Analysis of the Contagion of Financial Risks in Complex Financial Networks

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Abstract. Analysis for the area of complex networks and their applications to economics and finance is an important area of current research in finance. This paper carries out mathematical modelling of the financial system, aiming to study the contagion process of financial risk and to investigate the contagion effect of financial risk. It also analyses the subjects and paths in the process of financial risk contagion. Firstly, the paper abstracts the financial system as a financial network with scale-free nature. Based on the traditional SIRS virus contagion dynamics model, the financial risk contagion model (SIRD model) is built by adding bankruptcy nodes and bankruptcy rate parameters, which is in conjunction with the characteristics of the actual financial system. Then simulations are carried out to analyze the effects of risk propagation rate, recovery rate, resistance failure rate and bankruptcy rate on the steady state results in the model respectively. Finally, based on the simulation results, the characteristics of financial risk contagion under different scenarios are discussed, and the nature of financial risk contagion is summarized and suggestions are given.

Keywords: Complex Network, Scale-free Network Model, Financial Risk Contagion, Propagation Dynamics Models.

1 Introduction

In recent years, the global economic situation and financial markets have been greatly affected by the COVID-19 and war. The financial sector is desperately trying to avoid another global crisis. The financial system is at the heart of modern economic finance which is a system concerned with the concentration, flow, and distribution of funds. With the changes of the times, the development of information technology has made the financial world react more rapidly and violently to fluctuations and shocks than it did in the last century. The study of financial crises has shown that all economic actors can fall victim to them. Far from the previous century, one of the significant changes in the current financial crisis is the strengthening of the tendency towards "mutual shocks". As a result, the financial community is always looking for new financial models and theories of finance to study the real financial system. In recent years, the focus of the efforts of these physicists and mathematicians has gradually

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shifted to the field of network science. The increasingly obvious shift in the financial system from 'too big to fail' to 'too complex to fail' and 'too connected to fail' complex networks that characterized the financial crisis has made the case for complex This makes the case for the entry of complex network science into the field of finance even more natural.

The origins of complex network theory can be traced back to the study of the 'seven bridges problem'. Initial researchers focused on regular networks, and in the middle of the last century Erdos and Rnyi studied random graphs and proposed ER random networks [1]. Later, Watts and Strogatz studied real networks in society and proposed WS small-world networks [2]. Barabdsi and Albert discovered the scale-free character of networks and proposed BA scale-free networks [3]. Subsequently, complex network science has been widely used in the fields of computing, bioscience, and information science. Entering the 21st century, researchers have begun to dissect financial systems through the lens of complex networks. Early researchers approached complex network systems from a traditional economic equilibrium perspective, Allen, Gale [4] and Freixa [5] Stefano Schiavo and others [6] used a complex network approach to study the integration patterns of international trade networks and international financial network. Song and others use complex network theory as a tool to study and suggest that interbank networks have a double power law distribution [7]. D'Arcangelis A. M. and Rotundo G. use complex network theory to investigate the nature of geographic aggregation of fund management companies in Europe [8]. Kydros Dimitrios et al focus on the "shallowness" of financial markets, choosing correlation coefficients to measure the correlation between nodes in a financial network [9]. Wang and others use a minimum spanning tree network to describe the linkages in international foreign exchange markets [10]. In addition, extending more algorithms and models for risk contagion has also been a focus of researchers. Eisonberg and Noe proposed the EN algorithm, which constructs interbank correlations from information on banks' exposures at various points in time and clears the nodes in the network by an exposure matrix [11]. Battiston et al. proposed the DebtRank algorithm based on the PageRank algorithm and feedback center, which can recursively simulate the transmission process of decay [12]. Hu constructed the China interbank network and analyzed the large payment data of the People's Bank of China using virus transmission dynamics algorithms [13]. Huang focused on the DebtRank network infection algorithm and improves it [14].

