Acturial analysis of insolvency probability considering correlations among claims: case study for transmisstion tower failures

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Abstract. This research aims to develop an actuarial analysis framework to evaluate the probability of insolvency for the infrastructure property insurance of a specific region. The study focuses on the case of nationwide transmission tower failures. The proposed framework takes into account the correlations among claims, as well as the interdependence of individual infrastructure failures, in order to calculate revised insurance premiums. To achieve this, the research utilizes advanced statistical modeling techniques to capture the intricate relationships between claims and their correlation patterns. By integrating historical data on transmission tower failures, claims, and relevant economic factors, a comprehensive risk assessment model is constructed. This model estimates the probability of insolvency for the insurance portfolio, accounting for the interplay of correlated claims. Furthermore, the research investigates the interdependence among individual transmission towers, recognizing that failures in one tower can trigger a cascading effect throughout the network. This dynamic is captured through network analysis methods, allowing for a more accurate assessment of the risk and potential systemic repercussions. The findings of this study could provide valuable insights for insurance companies, policyholders, and risk managers involved in the infrastructure sector. The actuarial analysis framework incorporates correlations among claims and accounts for the interconnectedness of transmission tower failures. Such a comprehensive approach improves accuracy in estimating insolvency probabilities, enabling better risk pricing, underwriting, and decision-making in the insurance industry.

Keywords: Actuarial analysis. Insolvency probability, Correlations, Infrastructure, Property insurance, Nationwide transmission tower failures

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1 Introduction

Infrastructure plays a vital role in the socio-economic development of nations, serving as the backbone of transportation, communication, and power supply systems [1]. However, the risks associated with infrastructure failures can have severe consequences, both economically and socially[2]. Property insurance is a common mechanism used to mitigate these risks by providing financial protection against damage or loss. Insolvency probability, a key consideration in property insurance, refers to the likelihood that an insurance provider may become unable to honor its financial obligations due to a high volume of claims. Accurately assessing this probability is crucial for insurance companies to maintain solvency, appropriately price policies, and manage their risk exposure[3]. Traditional actuarial methods have primarily focused on estimating insolvency based on individual claim events. However, in complex infrastructure systems, such as nationwide transmission networks, there is a need to account for the interdependencies and correlations among claims.

The purpose of this study is to develop an advanced actuarial analysis framework that incorporates the correlations among claims and the interdependence of individual infrastructure failures, specifically focusing on the case study of nationwide transmission tower failures. By considering these factors, the framework aims to provide a more accurate estimation of the probability of insolvency and enable the calculation of revised insurance premiums.

Correlations among claims play a crucial role in understanding the collective risk faced by insurers. When claims exhibit dependencies or similarities, the occurrence of one claim event may increase the likelihood of subsequent events. For instance, extreme weather conditions or natural disasters can cause multiple transmission tower failures in close proximity or over a short period. By taking into account these correlations, insurers can better assess their risk exposure and make informed decisions to manage their portfolios effectively[4]. Furthermore, the interdependence of individual infrastructure failures within a nationwide transmission network is a critical aspect to consider. Transmission tower failures can trigger a cascade of consequential damages, leading to widespread network disruptions and potentially catastrophic events. Analyzing the interplay between individual tower failures allows for a more comprehensive assessment of the overall risk and potential systemic repercussions. This analysis can provide valuable insights into the network resilience, vulnerability, and potential areas for infrastructure improvement and risk reduction[5].

To achieve the objectives of this study, advanced statistical modeling techniques will be employed, integrating historical data on transmission tower failures, claims, and relevant economic factors. This comprehensive risk assessment model will account for the correlations among claims, enabling a more accurate estimation of the probability of insolvency for the insurance portfolio. Network analysis methodologies will be utilized to capture the interdependence among transmission towers, incorporating the dynamics of cascading failure events. The findings of this research are expected to contribute to the field of insurance risk management, providing valuable insights for insurance companies, policyholders, and risk managers involved in the infrastructure sector. The actuarial analysis framework developed in this study, considering correlations among claims and the interdependence of transmission tower failures, can improve the accuracy of estimating insolvency probabilities. This, in turn, facilitates more effective risk pricing, underwriting, and decision-making in the insurance industry, ultimately enhancing the resilience and sustainability of infrastructure systems.

In the following sections, the methodology employed for data collection, statistical analysis techniques, and network modeling will be discussed, followed by the presentation and analysis of results and concluding remarks.

2 Probability of insolvency

In actuarial science, the probability of insolvency refers to the likelihood or probability that an insurance company or financial institution may become unable to meet its financial obligations or honor its policyholder claims due to inadequate reserves or an excessive amount of liabilities. It is a measure of the financial vulnerability of an insurer.

