# The Application of the Box-Jenkins (BJ) Method for Process Identification of the Batch Milk Cooling System

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**Abstract.** The Box-Jenkins (BJ) method is a well-known system identification method that has been applied in several fields. Engineers use the Box-Jenkins method for quality control and process optimization in manufacturing. It can identify patterns and trends in production data, leading to improvements in product quality and efficiency. This study presents the application of the BJ method for the identification of a batch milk cooling process. The primary goal of this research is to create a model for the batch milk cooling process, intending to ease the control and enhance the optimization of milk cooling procedures. The data used in this study was collected from a batch milk cooling process over a period of time generated from Simulink. The BJ method was applied to the data to obtain the process transfer function. The model was then used to simulate the batch milk cooling process and tested for different perturbations. The findings indicated that the BJ method effectively represented the batch milk cooling process with high accuracy. The final result obtained from the BJ method is: $\frac{0.02361}{s+0.02361}$ which corresponds to $\frac{1}{42.3549s+1}$.

**Keywords:** Box-Jenkins, Simulation, Dynamic Study, Milk Cooling.

## 1 Introduction

The dairy sector is in charge for the production and the handling of milk and its derived products, encompassing items like cheese, yogurt, butter, and ice cream. [1]–[3]. The dairy industry plays a vital role in the worldwide food sector, and it provides a wide range of dairy products for human consumption and industrial use. Milk production is a primary activity in the dairy industry, and it involves the collection and processing of raw milk from dairy farms. Stringent regulations govern the dairy industry to uphold the wellbeing and excellence of dairy products, with strict standards in place for their production, processing, and marketing [4], [5].

The dairy industry is regulated to guarantee the safety and quality of dairy products, safeguarding consumers from potential harmful contaminants or pathogens. Government agencies and organizations, like the FSANZ (Food Standards Australia New Zealand) in Australia and New Zealand, the EFSA (European Food Safety Authority) in


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the EU, the FDA (Food and Drug Administration) in the US, are usually responsible for overseeing the regulation of the dairy industry.

The regulations in the dairy industry typically cover a wide range of areas, including:

1. Milk production: Regulations for dairy farming, including guidelines for the care and treatment of dairy animals, and standards for the collection and storage of raw milk.
2. Milk processing: Regulations for the processing and handling of raw milk to produce dairy products, including standards for pasteurization, homogenization, and the addition of other ingredients.
3. Product labeling: Regulations for the labeling of dairy products, including information about ingredients, nutrition, and allergen warnings.
4. Product testing: Regulations for the testing of dairy products to ensure that they meet established standards for safety, quality, and composition.
5. Food safety: Regulations for the prevention of food-borne illness, including standards for sanitation, packaging, and storage of dairy products.

Milk cooling is an important process in the dairy industry [6], as it affects the quality and safety of the final product. The batch milk cooling process is a dynamic process that is difficult to control and optimize. Therefore, there is a need for a model that can accurately represent the batch milk cooling process.

The BJ method is a well-known system identification method that has been applied in several fields, and it has been shown to be effective in modeling dynamic processes. In their study, Serfidan et al. introduced an automated tool that applies the Box-Jenkins methodology to predict crude oil prices and its primary products for the upcoming two months at the start of each month [7]. Guleryus constructed a Box-Jenkins forecasting model, utilizing statistical data for Turkey aiming to manage and mitigate the transmission of the COVID-19 virus [8]. Shadab et.al examined the application of the Box-Jenkins method to model solar radiation under varying sky conditions, providing insights into predicting and analyzing future trends of monthly insolation through the examination and study of temporal data [9]. In this study, the BJ method was applied to the batch milk cooling process to develop a model that could be used to control and optimize the process. This paper extends the works in [10], which derived the milk cooling transfer function through the heat balance equation, and [11] which identified the system through the ARX (Auto Regressive eXogenous) method. Agustriyanto and Mochni [10] have developed a model for batch milk cooling that will be used in this research to generate data. The BJ identification model has not been previously used to analyze the data and identify this process, and this is the novelty of this paper.

## 2 Batch Milk Cooling Process

The subject of examination is the milk cooling system owned by Cooperative SAE Pujon, illustrated in Figure 1, [10]– [14]. This system includes a rectangular tank with an integrated half-cylindrical tank used to hold the milk. The box-shaped tank is responsible for water-cooling, while the half-cylindrical tank serves as the milk container. The design associated with the tank can be observed in Figure 2.

The transfer function of the process was derived previously [12], [13]and the results obtained are as follows:

$$\frac{T_o(s)}{T_1(s)} = \frac{1}{42.3548s + 1} \tag{1}$$

Equation (1) represents a first-order transfer function in the Laplace domain, characterized by a gain of 1 and a time constant of 42.3548 minutes.