As the various actors in the financial system are connected to each other through funds, contracts, business transactions, etc., this connection acts as a network covering the entire financial system and as more and more mathematicians and physicists become involved in the study of finance, they find that this network has very similar properties to the complex networks they are familiar with. Naturally, in the face of today's increasingly complex financial system and unstable international forms, complex network science gives the academic community a new perspective to study finance system and financial risks. In order to further explore the network characteristics of the financial system and study the spread characteristics of financial risks, this paper intends to carry out abstraction mathematical modeling of the real financial system, try to establish financial risk contagion in complex financial networks, And adopt a propagation dynamics model that is more in line with the actual financial network to study the process of financial risk propagation, and simulate it.

2 Methodology

This study requires a combination of mathematics, network science, finance, and economics in constructing financial networks and summarizing the effects of financial risk propagation, and considers the reality of the situation. Based on the results of scale- free networks, this paper mathematically models the financial system and abstracts it into a scale-free financial network. This paper also builds on and improves the traditional SIRS virus propagation dynamics model by mathematically modelling the process of financial risk propagation in the network, considering the actual possible bankruptcy scenarios, introducing bankruptcy nodes and bankruptcy rate parameters, and establishing a financial risk contagion model (SIRD model).

Due to the complexity of actual financial systems, not only are data and information confidential and difficult to access, but also the connections between subjects are often territorial and business networks are often subjective in nature, factors that are detrimental to the study of the overall characteristics of financial networks. This paper therefore assumes an ideal financial system based on the scale-free nature of financial networks, and solves the established SIRD differential equation model with the help of Python's odeint function as the core to simulate the steady-state results of risk propagation using the Longo-Kutta method. In the process, the parameters are repeatedly adjusted and the experiment is repeated to investigate the effect of different parameters on the steady-state results of the model.

This study focuses on the foundations of the propagation of financial risk in financial networks. The abstraction of a financial system into a financial network is a process of abstracting the participants of the financial system into nodes and the links between them into connected edges. Since the participants in a financial system are complex and have different functions, the nodes into which they are abstracted should also have different properties, and one has to filter the participants in order to build the ideal model. Since this paper focuses on the process of financial risk transmission in financial networks, the economic agents selected as nodes should be the main components of the financial risk transmission process, i.e., the micro-foundations of the financial risk contagion effect. And the study of risk contagion paths reveals the aggregation and centrality of nodes in a network. This research improves the traditional virus transmission model. The traditional SIRS virus propagation dynamics model is improved by constructing a financial risk propagation model (SIRD model) that considers the specificity of financial network nodes and the differences between financial risks and viruses in the propagation process, introducing bankruptcy nodes and bankruptcy rate parameters, adding a new removal state (bankruptcy state) to the circular evolution of the original SIRS model, and the proportion of bankruptcy. The proportion of bankruptcy nodes reflects to a certain extent the destructiveness of risk propagation to the financial system.

3 Financial Risk Contagion Models

Research on financial risk contagion models is now relatively mature, with most studies showing that financial networks have significant scale- free network characteristics, and the research in this paper is based on this assumption. One of the more mainstream studies is to apply the traditional virological three-state contagion model, i.e., the SIR viral contagion model, to financial networks. This paper focuses on improving the traditional SIRS viral contagion dynamics model to obtain the SIRD model of financial risk contagion.

3.1 Traditional SIRS infectious disease transmission model

In traditional SIRS models, there are three states, with each individual in one state. The basic states include susceptible state (S), which refers to individuals in a healthy state but may be infected; infectious state (I), which refers to individuals in an infectious state who are infectious; recovery state(R), also known as the state of removal, refers to the state of recovery and acquisition of certain immunity after infection. Individuals in a cured state in the SIRS model still have the possibility of becoming susceptible again and will not be removed from the system. The infection mechanism of the SIRS model can be described as: a portion of infected nodes in the early stages of transmission β The probability of (infection rate) transmitting the virus to susceptible nodes, while the infected nodes will γ The probability of (cure rate) becoming a cure node, and the cure node will α The probability of the cure failure rate becoming a susceptible node again. The infection diagram is as shown in Fig. 1.

