Investigation of the Effect of Tuned Mass-Damper System on the External Vibrations Endured by a Vehicle

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Abstract. This report presents the application of a Tuned-Mass Damper system to effectively reduce external vibrations experienced by the driver, caused by the random vibrations of the track conditions. The aim of this study is to investigate the impact of a tuned mass-damper (TMD) system on external vibrations endured by a vehicle under random vibration conditions. A quarter car model is used to simulate vehicle vibrations under random conditions and the TMD system is modeled as an additional mass-spring-damper system connected to the vehicle body. Simulation results show that the TMD system effectively reduces root mean square (RMS) value of the vehicle's acceleration by 19.804%, and its effectiveness increases with the TMD mass. The study's results indicate that a TMD system can be an effective method to improve a vehicle's ride comfort under random vibrations. The analysis was performed by modeling the system on Simulink, and the results were derived from the same model.

Keywords: Tuned Mass Damper (TMD) · Quarter Car Model · Random Vibrations

1 Introduction

Vibration control is a crucial aspect of vehicle design and development, as vibration affects the ride comfort and safety of the vehicle. One of the most commonly used passive control methods for reducing the vibrational amplitudes in the structure is the tuned mass-damper (TMD) system. A TMD system is a mechanical device that is designed to reduce the amplitude of vibrations in a structure by absorbing and dissipating the energy of the vibrations. The TMD system consists of a mass, spring and damper, which are designed to resonate at a specific frequency to maximize their damping effect. Alternatives to TMDs include Active suspension, Electromagnetic suspension, Pneumatic dampers etc. However, TMDs are advantageous as they do not require any external controlling system or any power source to control vibrations.

TMD systems have been demonstrated to be effective in reducing the vibrations in various structures such as bridges and buildings. However, the effectiveness of TMD

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systems for a vehicle under random vibrations has not been well studied. Random vibrations refer to vibrations that have no discernible pattern in their frequency or amplitude. It includes vibrations caused by road roughness, irregularities and other external excitations which can lead to discomfort and fatigue for vehicle occupants.

1.1 Literature Review

Baki Ozturk et al. [1] This research investigates the best configuration for a tuned mass damper (TMD) throughout all frequency range to lower the dynamic response of cantilever beams. To achieve this, random vibrations are used to identify the objective function to be decreased, which is the RMS acceleration of the end point of the cantilever beam.

Marian Sikora [2] The author models and simulates a damper with the TMD concept under oscillatory inputs, and provides a critical analysis of the results.

Anand S et al. [3] The efficacy of Tuned Mass Dampers in passenger cars is evaluated in this paper, and it is found that the addition of a tuned mass to the suspension system significantly reduces the transmissibility. A study was conducted to examine how modifying different parameters, such as the damping of the suspension, tuned mass, and spring stiffness, would impact the outcome.

1.2 Quarter Car Model

A quarter car model is a mathematical model that represents the dynamic behavior of a suspension system. It includes two masses ie- sprung and unsprung mass that illustrates one quarter of the vehicle, two springs that represent the suspension spring and the tyre stiffness, and two dampers that represent the damping in the suspension and the damping of the tyre. Figure 1 represents a quarter car model.

The quarter car model which is used to study the behavior of a suspension system and predict how it will respond to different inputs, such as road inputs and braking and acceleration forces. It does not represents angular roll, pitch and yaw however, it forms the basis upon which the above quantities are evaluated using the half and full car model.

While the quarter car model is a simplification of an actual vehicle, its importance comes from the possibility to analyze and optimize the suspension system in an efficient way and for a lower cost than using a full-car model, which is more complex and time-consuming to analyze.

1.3 Quarter Car Model with TMD

Figure 2 shows use of a TMD on a quarter car model.

For the following section, notations used are:

m_1 = mass of unsprung mass	c_2 = damping coefficient of sprung mass
m_2 = mass of sprung mass	c_d = damping coefficient of TMD
m_d = mass of TMD	u_g = displacement of ground

(*continued*)

(*continued*)

k_1 = spring stiffness of unsprung mass	u_1 = displacement of unsprung mass
k_2 = spring stiffness of sprung mass	u_2 = displacement of sprung mass
k_d = spring stiffness of TMD	p_1 = force on unsprung mass
c_1 = damping coefficient of unsprung mass	p_2 = force on sprung mass

