's law,

$$q_{01} = k_{01} \left(\frac{h_0 - h_1}{b} \right) b = k_{01} (h_0 - h_1) \quad (2)$$

where b is the pixel size, h_0 and h_1 are the total heads at P_0 and P_1 and k_{01} is the horizontal coefficient of permeability between the pixels. Assuming flow positive in the global directions of axes, the flow in other directions will be similarly determined,

$$q_{03} = k_{03} (h_3 - h_0) \quad (3)$$

$$q_{04} = k_{04} (h_0 - h_4) \quad (4)$$

$$q_{02} = k_{02} (h_2 - h_0) \quad (5)$$

The net flow through a pixel shall be zero to maintain the steady state, hence,

$$\Delta q = k_{01} (h_0 - h_1) - k_{03} (h_3 - h_0) + k_{04} (h_0 - h_4) - k_{02} (h_2 - h_0) = 0 \quad (6)$$

The equation of seepage analysis may be re-formed as,

$$h_0 = \frac{k_{01} h_1 + k_{02} h_2 + k_{03} h_3 + k_{04} h_4}{k_{01} + k_{02} + k_{03} + k_{04}} \quad (7)$$

This is similar to a finite difference equation in numerical seepage analysis, cf. [6].

To determine the coefficients of permeability at the borders between pixel P_0 and each one of the surrounding pixels, it can be assumed that the flow velocity in two adjacent pixels across a border is constant, hence, the intermediate value for the coefficient of permeability k_{0n} can be easily determined from the values of k_0 and k_n where n is 1, 2, 3, or 4, referring to the adjacent pixel,

$$k_{0n} = \frac{2k_0 k_n}{k_0 + k_n} \quad (8)$$

where k_0 and k_n are taken as the horizontal permeabilities of the pixels if the flow is horizontal and the vertical permeabilities if the flow is vertical.

Thus, Eq. 8 can now be used to construct the matrix equation of the flow, but the matrix equation is still singular, and the boundary conditions must be added.

6.2 Boundary Pixels

In confined flow there are three types of boundary pixels, and each type is handled in a different way,

1. A boundary where the total head is fixed. In this case the equation of the flow is adjusted to one term, $h_{i,j} = h_{\text{specified}}$.
2. An internal boundary between different zones of soil where flow is allowed through. In this case, either the boundary pixel is given a soil type from the beginning with the DDA or the permeabilities are averaged between the surrounding soils.
3. A boundary where the flow is not allowed through, i.e., an impervious pixel. In this case the coefficient of permeability in Eq. 10 for any impervious pixel will be zero.

In unconfined flow, in addition to the above, there will be a phreatic line. The phreatic line is assumed first, see for example Fig. 3(C). The pixels of the line will be given a prescribed head equal to their elevation head $h_{i,j} = j \times b$. The solution must be iterative in this situation. After each iteration, the phreatic line pixels are checked for the heads. If the heads obtained are higher than their elevations the above pixel will be the new estimate for a phreatic pixel and vice versa. The solution is repeated until the difference between the heads of the two solutions reaches an acceptable accuracy limit.

It should be noted that air pixels, impervious pixels, or pixels above phreatic pixels will be excluded from the matrix equation.

7 Solution of the Matrix Equation and Results

Although the matrix equation may be solved by direct methods, iterative solutions are favored due to the high sparsity of the matrix elements. An initial guess for total heads of all flow pixels could be their elevation heads. Values are corrected in each iteration according to Gauss Seidel method [7].

To stop the iterations and finalize the solution, an accuracy limit $\varepsilon = 0.001$ is chosen and the check may be in the form of

$$\max \left| h_{t,ij}^{k+1} - h_{t,ij}^k \right| < \varepsilon \quad (9)$$

where k refers to the iteration number. In this case, ε will have units. This is found acceptable especially when you have zeros in the solution. Sometimes, Euclidean norm of the error vector or relative error may be used.

After total heads reach acceptable accuracy, the solution is stopped, and other results may be computed such as flow quantity or pressure heads and porewater pressures.

Through any chosen line of pixels on the pixel matrix the flow through that line may be easily determined by calculation the flow velocity through each pixel by discretized Darcy's law. For example, if a flow is from left to right the horizontal velocity of flow through a pixel on a query line, will be

$$v_{i,jtoi+1,j} = k_{i,jtoi+1,j} \frac{h_{ti,j} - h_{ti+1,j}}{b} \quad (10)$$

The flow rate may be then calculated through the pixel of width b as

$$q_{i,jtoi+1,j} = k_{i,jtoi+1,j}(h_{ti,j} - h_{ti+1,j}) \quad (11)$$

And the total quantity of seepage in the horizontal direction will be the summation of all flow rates through the pixels of the line.

A similar approach is applied for the vertical flow and the resultant flow.

Another result from the analysis is the porewater pressure $u_{i,j}$ which is equal to pressure head multiplied by the unit weight of water,

$$u_{i,j} = h_{pi,j} \times \gamma_w = (h_{ti,j} - j \times b)\gamma_w \quad (12)$$

8 Comprehensive Verification Example

Many verifications were performed in fact. Some were on confined flow problems, and some were on unconfined flow problems. The example shown here is based on a problem given in [8]. The dimensions shown in Fig. 6(A), as sketched in Slide2 software. The total head at the upstream is 60 m, and the shell and the core are made of anisotropic permeability. For the shell material the horizontal coefficient and the vertical coefficient of permeability are $3E-7$ m/s and $3E-8$ m/s respectively while for the core they are $5E-8$ m/s and $5E-9$ m/s respectively. In Slide2, finite element analysis with 3000 triangular elements and accuracy of 0.001 were implemented. While in the raster scan method, this study, the square pixels were of 0.1 m sides and the accuracy was 0.001 as well. The results are shown in graphical form in Fig. 6, showing the phreatic lines and equipotential lines with 6 m drop in head between successive lines.

The results have shown an acceptable match. Also, two vertical lines were added to each model to get the flow rate through them. The results for each model are given on the plots. The raster scan method showed smoother transition in the phreatic line at the boundary between the right shell and the core, and closer to the flow net expectation given in the original reference. The equipotential lines in raster scan method are thicker due to including more pixels within a range and not averaging as in Slide2.

9 Conclusion

The concept of representing the soil media by square pixels and applying raster scan algorithms to define the pixel types and properties is a genuine idea that can be applied to all fields of soil mechanics after modifying the traditional equations into pixel-based equations. The method is applied by the author to seepage analysis in this paper and to slope stability elsewhere. The method is found powerful in programming and solution.

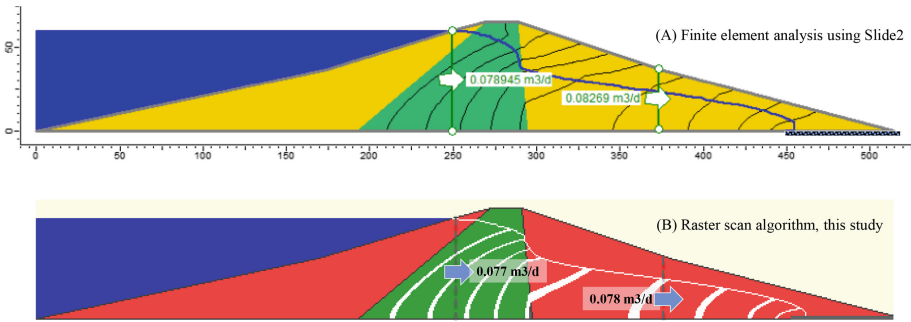


Fig. 6. Comparison of Raster Scan Method with FEM using Slide2

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