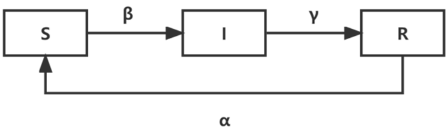


Fig. 1. Schematic diagram of infection process in traditional SIRS model.

3.2 SIRD financial risk contagion model

There are four types of economic entities in the financial risk contagion model (also known as the SIRD contagion model), including health status (S), which refers to economic entities that have not yet been affected by the crisis, but may be infected by related economic entities that have already fallen into crisis; crisis state, which refers to the economic entities severely affected by the crisis, and these economic entities will transmit risks to relevant entities through transactions, settlements, investment and financing, and other connections; recovery state (R), which refers to an economic entity that has emerged from a crisis or has a certain level of risk resistance; bankruptcy state (D), which refers to the economic entity that cannot resist the crisis and goes bankrupt. Nodes in bankruptcy state will no longer enter the SIR triple state cycle. The contagion mechanism of the SIRD model can be described as follows: at the beginning of a financial crisis, a small proportion of nodes will be the first to fall into the crisis, and a large number of healthy nodes will be connected to the crisis nodes because they have related businesses and will fall into the crisis nodes with a probability of β (risk contagion rate); the crisis nodes will enter the removal state with a probability of γ (recovery rate) due to spontaneous market regulation or internal risk management; the nodes in the removal state may lose their risk resistance with a probability of α (resistance failure rate) and become healthy nodes again. Nodes in the removed state may lose their risk resistance with a probability of α (resistance to failure) and become healthy nodes again; some economic agents in crisis with small size or poor risk management may enter a state of bankruptcy with a probability of δ (bankruptcy rate). The risk contagion diagram of the model is shown in Fig. 2. Therefore, there is a property: in financial networks with scale-free characteristics, infected nodes will always occupy a certain proportion, and risk contagion in financial networks will almost always exist.

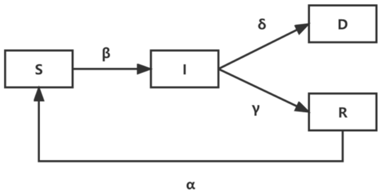


Fig. 2. Schematic diagram of the contagion process of the SIRD financial risk contagion model.

4 Results & Discussion

The micro entities involved in risk propagation during financial crises have been discussed earlier, and their respective characteristics have been discussed. There is heterogeneity among these economic entities, with different entities occupying different positions in the financial system, possessing different business capabilities, risk management capabilities, and risk bearing capabilities, playing different roles in the process of risk dissemination. This means that the proportion of each economic entity in the nodes constituting different financial networks is also different, which is generally characterized by setting Degree distribution. For different nodes, their infection rate, recovery rate, resistance failure rate, and bankruptcy rate also vary. This article simplifies this process by taking parameters that represent the average level of node features. When studying the SIRS model, it was mentioned that nodes in a cured state will acquire certain immune abilities. In the application process of some SIRS models, a coefficient is added to reflect the duration of immunity, usually in the form of the reciprocal of immune duration. Considering the particularity of the financial system, this coefficient is not used in the modeling process in this article. The probability of a node transitioning from a removed state to a healthy and susceptible state, known as the resistance failure rate, is directly used as the coefficient of the model.

