Research on frequency calculation of short suspender with fixed support at both ends

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Abstract. In order to analyze the influence of flexural stiffness on the frequency of short boom, the calculation accuracy of natural vibration frequency of short boom cable force is low. Based on the existing formula for calculating natural vibration frequency of the cable with fixed supports at both ends and considering the non-linear vibration of the cable, the finite element model considering the flexural stiffness of the derrick is established in combination with the finite element software in a practical project, and the influence of the flexural stiffness on the cable force of the derrick of different lengths is analyzed. The modified formula of theoretical frequency is fitted. The analysis shows that the flexural stiffness has a great influence on the recognition of the natural vibration frequency of the short boom, which leads to the low recognition accuracy in the short boom, and can not be directly used to identify and judge the frequency of the short boom. According to different boom lengths, the theoretical frequency is fitted with the measured natural vibration frequency, and the correction formula of the first order natural vibration frequency is obtained. Compared with the traditional boom frequency calculation formula, the error of natural vibration frequency identification can be controlled within plus or minus 5% by using the revised formula.

Keywords: determined short suspender frequency; frequency method; boundary conditions; flexural stiffenss

1 Introduction

Suspender is one of the key components that bear forces in a tied-arch bridge with a middle and lower bearing system. It directly carries and transfers loads from the main girder and arch ribs. Due to the multiple statically indeterminate nature of a tied-arch bridge, any change in the suspender force will result in a redistribution of internal forces and alter the structural load distribution. Therefore, accurate identification of the suspender force is crucial. The main methods for testing suspender forces include the frequency method, magnetic flux method, and hydraulic pressure gauge method.

Currently, the frequency method is the primary approach for testing suspender forces, as it offers simplicity, cost-effectiveness, practicality, and applicability to various types of suspenders. It is widely used in bridge health monitoring. A significant

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amount of research has been conducted by domestic and international scholars on the calculation formulas and frequency identification of suspender forces based on the frequency method. Yan Qiqing et al.[1] analyzed the factors influencing the accuracy of suspender force testing and proposed a force calculation formula by introducing boundary correction coefficients and force deviation coefficients. Dai Jinpeng et al.[2] introduced the concept of suspender length correction coefficient and fitted a calculation formula for the length correction coefficient of suspenders. Zhang Rongling et al.[3] established a suspender force calculation formula considering the coupling effects of bending stiffness, shear deformation, and rotational inertia under the boundary conditions of hinged supports at both ends. He Rong et al.[4] considered complex boundary conditions such as temperature effects and established an implicit equation for suspender force by introducing boundary influence coefficients to improve the accuracy of practical calculation formulas. Ai Yulin et al.[5] analyzed the influence of fork ears on suspender forces and introduced the boundary conditions of one end hinged and one end fixed, deriving an explicit expression for suspender force and natural frequency using the energy method. Ran Zhihong et al.[6] erived the frequency calculation formula for cable-stayed bridges based on the boundary conditions of both ends fixed using the singular perturbation method. Armin et al.[7]derived the frequency equation for suspenders based on their bending stiffness and sag extension characteristics. Kim[8] improved the accuracy of suspender natural frequency testing and identification using digital processing techniques. Yan[9]optimized the relationship between cable force and natural frequency through visual tracking and analysis, making it applicable to any boundary conditions.

In order to accurately determine the natural frequency of the suspender when it is tensioned to the target force and thus monitor the tensioning of the suspender accurately, the existing suspender force calculation formulas are combined. The influence of bending stiffness on suspender force calculation for different suspender lengths is taken into account. The suspender length is divided into three calculation ranges, and by fitting the theoretical frequency with the measured frequency, a correction formula for the theoretical frequency is obtained. This formula is applied to the construction monitoring of suspender tensioning in a certain steel tube concrete suspenders of an underslung arch bridge, resulting in precise testing results.

2 Force analysis of suspender and solution of vibration equation

Due to the influence of the lateral bending stiffness on the calculation of suspender forces, the following basic assumptions are made for the suspender: ①. The suspender is made of flexible homogeneous material, and the effect of suspender forces on suspender linear density is disregarded.②. The suspender undergoes parabolic vibrations only in the lateral plane, and the effects of self-weight shear deformation and damping are neglected.③. The suspender is not subjected to lateral forces, and the axial displacement is very small and can be neglected. Due to the relatively small and short suspender inclination compared to cable-stayed bridges, the influence of suspender sag can be disregarded. Based on the above analysis and assumptions, the force analysis of the suspender is established as shown in Figure 1. By applying the Euler-Bernoulli beam theory, the undamped vibration differential equation for the suspender is derived as follows:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} - T\frac{\partial^2 y(x,t)}{\partial x^2} + m\frac{\partial^2 y(x,t)}{\partial t^2} = q(x,t) \tag{1}$$

When neglecting external forces acting on the suspender, the equation can be simplified to:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} - T\frac{\partial^2 y(x,t)}{\partial x^2} + m\frac{\partial^2 y(x,t)}{\partial t^2} = 0 \tag{2}$$

In the above formula, T is the cable force of the boom, EI is the flexural stiffness of the boom, $y(x,t)$ is the lateral displacement function of the boom vibration, m is the linear density of the boom, and l is the length of the boom.