**Fig. 1.** The Packo milk cooling system, provided by KUD SAE Pujon.

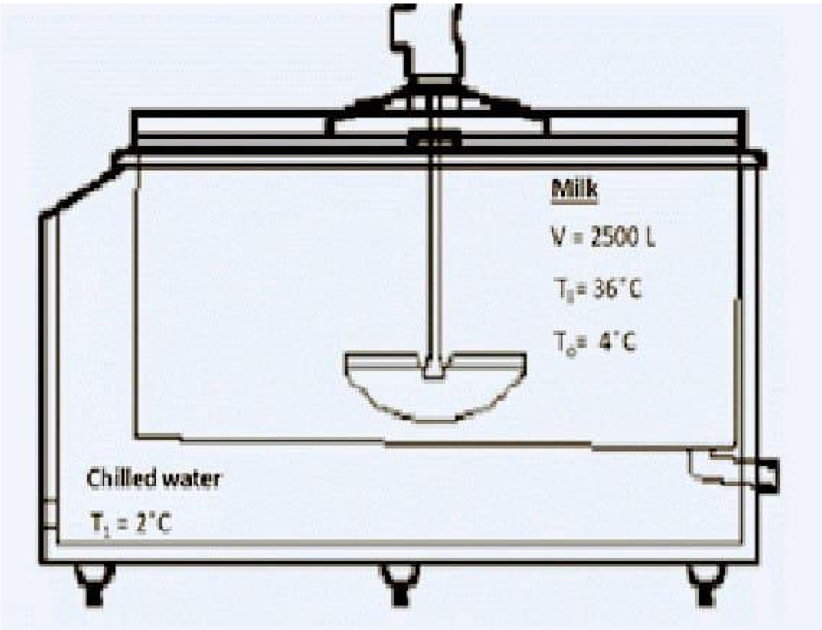


**Fig. 2.** A perspective from the side of the milk cooling system.

Table 1 displays process specifications for the simulation.

**Table 1.** Process specifications

| Variables | Description | Data points | Measurements |
|---|---|---|---|
| **V** | The tank volume | 2,500 | L |
| **Cp** | Heat capacity of the milk | 3.93 | kJ/kg.K |
| **ρ** | Density of the milk | 1,027 | $kg/m^3$ |
| **Q** | The heat dissipation from hot fluid (milk) to cold fluid (water) | 2,690.74 | kJ/min |
| **M** | The milk quantity (mass) inside the tank | 2,567.5 | kg |
| **U** | The factor that represents the overall heat transfer rate | 274.4613 | $kJ/min.m^2.^{\circ}C$ |
| **A** | The heat transfer area | 0.8680 | $m^2$ |
| **$T_i$** | The inlet milk temperature | 36 | ˚C |
| **$T_1$** | The chilled water temperature | 2 | ˚C |
| **$T_o$** | The milk temperature inside the tank | 4 | ˚C |

## 3 Box-Jenkins (BJ) System Identification

The method involves four steps [15]:

1. Model Selection: Identifying a suitable mathematical model that can describe the behavior of the system being analyzed.
2. Model Estimation: Determining the parameters of the chosen model by utilizing past data on the system's response.
3. Model Checking: Ensuring the reliability of the estimated model by comparing the actual and estimated values of the system's response.
4. Model Forecasting: Using the estimated model to make predictions of the system's future behavior.

Box-Jenkins model is represented by [16]–[18]:

$$y(t) = \frac{B(q)}{F(q)} u(t-k) + \frac{C(q)}{D(q)} e(t) \qquad (2)$$

in which:

B, F, C, D are polynomials.
$n_b$ = the degree of the B +1 ($N_y$ by $N_u$ matrix)
$n_f$ = the degree of the F ($N_y$ by $N_u$ matrix)
$n_c$ = the degree of the C (column vector of $N_y$ entries)
$n_d$ = the degree of the D (column vector of $N_y$ entries)
$n_k$ = the delay in input (in number of samples, $N_y$ by $N_u$ matrix)
$N_u$ = the count of inputs
$N_y$ = the count of outputs

From the input and output data, the Box-Jenkins models to be examined are BJ22221 and BJ11111. As soon as both models are obtained, they can be simulated (either as discrete or continuous models). Converting a discrete model into a continuous one is a straightforward process in MATLAB environment. Continuous transfer function models can be compared immediately with those obtained from mechanistic model, and the simulation results can be visualized through graphs

## 4 Results and Discussion

Assume that the process transfer function is unknown, and we intend to identify it using the Box-Jenkins model. To achieve this, Equation (1) was simulated in Simulink, where disturbances were introduced to the input (the temperature of the cooling water.) as depicted in Figure 3. The corresponding output (the temperature of the milk contained in the tank) is presented in Figure 4. Subsequently, the input-output data was subjected to identification using the Box-Jenkins method.