For traditional virus transmission models, the initial proportion of susceptible nodes $S(0)\approx 1$, the initial proportion of infected nodes $I(0)\approx 0$, and the initial proportion of cured nodes $R(0)=0$ [15] are usually set during the application process. Because when studying the process of virus transmission, the overall population is generally a large value (such as the resident population of Guangzhou, which is over 18 million), and the initial infected population is usually only a few or dozens, which is a value close to zero in proportion. In reality, when studying a financial system, the number of economic entities involved is much smaller than the population that needs to be considered in the virus infection model, so the initial infection node proportion cannot be ignored as zero. In addition, in the early stages of virus transmission, it is generally believed that nodes do not have self-healing and immune capabilities. However, in the financial system, excellent risk management can help economic entities effectively cope with risks, thereby gaining a certain level of risk resistance. Therefore, when setting parameters, it should be considered to assign a non-zero initial value to the healing node. In addition, this article argues that no economic entity has already gone bankrupt in the early stages of risk outbreak, indicating that D(0)=0.

4.1 Model parameter settings

Based on the properties summarized from previous research, it has been demonstrated that the specific numerical values of the model do not have a decisive impact on the model results, and the model results are only determined by the proportion between parameters. Therefore, based on existing research results and experience, the node ratios for the four initial states are set as S(0)=0.85, I(0)=0.05, R (0)=0.10, and D(0)=0. A reference group parameter group is set (β, γ, α, δ)=(0.80,0.20, 0.05, 0.01) were simulated and the following sets of parameters were set for comparison to explore the impact of different parameter changes on model evolution. First, we study the impact of changes in risk transmission rate, we determine (γ, α, δ)= (0.20, 0.05, 0.01), β takes the values listed in Table. 1 As for the impact of changes in recovery rate, we determine (β,α,δ)= (0.80, 0.05, 0.01), γ takes the values listed in Table. 2. To study the impact of changes in resistance failure rate, we determine (β,γ,δ)= (0.80, 0.20, 0.01), α takes the values given in Table. 3. For the impact of changes in bankruptcy rates, we determine (β,γ,α)= (0.80, 0.20, 0.05), δ takes the values in Table. 4.

Table 1. Value of risk propagation rate.

	(a)	(b)	(c)	(d)
Beta=	0.90	0.60	0.40	0.20

Table 2. Value of recovery rate.

	(a)	(b)	(c)	(d)
Gamma=	0.95	0.80	0.40	0.16

Table 3. Value of resistance failure rate.

	(a)	(b)	(c)	(d)
Alpha=	0.01	0.025	0.16	0.40

Table 4. Value of Bankruptcy Rate.

	(a)	(b)
Delta=	0.05	0.10

4.2 Analysis of simulation results

Firstly, we analyze the reference group. The simulation results of the financial risk propagation model (0.80,0.20, 0.05, 0.01) are shown in Fig. 3. Accordingly, local nodes in a scale-free financial network fall into crisis, financial risks quickly spread throughout the network, and a large number of healthy nodes quickly become crisis nodes. It only takes about 7 to 8 time units for the proportion of crisis nodes to reach its peak, and the proportion of bankruptcy nodes during this period also increases rapidly. At the same time, economic entities in the network will carry out self-help actions (such as interbank lending, rapid adjustment of business by physical enterprises, and financial institutions reselling assets to increase cash flow), and the situation of powerful or risk management capable economic entities will recover, indicating that they have entered the recovery state. From the graph, it can be seen that the proportion of healthy nodes reaches its lowest value at about 12 time units, then slightly rebounds, and finally stabilizes at about 40 time nodes. The proportion of removing state nodes at the same time reaches its highest value in about 15 time units, then drops back and gradually decreases at an extremely low rate. This phenomenon indicates that a certain proportion of economic entities in the financial system are in a sensitive state to crises, which is likely due to their own strength and risk management capabilities being insufficient to support them in safely navigating crises. Meanwhile, although the proportion of infected nodes will decrease at a low rate in the long run, the proportion of bankrupt nodes will also increase at a low rate, which means that as long as risks continue to spread in the financial network, they will continue to cause damage to the financial system. It can be considered that the peak and steady-state values of the proportion of crisis nodes can reflect the degree of contagion of financial risks, while the proportion of bankruptcy nodes can reflect the degree of damage to the financial system caused by the crisis. The conclusion drawn above is that when a financial crisis erupts, it is necessary for regulatory and government departments to aid economic entities in the financial system.