The equations of motion for a quarter car model without Tuned Mass Dampers are:

$$m_1\ddot{u}_1 + c_1\dot{u}_1 + k_1u_1 - k_2(u_2 - u_1) - c_2(\dot{u}_2 - \dot{u}_1) = p_1 - m_1\ddot{u}_g \quad (1)$$

$$m_2\ddot{u}_2 + c_2(\dot{u}_2 - \dot{u}_1) + k_2(u_2 - u_1) = p_2 - m_2\ddot{u}_g \quad (2)$$

The equations of motion for a quarter car model with Tuned Mass Dampers are:

$$m_1\ddot{u}_1 + c_1\dot{u}_1 + k_1u_1 - k_2(u_2 - u_1) - c_2(\dot{u}_2 - \dot{u}_1) = p_1 - m_1\ddot{u}_g \quad (3)$$

$$m_2\ddot{u}_2 + c_2(\dot{u}_2 - \dot{u}_1) + k_2(u_2 - u_1) - k_du_d - c_d\dot{u}_d = p_2 - m_2\ddot{u}_g \quad (4)$$

$$m_d\ddot{u}_d + k_du_d + c_d\dot{u}_d = -m_d\left(\ddot{u}_2 + \ddot{u}_g\right) \quad (5)$$

By combining the above equations and expressing them in a form similar to a Single Degree of Freedom (SDOF) system, the problem can be reduced to an equivalent SDOF system using matrix notation. A Simulink model was created based on the above equations and the values of masses, spring stiffness and damping coefficients were given according to Table 1. These values are based on a small capacity passenger car in the Indian market.

Note that the mass of the original car is 700 kg but a mass of 175 kg is taken due to the fact that in the quarter car model, only one-fourth of the mass is assumed to be acting on the suspension system [6].

2 Force Input

Random vibrations refer to vibrations or oscillations that occur in an irregular or unpredictable manner. Unlike periodic vibrations that occur at regular intervals, random vibrations do not follow a predictable pattern and can vary in amplitude, frequency, and direction. They are typically caused by external factors such as wind gusts, uneven road surfaces, or machinery vibrations and can be difficult to predict or control. Random vibrations can have a significant impact on structures, machines, and vehicles, causing damage, discomfort, or instability. Therefore, it is important to study and analyze random vibrations to develop effective methods of mitigating their effects (Fig. 3).

The road surface is assumed to have undulations of up-to a maximum of ± 10 cm in height, which is the standard height of a speed-breaker. The corresponding limits of force acting on the suspension system was calculated. The input was given a random number with Gaussian Distribution of mean = 0, variance = 1010416.667 and sample time = 0.1 s. All the results obtained for the model are all based on the above input function.

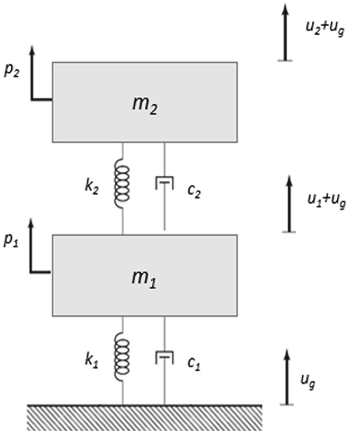

Fig. 1. Quarter Car Model Representation

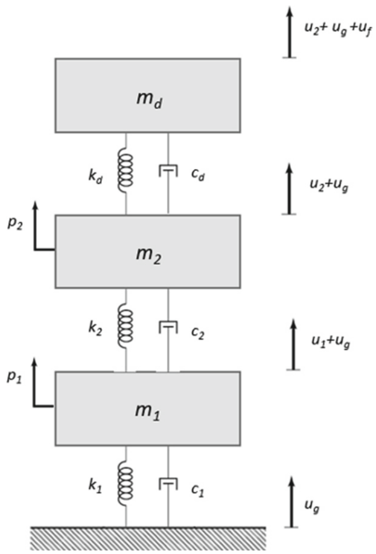

Fig. 2. Quarter Car Model with attached TMD [4]

3 Design of the TMD

In the formulation of the governing equations, it can be clearly noticed that the spring and damper of the Tuned Mass Damper system is heavily dependent on the following variables:

1. Mass ratio (μ)
2. Stiffness of the spring of the strut, (k_1)

Table 1. Parameters for quarter car model [5]

Parameters	Symbol	Value
Sprung mass	m_2	175 kg
Spring stiffness of the suspension system	k_2	14,000 N/m
Damping coefficient of suspension	c_2	341.88 Ns/m
Unsprung mass	m_1	43.76 kg
Stiffness of tyre	k_1	201021 N/m
Damping coefficient of tyre	c_1	14.6 Ns/m