Using the MATLAB Identification Toolbox, by default the order of the polynomials Box Jenkins method is BJ22221, which means that $n_b = n_c = n_d = n_f = 2$ and $n_k = 1$. We can also identify the data by different order such as BJ11111 ($n_b = n_c = n_d = n_f = n_k = 1$). This will result in a first-order transfer function in the s-domain. The comparison for the two orders is shown in Table 2.

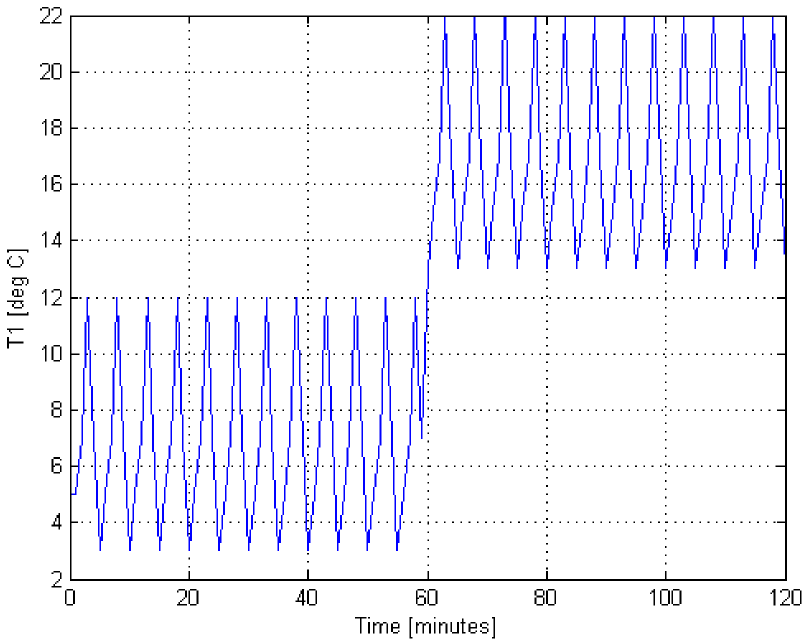


**Fig. 3.** Input profile.

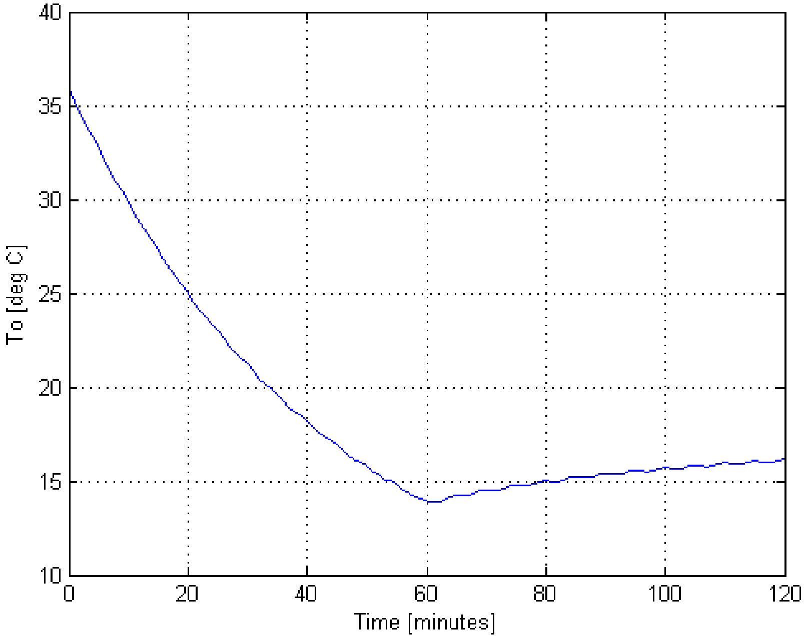


**Fig. 4.** Output profile.

**Table 2**. Box-Jenkins Model

| Model | Transfer Function (z) Discrete | Transfer Function (s) Continuous |
|---|---|---|
| **bj22221** | $\frac{0.02333z^{-1}+0.00746z^{-2}}{1-0.6569z^{-1}-0.3123z^{-2}}$ | $\frac{0.02361s^{2}+0.05385s+0.2637}{s^{3}+2.304s^{2}+11.22s+0.2637}$ |
| **bj11111** | $\frac{0.02333z^{-1}}{1-0.9767z^{-1}}$ | $\frac{0.02361}{s+0.02361}$ |

A discrete model (in the z-domain) is essentially the same as a continuous model (in the s-domain) and only differs in the sampling time used to obtain data.