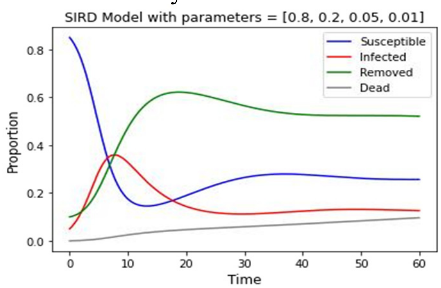
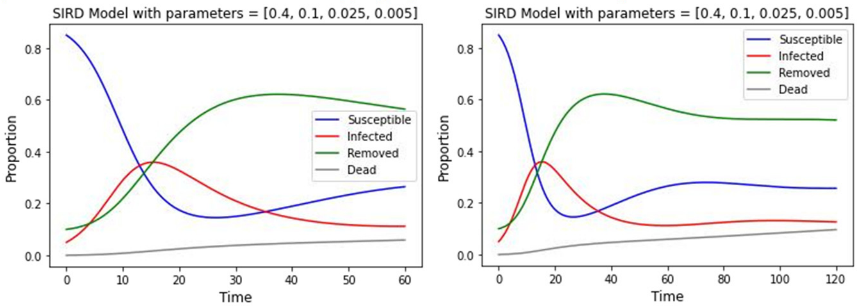


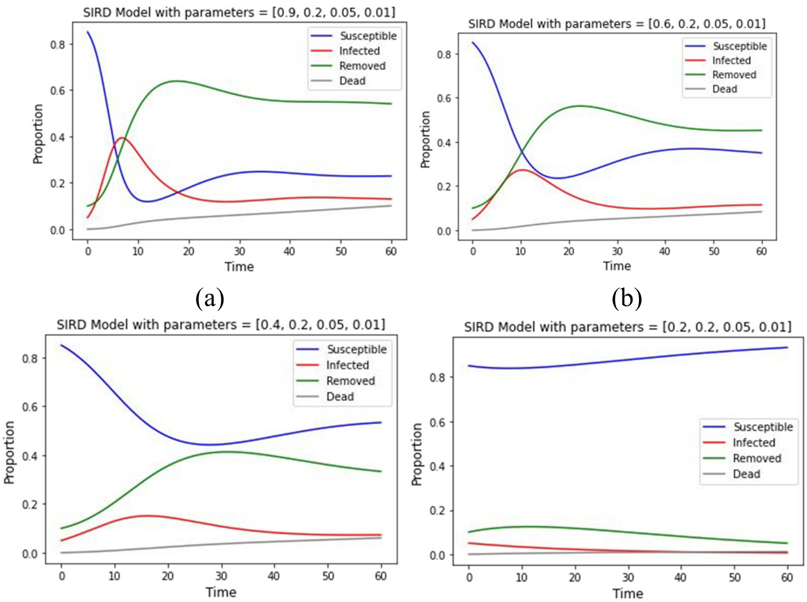
Fig. 3. Simulation results of SIRD model (reference group).

The basic ideas can include reducing propagation rate, increasing recovery rate, reducing resistance failure rate, and reducing bankruptcy rate. Specifically, if the four parameters are proportionally reduced by twice, the simulation results of the model are as shown in Fig. 4. Among them, Fig. 4 (a) shows the results at the same time scale as Fig. 3, while Fig. 4 (b) shows the results at twice the time scale. It can be observed that Fig. 4 (b) and Fig. 3 are completely consistent in shape. This verifies that in the financial risk propagation model, if the proportions of the four parameters are the same, the results when the model reaches steady state are also the same, which only affects the time when the risk propagation reaches steady state.



(a) (b)

Fig. 4. Simulation results of proportional changes in reference group parameters.



(a) (b) (c) (d)

Fig. 5. Simulation results on risk propagation rate.