The probability of insolvency is a key metric used to assess the financial stability and solvency of an insurance company. It quantifies the risk of the insurer's financial resources being insufficient to cover its liabilities, resulting in potential bankruptcy or inability to fulfill its contractual obligations. Let R(t) be the capital of an insurance company at the time *t*, therefore, R(t) can be represented as,

$$R(t) = C + \mathrm{T}(t) - X(t) \tag{1}$$

where C denotes the initial capital of an insurance company, $\mathrm{T}(t)$ denotes the total premium income functionalized with t and $X(t)$ is the aggregate claim amount up to time t which can be also defined as risk process. Moreover, let $N(t)$ be a stochastic process that denotes the total number of claims up to time t and $X(t)$ can be further calculated as,

$$X(t) = \sum_{i=1}^{N(t)} X_i \tag{2}$$

Mathematically, the probability of insolvency or bankruptcy can be defined through the following equation:

$$P_{ins} = \Pr(R(t) \leq 0,\ \ \mathrm{w.r.t}\ \ t \in [0, \mathrm{T}]) \tag{3}$$

Therefore, the probability of bankruptcy quantifies the evolving risk of the bank insolvency. Actuarial analysis considers various quantitative and qualitative factors to estimate the probability of insolvency accurately. These factors include historical claim experience, risk management practices, underwriting standards, investment performance, regulatory requirements, and economic conditions. Advanced modeling techniques, such as stochastic simulations and stress testing, are often employed to assess the likelihood of insolvency under different scenarios and market conditions. Insurance regulators and industry stakeholders closely monitor the probability of insolvency to ensure the financial stability of insurance companies and protect policyholders. Adequate capital requirements and solvency regulations are implemented to minimize the risk of insolvency and maintain the long-term viability of insurers in the market. Understanding and managing the probability of insolvency is crucial for insurers' risk management strategies, ensuring they maintain sufficient reserves, appropriate pricing, and sound investment practices to mitigate the risk of financial instability. By accurately assessing and monitoring the probability of insolvency, insurers can safeguard their financial soundness and provide policyholders with the confidence that their claims will be honored in the event of losses or unforeseen events[6].

3 Estimate of premium considering dependent claims

As is widely known, the calculation of premiums for clients is based on the application of the law of large numbers in statistics[7]. In other words, under the current definition of premiums, the probability of insurance companies making a profit must not fall below a certain threshold. In actuarial science, the Central Limit Theorem (CLT) plays a crucial role in various analytical processes and statistical modeling. The CLT states that the distribution of the sum (or average) of a large number of independent and identically distributed random variables will tend towards a normal distribution, regardless of the shape of the original distribution. In the context of insurance actuarial analysis, this theorem is of immense significance. It allows insurance professionals to make probabilistic inferences and predictions based on a limited amount of data. The insurance industry deals with uncertainty and risk, and the application of the Central Limit Theorem helps in understanding the behavior of various insurance-related variables[8]. Let Σ_X be the covariance of the dependent claims:

$$\Sigma_X = \begin{bmatrix} {\sigma_X}^2(x_1) & \rho_{1,2}\sigma_X(x_1)\sigma_X(x_2) & \cdots & \rho_{1,N}\sigma_X(x_1)\sigma_X(x_N) \\ \rho_{2,1}\sigma_X(x_2)\sigma_X(x_1) & {\sigma_X}^2(x_2) & & \rho_{2,N}\sigma_X(x_2)\sigma_X(x_N) \\ & \vdots & \ddots & \vdots \\ \rho_{N,1}\sigma_X(x_N)\sigma_X(x_1) & \rho_{N,2}\sigma_X(x_N)\sigma_X(x_2) & \cdots & {\sigma_X}^2(x_N) \end{bmatrix} \tag{4}$$

where ${\sigma_X}^2$ denotes the covariance between two dependent claims x_i and x_j Therefore, the variance of the probability distribution of the aggregate claims can be calculated as follows,

$$\sigma_X^2 = \sum_{i=1}^{\mathrm{N}} \sum_{j=1}^{\mathrm{N}} \boldsymbol{\Sigma}_{X_{i,j}} \tag{5}$$

According to CLT, the aggregate claim amounts up to time t, $X(t)$, can be defined as a normal distribution with the mean value, μ_X, and variance, σ_X^2,

$$X(t) \sim N(\mu_X, \sigma_X^2\ ) \tag{6}$$

Therefore, combining Eq. (1) and (3), the optimal premium can be subsequently defined as,

$$\Psi^* = \underset{X_p \in \Gamma,\ P_{ins} \geq P_{thr}}{\arg\max} \quad R(t) \tag{7}$$

where Ψ^* can be defined as the optimal premium, which maximizes the capital and simultaneously ensures the probability of insolvency not exceeding the prescribed value, P_{thr}.

4 Case study

In this section, we will examine an imaginary numerical case to demonstrate the proposed premium pricing scheme. In this case, we will examine the insurance scheme for the transmission towers of New York state's infrastructure system. The transmission towers, as the main structures supporting the entire power transmission network, play a critical role in ensuring the supply of the national energy system. The damages and failures of individual transmission towers are interrelated, as the collapse of a transmission tower caused by natural disasters such as hurricanes and strong winds is interconnected. In other words, the failure of transmission towers in a particular region will have an impact on the health of adjacent transmission towers, and the insurance scheme should also take into account the interdependence of individual premiums.