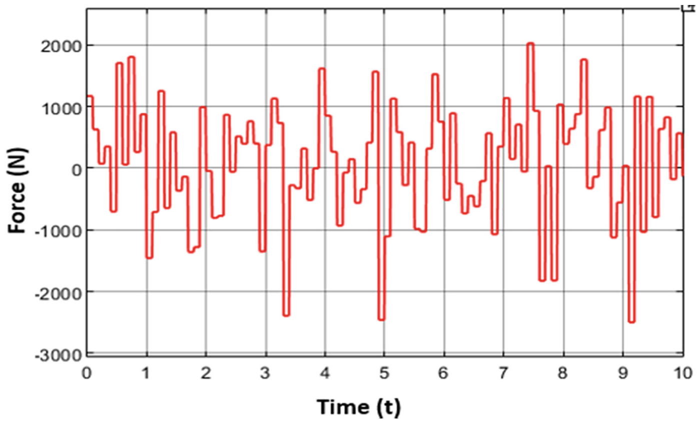

Fig. 3. Graph showing the input force

The combinations of these variables lead to different outcomes for the vibrations of the vehicle's sprung mass. A vehicle experiences a very wide variety of loading conditions in terms of frequency, amplitude, jerk etc. Each different loading condition has a unique response from the vehicle and the vehicle experiences vibrations. In order to account for all these ranges of vibrations, a suitable method must be chosen for determining the stiffness of the spring of the Tuned Mass Damper and also its damper.

The following table shows a list of options for selecting the stiffness and damping coefficient of the tuned mass damper based on mass ratio and stiffness of the spring of the strut (Table 2).

Since the objective of this study is to minimize the vibrations of the vehicle for all kinds of road conditions, the second option of "**Minimizing the total displacement of sprung mass over all frequency**" was chosen.

The stiffness and damping ratio of the TMD spring and damper are given by the formulae-

$$k_d = \frac{k_2\mu}{(1+\mu)^2}\left(\frac{2+\mu}{2}\right)\xi = \sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$$

Table 2. Spring constant and Damping Coefficient for different objectives.

Objective	The TMD's damping ratio and stiffness
Minimizing max. Displacement of sprung mass [7, 8]	$k_d = \frac{k_2\mu}{(1+\mu)^2}\xi = \sqrt{\frac{3\mu}{8(1+\mu)}}$
Minimizing the total displacement of sprung mass over all frequency [9, 10]	$k_d = \frac{k_2\mu}{(1+\mu)^2}\left(\frac{2+\mu}{2}\right)\xi = \sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$
Reducing the system's transient vibration. [11, 12]	$k_d = \frac{k_2\mu}{(1+\mu)^2}\xi = \sqrt{\frac{\mu}{(1+\mu)}}$
Minimization of displacement of the sprung mass and relative displacement [13]	$k_d = \frac{k_2\mu}{(1+\mu)^2}\xi = \sqrt{\frac{\mu}{2(1+\mu)}}$
Minimizing total KE of the sprung mass over all frequencies [9]	$k_d = \frac{k_2\mu}{(1+\mu)}\xi = \frac{\sqrt{\mu}}{2}$

An initial TMD mass (m_d) of 10 kg was taken for the study however, the effectiveness vs. mass ratio (refer Fig. 6) has also been studied, varying the TMD mass over a range of values.

4 Results and Discussions

The following section shows the results obtained on Simulink for the given inputs:

1. The effectiveness was found to be 19.804%
2. The percentage change in transmissibility due to the TMD was found to be 19.314%

4.1 Comparison Between the Position of TMD and Sprung Mass

The inferences from the above graph (Fig. 4) are as follows:

1. The tuned mass damper and the vehicle sprung mass initially move upwards as the suspension system is excited. The TMD quickly moves out of phase from the vehicle mass owing to its construction and spring stiffness and damping value.
2. This out of phase vibration of the two bodies creates a destructive interference between two waveforms and thus, the amplitude of the vibrations of the sprung mass of the vehicle as well as that of the tuned mass damper reduce at that range of time when the destructive interference takes effect. This creates negative feedback for both the masses, reducing the amplitude of vibrations for both.