When simulating with different values of β, the results are shown in Fig. 5. From the comparison, reducing the risk propagation rate can significantly improve the proportion of healthy nodes in steady-state. The effective transmission rate decreases, the peak value of crisis nodes will significantly decrease, and the time to reach the peak will also shift back. This means that when a crisis occurs, if relevant departments or economic entities within the system can effectively block the spread of risks, i.e., reduce the spread rate, it can significantly prevent the crisis from further expanding. It can be seen from the graph that when the effective propagation rate decreases, the proportion of crisis nodes developing into bankruptcy nodes will also significantly decrease due to the low proportion of crisis nodes. As shown in Fig. 5 (d), when λ= 1 hour, it can be considered that when the infectious ability and recovery ability are equivalent, the risk will hardly spread, and the proportion of crisis nodes will not peak, presenting a monotonic decreasing function. The proportion of healthy nodes will continue to increase, ultimately occupying the vast majority in steady-state. At this time, few economic entities go bankrupt. Especially, when the effective propagation rate decreases, the proportion of removing the steady-state state also significantly decreases, because the number of nodes in crisis state significantly decreases, resulting in a significant decrease in the number of nodes transitioning from crisis state to recovery state. When the ability of risk propagation in the network is small, we can assume that the likelihood of healthy nodes being affected by the crisis is also reduced, which means that these healthy nodes can be considered safe.

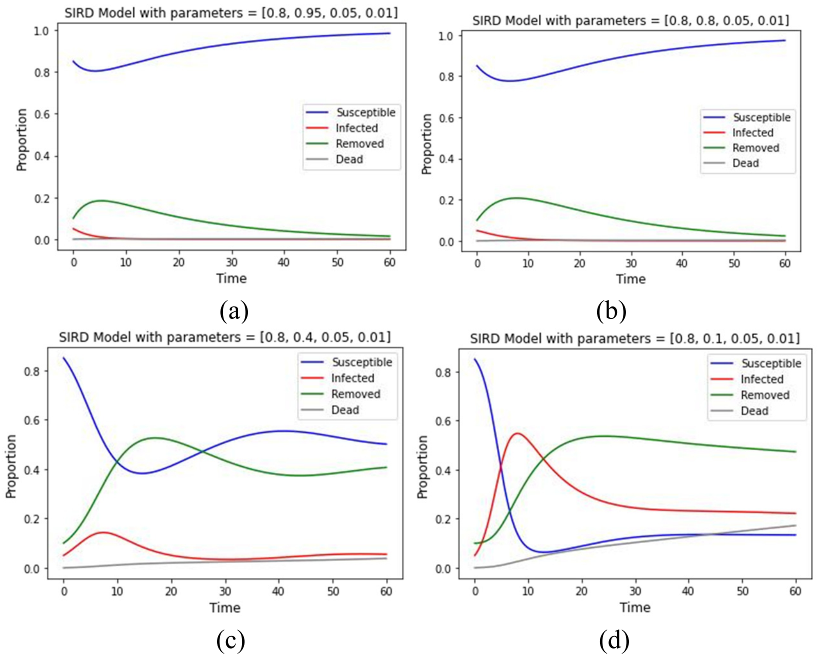
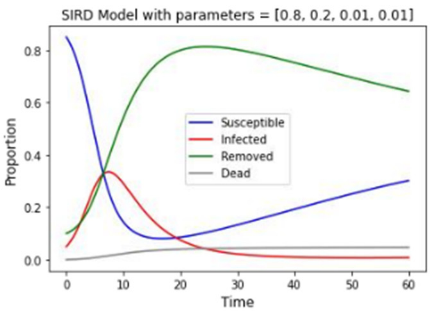


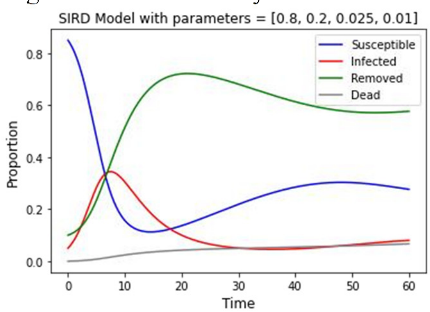
Fig. 6. Simulation results on recovery rate.