As shown in the Fig. 1 below, transmission towers are distributed across various regions in the state of New York, with the red dots representing the areas where the transmission towers are located. However, their damages and failures are interconnected, which can be expressed through the following formula:

$$\rho_{i,j} = min\left[1 - \gamma \frac{\Delta D}{D_{thr}} \quad , \quad 1\right] \tag{8}$$

where $\rho_{i,j}$ represents the geographical correlation between transmission tower i and j, D_{thr} is a constant, ΔD is the geographical distance between transmission tower i and j, and γ is the corresponding correction coefficient. In this case, D_{thr} is set to 500 km, and γ is assumed to be 0.78. In addition, $\mathrm{N}(t)$ follows a Poisson random process, the initial capital, C, is defined as 500k \$, and the client's premium is collected annually.

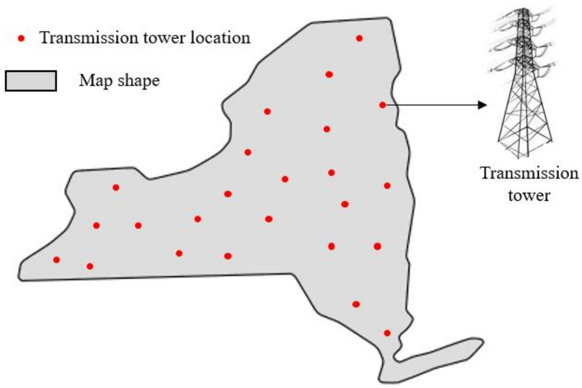

Fig. 1. Topology of the location of transmission tower in New York City

By combining Eq. (5), (6) and (7), it is estimated that the optimal premium, Ψ^*, is finally calculated as 3.84×10^2 \$ per year. However, it is also estimated that Ψ^* is equal to 2.97×10^2 \$ if the correlations among the claims are not considered. This is partially attributed to the reason that the dependence among claims increase the chance of failure risks of transmissions tower. This substantial difference can be attributed to the fact that the dependence among claims greatly influences the risk of failure in transmission towers. When correlations among claims are taken into account, it becomes evident that the potential failure of one tower can more significantly impact the likelihood of failures in other towers within its vicinity. This interconnectedness increases the overall risk exposure, which subsequently affects the optimal premium calculation. One possible explanation for this increased risk is the potential for cascading failures. When one transmission tower fails, it can impose additional stress on nearby towers, leading to a higher likelihood of subsequent failures. The correlations among claims capture this spreading effect and emphasize the importance of considering them in the premium estimation process. Furthermore, by incorporating the correlations among claims, insurers can more accurately assess the overall risk profile of a portfolio of transmission towers. Understanding the interplay between individual tower risks and their collective impact on the entire portfolio is crucial for setting appropriate premiums. Failing to account for these correlations may result in underestimating the true risk exposure, leading to lower premiums that might not adequately cover potential losses.

5 Conclusion

This research developed an advanced actuarial analysis framework to evaluate the probability of insolvency in infrastructure property insurance, focusing on nationwide transmission tower failures. By incorporating correlations among claims and considering the interdependence of infrastructure failures, this framework enhances the accuracy of estimating insolvency probabilities. The findings provide valuable insights for insurance companies, policyholders, and risk managers, enabling them to make informed decisions and effectively manage their risk exposure. Future work should explore additional factors, advanced statistical techniques, and expand the scope of analysis to further improve risk assessment in the infrastructure insurance sector. Firstly, incorporating additional factors into the model could yield more accurate predictions. Factors such as weather patterns, maintenance records, and technological advancements in infrastructure systems could significantly influence the likelihood of failures and subsequent insurance claims. By integrating these factors into the framework, insurers could better understand and manage their risk exposure. Secondly, exploring more sophisticated statistical techniques could enhance the modeling capabilities. Machine learning algorithms, such as neural networks or random forests, could be employed to identify complex patterns and non-linear relationships between variables.

These techniques could provide deeper insights into the interdependencies and correlations among claims, leading to improved risk assessment and more precise estimations of insolvency probabilities. Additionally, expanding the study to cover a broader range of infrastructure systems and geographical regions would contribute to a more comprehensive understanding of the challenges and risks in property insurance [9]. Different types of infrastructure, such as bridges, tunnels, or power plants, exhibit unique characteristics and failure patterns. By extending the analysis, insurers can gain insights into industry-specific risks and develop tailored risk management strategies. Addressing the limitations of this research is crucial for future studies. One limitation is the availability and quality of data, particularly regarding past claims and infrastructure failure events. Improved data collection methods, including collaboration with industry stakeholders and utilizing emerging technologies such as Internet of Things (IoT) sensors, could overcome this limitation [10]. Additionally, the framework's assumptions and simplifications may require further refinement to capture the complexity of real-world scenarios more accurately.

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