In the above figure, the out of phase nature of the two waves can be seen as, at the instant of time when one of the waves is rising to its peak, the other wave is falling to its crest. This phenomenon is responsible for the destructive interference between the two waves. This phenomenon can be clearly noticed between t = 2.25 s to t = 3.5 s. It is worth noting that the destructive interference phenomenon does not occur over the entire range of time. This is because the force input is random and it has no particular defined frequency or amplitude as it simulates real-world road conditions. Since the system has

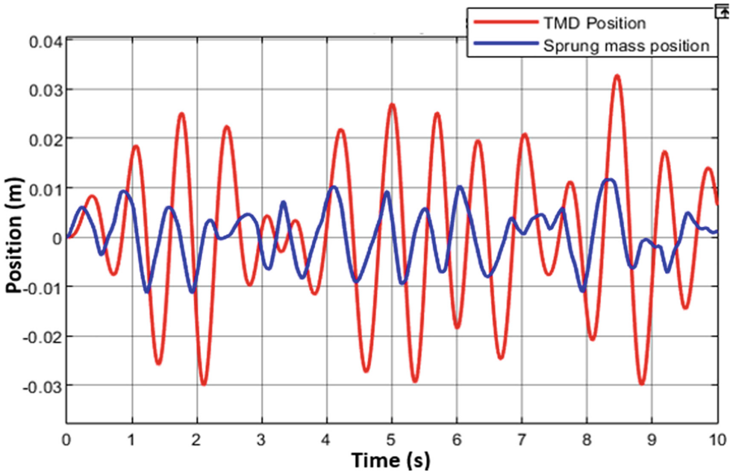

Fig. 4. Amplitude vs time graph of TMD and Sprung mass

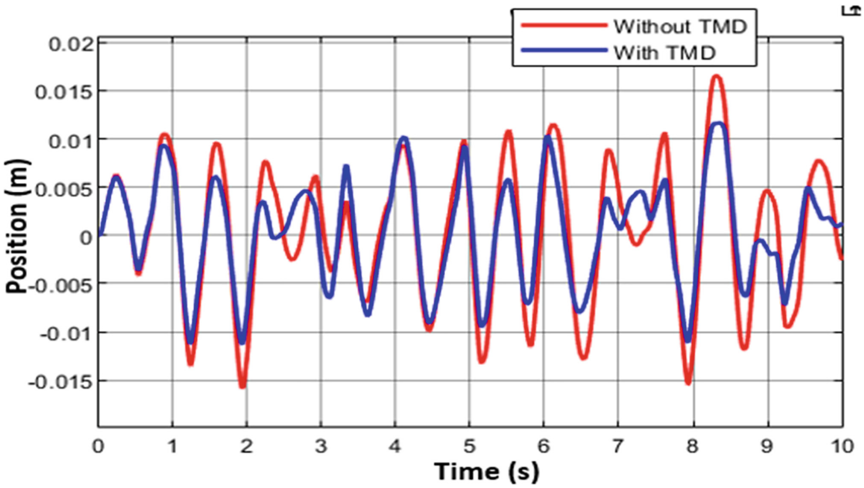

Fig. 5. Amplitude vs time graph of a system with and without a TMD

no active control system, the TMD mass cannot always be out of phase to the unsprung mass. Thus, for maximum comfort and handling of the vehicle over all road conditions, the TMD was designed to minimize the overall displacement of the sprung mass across all frequencies. Had the input force been more periodic in terms of force applied, with a constant frequency and amplitude, the vibration absorbing characteristics could have been better.

4.2 Comparison Between a System with and Without a TMD

In this section of the results, a direct comparison is made between two different models, one in which Quarter Car model is retrofitted with a TMD and another in which the Quarter Car does not have any Tuned mass Damper attached to it (Fig. 5).

1. 1.It can be clearly observed that at almost all points in the time domain, amplitude of vibration of the model with attached Tuned Mass Damper lies below the model without the TMD. This means that the model with the attached Tuned Mass Damper always experiences less disturbance in terms of vibrational amplitude.
2. The peaks values of vibration are almost always less in case of the model with an attached Tuned Mass Damper as compared to the model without the Tuned Mass Damper. This reduction in peak value has been quantified as the percentage change in value of the peaks of vibrations of models with and without an attached Tuned Mass Damper.

4.3 Effectiveness of TMD

The effectiveness of a TMD refers to the ability of the TMD to reduce the amplitude of vibrations. The effectiveness of a TMD can be defined in many different ways like force effectiveness, acceleration effectiveness, velocity effectiveness amplitude effectiveness time effectiveness etc.