When γ take different values for simulation, and the results are shown in Fig. 6. From the simulation results, increasing the recovery rate is also very significant in helping the financial system escape the crisis. When the effective transmission rate λ When it is less than or equal to 1, it also shows that the risk will not propagate on a large scale in the network. In actual financial networks, the transmission rate is often maintained at a high level, and the recovery rate is generally lower than the transmission rate, which is generally manifested as λ> 1. Therefore, compared with the results in Fig.5 (d), Fig. 6 (a) and (b) are the main goals to be achieved by the relevant departments in the crisis, because the marginal revenue to increase the recovery rate is generally higher. In addition, Fig. 6 (d) shows the results when the rescue rate is much lower than the transmission rate, indicating that the proportion of crisis nodes quickly reaches its peak and nearly 60% of nodes will fall into crisis. The proportion of healthy nodes quickly bottoms out and will stabilize at a low level, while the proportion of removed state nodes continues to decrease after increasing, while the proportion of bankrupt nodes rapidly increases. If relevant departments do not take measures at this time, the result of subsequent development will be that crisis nodes and bankruptcy nodes occupy the main part of the network, and the financial system will be severely damaged. When α take different values for simulation, and the results are shown in Fig. 7. The results differ significantly from those in Fig. 6 and Fig. 5, which can be intuitively observed from the images. As shown in Fig. 7 (a) and Fig. 7 (b), when the resistance failure rate decreases, although the proportion of crisis nodes still shows a peak similar to the peak in Fig. 3, the subsequent development is more significant: we obtained a higher proportion of nodes in the recovery state, as well as a lower proportion of crisis nodes and bankruptcy nodes. This indicates that reducing the failure rate of resistance is effective.

On the contrary, when the failure rate of resistance increases (as shown in Fig. 7 (c) and Fig. 7(d)), its negative impact is also quite significant. The proportion of healthy nodes and removed nodes will remain low, while nodes in crisis will become the mainstream in the network. Over time, the proportion of healthy nodes will stabilize, but the proportion of crisis nodes and removed nodes will continue to decrease, while the proportion of bankrupt nodes will rapidly increase. When the actual financial system encounters a crisis, it is very difficult to avoid economic entities that have already emerged from the crisis from falling into the crisis again, and often presents a situation where the resistance failure rate is higher than the recovery rate.



(a)



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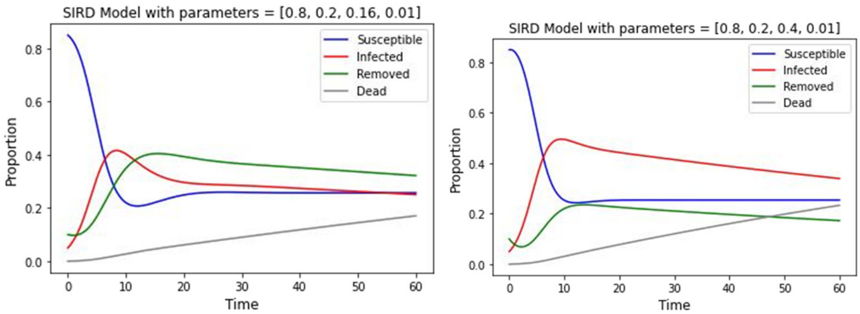
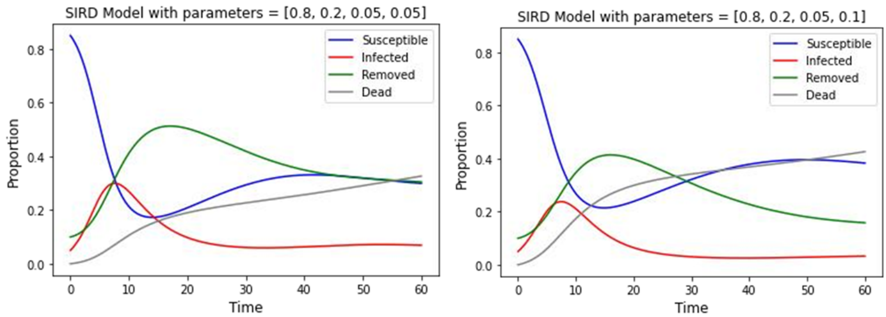