The displacement function $y(x,t)$ can be expressed as:

$$y(x,t) = \varphi(x) + \sin(\omega t + \theta) \tag{3}$$

using the method of separating variables and trigonometric function and shape function.

In the above formula, ω is the natural vibration circle frequency of the boom; θ is the phase angle; The relation between circular frequency ω and engineering frequency f is as follows: $\omega = 2\pi f$; t is the time.

Substituting equation (3) into equation (2), we obtain:

$$EI\varphi^{(4)}(x) - T\varphi''(x) + m\varphi(x)\omega^2 = 0 \tag{4}$$

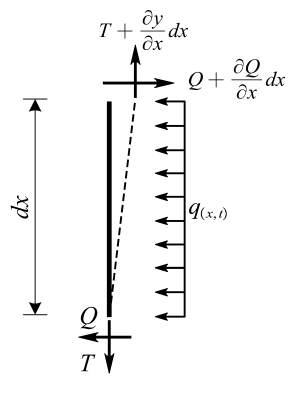

Fig. 1. Force diagram of the micro-element of suspender

3 Practical formula for calculating the cable force of the suspender of tied arch bridge

Assuming the boundary condition of the hanger rod is hinged and considering the effect of flexural stiffness on the tension of the hanger rod, solving equation (4) yields the vibration mode equation of the hanger rod as follows:

$$\varphi_{(x)} = A_1 \sin(\alpha x) + A_2 \cos(\alpha x) + A_3 \sinh(\beta x) + A_4 \cosh(\beta x) \tag{5}$$

FANG[10] and others obtained the frequency equation for the first-order free vibration of the hanger rod by solving equation (5) as follows:

$$f_T^2 = \frac{T}{4ml^2} + \frac{EI\pi^2}{4ml^4} \tag{6}$$

In the equation, A_1, A_2, A_3, A_4 represents an undetermined constant; $\alpha^2 = \sqrt{\sqrt{T^2/(4(EI)^2) + 4\pi^2 m f_n^2/(EI)} - T/(2EI)}$; $\beta^2 = \sqrt{\sqrt{T^2/(4(EI)^2) + 4\pi^2 m f_n^2/(EI)} + T/(2EI)}$

Based on the energy method, substituting the boundary condition of one end hinged and one end fixed into equation (5), the frequency equation for the first-order free vibration of the hanger rod is obtained as[11]:

$$f_T^2 = \frac{5.75T}{8m\pi^2 l^2} + \frac{118.77EI}{8m\pi^2 l^4} \tag{7}$$

Based on the boundary condition of both ends being fixed, introducing the eigenvalue λ and the high-order small quantity bending stiffness coefficient α, we substitute them into equation (5), yielding:

$$\varepsilon^2 \varphi_{(x)}^{(4)} - \varphi_{(x)}'' - \lambda \varphi_{(x)} = 0 \tag{8}$$

Due to the high-order small quantity nature of the bending stiffness coefficient α, which appears in higher-order terms, it introduces singular perturbations to the governing equation (8) and results in boundary layer effects [12][13]. Building upon this, in reference [6], the boundary layer type function singular perturbation method is applied to solve equation (8), resulting in the calculation of cable frequencies as follows:

$$\omega_n^2 = \frac{Tn^2\pi^2}{ml^2} + \Delta\lambda(n,\xi)\frac{T}{ml^2} + \frac{4n^2\pi^2\sqrt{EIT}}{ml^3} + \frac{EIn^4\pi^4}{ml^4} + \frac{12EIn^2\pi^2}{ml^4} \tag{9}$$

In the equation, represents the difference between the modified eigenvalue T and β, which is only related to the sag, while the meanings of other symbols remain the same. Equation (9) represents the formula for calculating the cable tension under the condition of both ends being fixed. For the hanger, the correction value can be ignored. Therefore, the calculation formula for the engineering frequency equation of the 1st-order free vibration of the hanger under the condition of both ends being fixed is as follows:

In the equation, $\Delta\lambda$ represents the modified characteristic value considering the sag effect, and it is the difference between $n^2\pi^2$ and λ_0, which is only related to the sag. The meanings of other symbols remain the same as before. Equation (9) represents the formula for calculating the cable tension under the condition of both ends fixed. For the case of a hanger rod, the correction value $\Delta\lambda$ can be ignored. Therefore, the engineering frequency equation for the first-order free vibration of the hanger rod under the condition of both ends fixed is given by:

$$f_T^2 = \frac{T}{4ml^2} + \frac{EI\pi^2}{4ml^4} + \frac{3EI}{ml^4} + \frac{\sqrt{EIT}}{ml^3} \tag{10}$$

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4 Frequency calculation and analysis

4.1 Experimental Overview and Data Acquisition

The study focuses on an actual engineering project of a 64m simply supported lower-bound arch bridge with steel-concrete composite girders. The suspension cables used in the bridge are pre-stressed PES(C)7-109 strands with a diameter of φ7 mm. These high-strength galvanized steel strands have a standard strength of 1670 MPa and an elastic modulus of 195 GPa. The diameter of the suspension cables is 97mm, and the unit mass per length is 35.7 kg/m. The bending moment of inertia of the suspension cables is $I = 2.049\times10^{-6}$m4. The suspension cables are arranged in parallel with a spacing of 5.5m. The entire bridge is equipped with a total of 9 pairs of suspension cables, totaling 18 strands. The construction of the bridge follows the sequence of girder placement followed by arch construction. The installation and tensioning of the suspension cables take place when the concrete strength of the arch ribs reaches 95%. The elevation diagram of the bridge is shown in Figure 2.

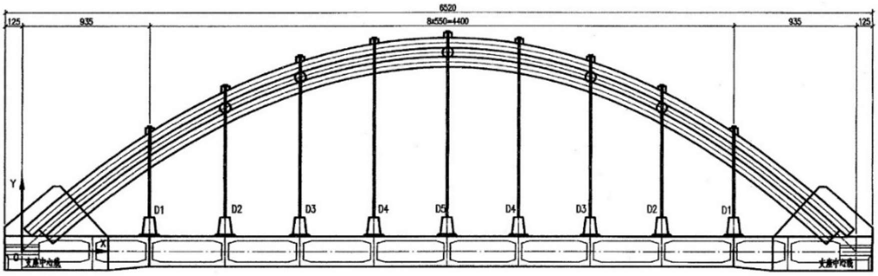


Fig. 2. Suspender number and arrangement

The order of tensioning the suspension cables is as follows: Suspension Cable 5, Suspension Cable 2, Suspension Cable 4, Suspension Cable 3, Suspension Cable 1. The results of the natural frequency readings during the tensioning and calibration of the suspension cables are shown in Table 1. (Due to space limitations, only partial test results are provided).

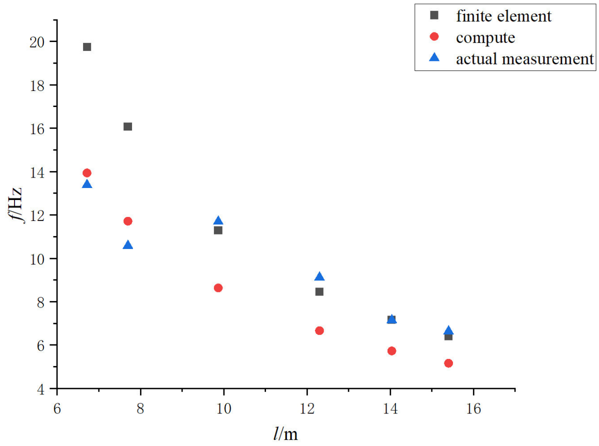
Table 1. The suspender tension and vibration frequency

suspender number	l/m	T/kN	Frequency order	f/Hz
1	7.699	1161.5	1	13.338
	7.699	1891.3	1	14.385
2	10.923	1142.5	1	12.974
	10.923	1740.8	1	15.437
3	13.207	1097.5	1	10.206
	13.207	1791.3	1	11.944
4	14.556	1165.9	1	8.418
	14.556	1937.1	1	10.252
5	14.974	1183.8	1	8.253
	14.974	1891.3	1	10.118