The **acceleration effectiveness** ($\in_{Acc}$) is a better indicator of the reduction of random vibrations than force ratio, as acceleration provides a more direct measure of the system's response to random vibrations, and it is also a more important parameter to evaluate the safety of the system. In this context of random vibrations, acceleration effectiveness of a Tuned Mass Damper is a measure of how well the TMD is able to reduce acceleration of the system when it is subjected to random vibrations. It is defined as—ratio of difference of the RMS acceleration of the system without TMD and RMS acceleration of the system with the TMD to the RMS acceleration of structure without the TMD. A higher acceleration effectiveness means that the TMD is more effective

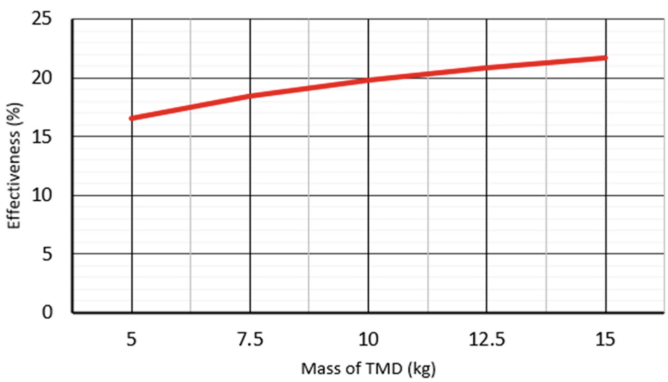

Fig. 6. Graph of Effectiveness vs Mass Ratio

in reducing the acceleration of the structure, and therefore reducing the amplitude of random vibrations.

$$\varepsilon_{Acc} = 1 - \frac{RMS\ acceleration\ With\ TMD}{RMS\ acceleration\ Without\ TMD}$$

For the above case of TMD mass 10 kg, the **effectiveness was found to be 19.804%** over the entire time domain of 10s.

This result was observed for the vehicle parameters given in Table 1 and a TMD of mass 10 kg.

The above graph in Fig. 6 illustrates how the effectiveness varies with an increase in the mass of TMD. The graph indicates that with an increase in the mass of the TMD, the effectiveness percentage increases monotonically but the rate of change in percentage effectiveness decreases. This is likely because, as the mass of the TMD increases, the sprung mass of the vehicle also increases since the TMD is directly bolted to the vehicle chassis. This can result in less effective vibration control. At a certain point, adding more mass to the TMD will not have a significant impact on vibration control and may even have a negative impact.

4.4 Transmissibility of TMD

Transmissibility (ε) is defined as the force transmitted to the force applied. In this case, the transmissibilities of the two cases- Transmissibility with the TMD and transmissibility without TMD need to be compared. Thus, their individual transmissibilities are found out and the percentage change in transmissibility due to the TMD is determined.

Transmissibility of TMD is a measure of how well the TMD isolates the structure from the vibrations. It is a dimensionless value, and a value of 100% means that the TMD is not reducing the vibrations, while a value less than 100% means that the TMD is reducing the vibrations. The closer the value is to 0, the more effective the TMD is at isolating the system from vibrations. Transmissibility is a useful measure to evaluate the isolation performance of the TMD, but it should be used in conjunction with other parameters such as acceleration effectiveness to evaluate the overall performance of the TMD and the safety of the system.

$$\varepsilon = \frac{ForcetransmittedtoSprungmass}{Forceapplied}$$

Transmissibility was found to be 23.775% without the TMD.

Transmissibility was found to be 19.183% with the TMD.

Thus, the **percentage change in transmissibility due to the TMD was found to be 19.314%** for the vehicle parameters given in Table 1 and a TMD mass of 10 kg.

5 Conclusion

The aim of this study was to develop a mathematical model of a Tuned Mass Damper applied to a quarter car model to simulate and analyze the vehicle's response to practical road conditions with random vibrations. Two models were compared: vehicles with and without a TMD. The study showed potential for improved vibration control and ride comfort, as evidenced by position-time response graphs of the sprung mass displaying reduced displacement and acceleration magnitudes over a broad frequency range. However, the trade-off is the motion of the TMD mass. Simulation results demonstrated the TMD system's efficacy in reducing the root mean square (RMS) value of the vehicle's acceleration, with effectiveness increasing with TMD mass. The conclusion is that a TMD system can effectively improve a vehicle's ride comfort under random vibrations. The study lays out a framework for analyzing and tuning TMDs with an added mass, based on the data inputs that are easily available. However, the authors believe that the use of an Active Tuned Mass Damper (ATMD) would be more beneficial for the vibration characteristics of the vehicle, compared to the traditional passive Tuned-Mass Damper (TMD) system. The ATMD system can actively adjust its parameters in real-time, to match the changing road conditions, which can lead to a more efficient vibration control and better ride comfort.

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