(c) (d)

Fig. 7. Simulation results on resistance failure rate.

When δ take different values for simulation, and the results are shown in Fig. 8. Certainly, we hope to see the lower the bankruptcy rate of financial networks during crises, the better. The analysis of bankruptcy rate mainly focuses on its destructive impact on the financial system. As shown in Fig. 8, the increase in bankruptcy rate has a significant impact on the financial network, with the proportion of bankruptcy nodes rapidly increasing and occupying an absolute position in the network.



(a) (b)

Fig. 8. Simulation results on bankruptcy rate.

In fact, the first set of parameters selected in this article serves as a reference combination $(\beta,\gamma,\alpha,\delta) = (0.80, 0.20, 0.05, 0.01)$ reflects a relatively healthy financial network with good risk management awareness. When financial crises erupt and spread on a large scale, the financial system often exhibits characteristics such as lack of regulation, poor overall risk management awareness, lack of market liquidity, and market panic. At this time, it will exhibit a high transmission rate, resistance failure rate, and even a very high bankruptcy rate. Fig. 9 roughly describes this trend, where many economic entities in the financial system will quickly go bankrupt. This is basically the case with the 2007 US subprime crisis and subsequent financial crises.

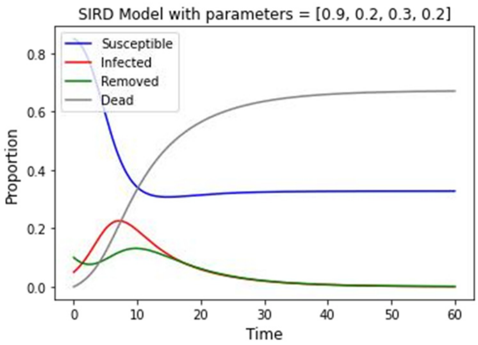


Fig. 9. Simulation results of poor financial system.

5 Conclusion

After analyzing the simulation results above, some properties of the financial risk contagion process can be summarized as follows. Primarily, the infection rate is directly proportional to the severity of the infection; The cure rate is inversely proportional to the severity of the infection; The cure failure rate is directly proportional to the severity of the infection. Once the virus (or crisis) begins to spread, the cure rate is generally lower than the infection rate and cure failure rate. Therefore, the marginal benefit of increasing the cure rate is higher than the marginal benefit of reducing the infection rate and cure failure rate. If the proportions of the four parameters are the same, the infection rate when the model reaches steady state is also the same. The steady-state values of infected nodes are only related to the relative ratio between parameters, and are not related to their numerical values. From the above properties, we can provide ideas and suggestions for dealing with financial risks.

Since the failure rate of resistance is not only related to the characteristics of crises and risks, but also to factors such as the economic strength and risk response strength of institutions or enterprises themselves, it is difficult to reduce the failure rate of resistance. So, reducing the transmission rate and improving the recovery rate are the best choices for relevant departments and institutions, and their essence is to reduce the effective transmission rate. In a scale-free financial network, the propagation threshold is close to zero, and the effective propagation rate is basically greater than the propagation threshold. It means that once a crisis occurs, the risk will spread in the network. From previous analysis, it can be concluded that if the risk spreads in the network, it will cause damage to the financial system. Therefore, in the early stages of risk transmission, regulatory and government departments should take action to reduce the effective transmission rate.

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