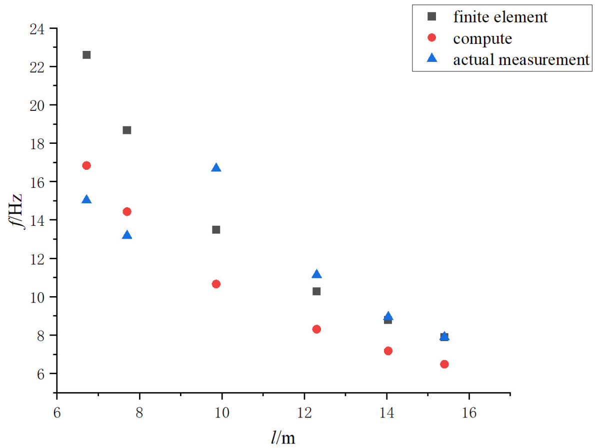
4.2 Finite Element Analysis

A finite element model was established using the ANSYS finite element software. The entire length of the suspension cable was simulated using beam elements, and the boundary conditions at both ends were set as fixed supports. By applying prestressing commands, the suspension cable was subjected to a specific tension. Modal analysis was performed on models with different lengths of the suspension cable under the same tension to extract the first natural frequencies of each suspension cable. These frequencies were compared with the measured frequencies of the suspension cables and the frequencies obtained from the modified formula in this study. The analysis focused on the natural frequencies of selected suspension cables at tension forces of 720 kN and 1200 kN. The comparison of frequencies is shown in Fig 3.

Comparing the frequencies obtained from finite element calculations with the measured frequencies, it can be observed that for shorter cable lengths, the influence of bending stiffness on the natural frequencies of the suspension cable is significant. The finite element method tends to overestimate the natural frequencies, resulting in lower accuracy. However, as the cable length increases, the influence of bending stiffness on the natural frequencies diminishes, and the finite element analysis can yield more accurate results. Comparing the calculated theoretical frequencies with the measured frequencies, it is evident that for shorter cable lengths (less than 7.699 meters), the frequencies calculated using Equation (10) are lower than the measured frequencies. For cable lengths between DG2 and DG3, there is a significant discrepancy between the calculated frequencies and the measured frequencies. However, as the cable length increases, the discrepancy between the calculated frequencies and the measured frequencies decreases. Based on the above analysis, the cable lengths are divided into three categories: short cables (0-7.699 meters), medium-length cables (8 meters to 13.207 meters), and long cables (13.207 meters and above).



(a) 720kN



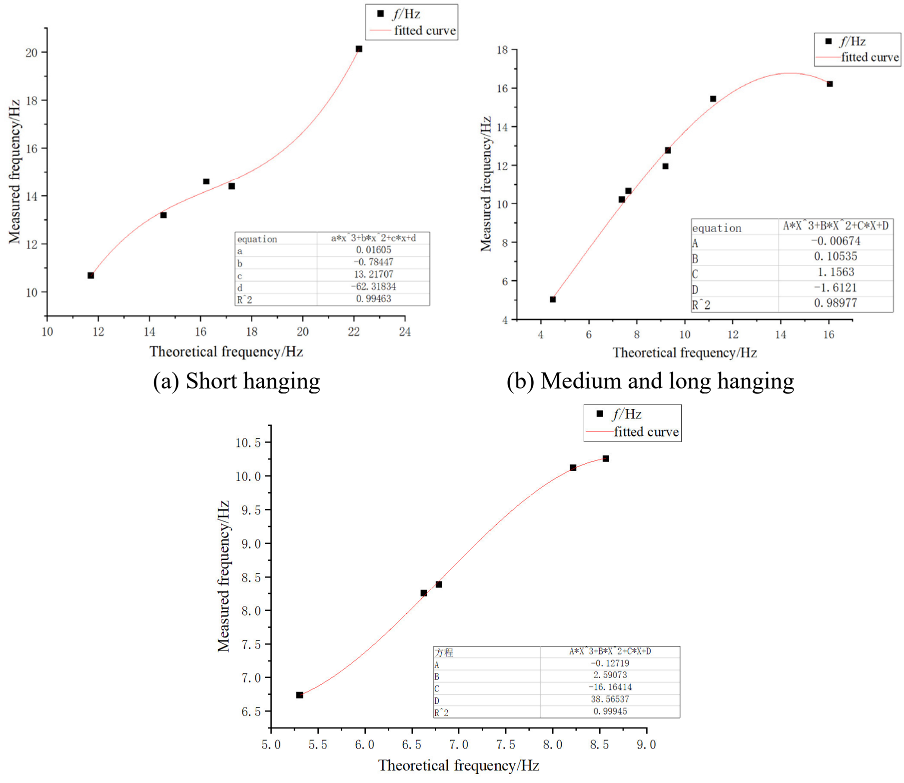
(b) 1200kN

Fig. 3. frequency changes with suspender length

4.3 Theoretical frequency calculation correction

Based on the analysis in Section 3.1, it is evident that the error between the theoretical frequency obtained from Equation (10) and the measured frequency is significant. To address this, a fitting approach is applied to modify the theoretical frequency. Based on the curve fitting analysis of the measured frequencies and theoretical frequencies as shown in Fig 4, the following formula is derived to modify the theoretical frequencies of the suspender:

$$f=\begin{cases}0.01605\times f_T^3-0.78447\times f_T^2+13.21707\times f_T-62.31834 & l\leqslant 7.699\mathrm{m}\\ -0.00674\times f_T^3+0.10535\times f_T^2+1.1563\times f_T-1.6121 & 7.699\mathrm{m}<l\leqslant 13.198\mathrm{m}\\ -0.12719\times f_T^3+2.59073\times f_T^2-16.16414\times f_T+38.56537 & l>13.198\mathrm{m}\end{cases} \tag{11}$$



(a) Short hanging (b) Medium and long hanging

(c) long hanging

Fig. 4. Fitted relationship between theoretic frequency and measured frequency

4.4 Data analysis Numerical simulation analysis

In order to verify the accuracy of formula (11) in identifying boom frequency in actual projects, the initial tension of part of the boom of the 80*m* concrete-filled steel tube simply supported tie pole arch bridge under actual projects is taken as the research object. The specific calculation parameters are consistent with the general situation of 3.1 experiment except the length of the suspender. Before the initial tensioning, the theoretical frequency under the target cable force is solved by the modified formula (11), and it is compared with the actual frequency. In this paper, the first-order natural vibration frequency analysis is adopted, and the specific analysis is shown in Table 2.

Table 2. The frequency of the suspender under different tension

number	l/m	T/kN	Measured frequency/Hz	(11)		(6)		(7)		(10)	
				f/Hz	error	f/Hz	error	f/Hz	error	f/Hz	error
DG1	6.716	1204.04	15.132	14.45	-4.51%	14.184	-6.26%	15.893	5.03%	16.861	11.43%
DG3	12.302	1203.21	11.148	11.415	2.40%	7.546	-32.31%	8.244	-26.05%	8.317	-25.39%
DG4	14.037	1206.53	8.974	9.005	0.35%	6.605	-26.40%	7.196	-19.81%	7.193	-19.84%
DG6	15.4	722.93	6.639	6.667	0.42%	4.676	-29.57%	5.112	-23.00%	5.169	-22.15%
DG4'	14.037	1201.26	8.959	8.985	0.29%	6.591	-26.43%	7.181	-19.84%	7.179	-19.87%
DG3'	12.302	1197.23	11.141	11.387	2.21%	7.528	-32.43%	8.224	-26.18%	8.299	-25.51%
DG1'	6.716	1217.13	15.153	14.482	-4.43%	14.256	-5.92%	15.968	5.38%	16.931	11.74%

Based on the analysis of the table results, it can be observed that the maximum relative error of the modified calculation formula in this study for shorter cable lengths reaches 4.43%, while the minimum error is 0.29%. All errors are below 5%, indicating that the derived correction formula has a high level of accuracy for short cables. For short cables with one end hinged and one end fixed boundary conditions, the error is smaller compared to the boundary conditions of both ends fixed or both ends hinged. Equation (10) in reference [6] demonstrates high accuracy with a relative error below 0.5% for calculating the natural frequencies of cables or rods longer than 91 meters, indicating its precision for long cables or rods. However, the consideration of the bending stiffness coefficient in the reference is suitable for slender cables and may result in larger errors in shorter structural elements such as suspension cables in arch bridges. Its values usually fall within the range of [0.02, 1][14]. Therefore, in this study, the errors are relatively large and not suitable for calculating the frequencies of short cables.

5 Conclusions

1). The frequency correction formula for the suspension rod's natural frequency can control the error within 5%, demonstrating a high level of accuracy. Compared to frequency calculation formulas based on different boundary conditions, the formula based on fixed-fixed boundary conditions yields smaller errors than those based on fixed-hinged boundary conditions.

2). Calculating the natural frequencies of suspension rods with different lengths before tensioning at the target prestress value, combined with the use of a dynamic measuring instrument, facilitates accurate control of the rod's tensioning force. As the most commonly used method for on-site tensioning of suspension rods involves measuring the rod's frequency using a dynamic measuring instrument, the frequency correction formula for the suspension rod's natural frequency has better practical applicability.

3). For suspension rods with lengths less than 7.699 meters, the flexural stiffness has a significant influence on the tensioning force and natural frequency. However, this influence diminishes as the length of the suspension rod increases.

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