The transfer functions obtained from Table 2 were then retested on certain other forms of perturbations (series of step up as depicted in Figure 5) of cold-water temperature. Figure 6 displays a comparison of the response of the Box-Jenkins model plot with Equation 1 (mechanistic model. It can be seen that the line of series 1 (___ obtained from transfer function), series 2 (+ from bj22221), and series 3 (bj11111) overlap. This overlapping indicates that both models (BJ22221 and BJ11111) can represent the process effectively It should be noted that the final result of bj11111 is also a single order process, equivalent to $\frac{1}{42.3549s+1}$ (comparable to equation 1 which is $\frac{T_o(s)}{T_1(s)} = \frac{1}{42.3548\,s+1}$ ). The two values are quite similar. Both of them have a gain of 1 and a time constant of around 42.355 minutes.

When compared to identification using ARX [13] (i.e $\frac{1}{42.3729s+1}$ ), BJ provides more accurate results.

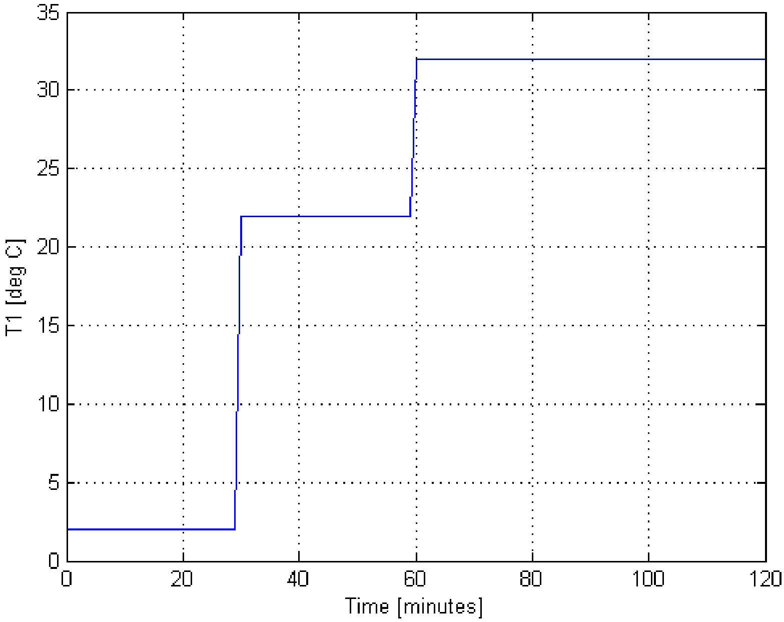


**Fig. 5.** Input testing.

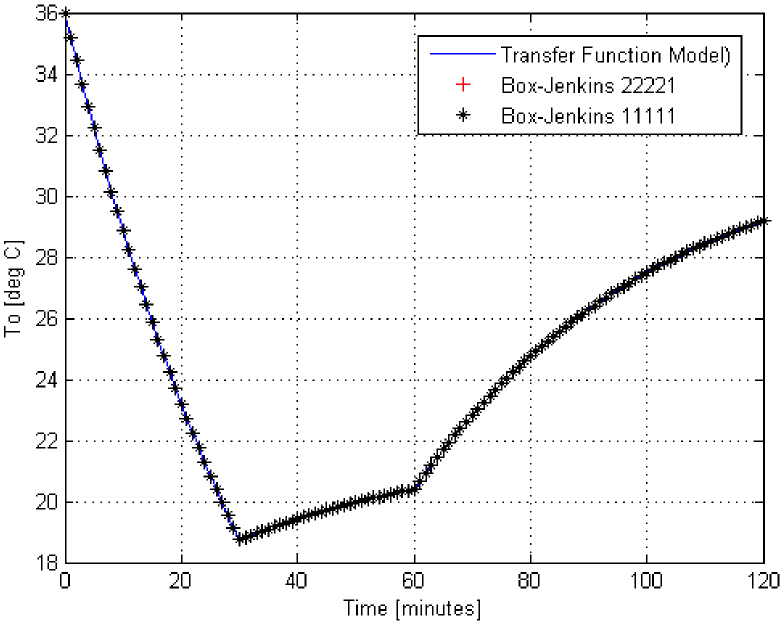


**Fig. 6.** Comparison of Transfer Function Model (___) with Box-Jenkins 22221 (+) and Box-Jenkins 11111 (*).

## 5 Conclusions

The BJ method was able to accurately model the batch milk cooling process. This is supported by the comparison of the obtained transfer functions with Equation 1 and the results from ARX system identification [13]. This study demonstrates the effectiveness of the BJ method in modeling dynamic processes, and it has potential applications in the dairy industry.

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