The analytical solution of water surface line of constant inhomogeneous flow with rectangular section

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Abstract. This paper aims at the complicated flow pattern of non-pressure spillway tunnel during operation. Based on the reconstruction project of Qingshan Reservoir spillway tunnel, this paper makes a mathematical derivation of the calculation and analysis of the constant gradient flow surface line of rectangular section in the non-pressure spillway tunnel, and gives the analytical solution between the flow S and the beginning and end water depth. It is more simple and convenient than the standard recommended piecewise trial algorithm. Finally, through the comparative analysis of 1:30 hydraulic model test results of Qingshan Reservoir spillway tunnel reconstruction project, it is considered that the analytical solution of water surface line calculation proposed in this paper is reasonable and reliable. It provides a basis for ensuring flood safety of spillway channel and downstream river, and can be used as reference for similar projects.

Keywords: Engineering hydraulics; Non-pressure flood relief tunnel; Water depth; Water surface line; Analytic solution.

1 Introduction

Rectangular section is the most common section type in the spillway, and its water surface line calculation needs to be solved by iterative trial calculation with the help of computers. The piecewise summing and difference trial of constant gradient flow differential equation[1-2] is used. Zhang Jianmin[3] proposed the iterative method, and some scholars also proposed Runge-Kutta, Newton iteration method and other solution methods[4]. However, the above methods are used to calculate the depth of each section in segments, which may lead to the gradual accumulation of trial depth errors of each section, and the greater the calculation error of the end depth. What's more, the above calculation methods need to be solved by computer programming, which is not easy to calculate by hand.

Based on the basic differential equation of constant gradient flow, the analytical solution of water surface line of constant gradient flow is derived by mathematical

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theory. Finally, based on the hydraulic model test results of the flood control capacity improvement project of Qingshan Reservoir in Hangzhou City, Zhejiang Province, China, the research results can provide reference for similar projects.

2 Water surface line and energy dissipation calculation of spillway tunnel

According to the basic differential equation of constant gradient flow[6,7,8,9,10]:

$$\frac{\mathrm{d}h}{\mathrm{d}s}=\frac{i_0-J}{1-F_r^2} \tag{1}$$

Among them,

with rectangular section:

$$A=bh\ ,\quad R=\frac{bh}{b+2h}\ ,\quad B=b \tag{2}$$

b is the base width, m; h is the depth of water, m; B is the water surface width, m; A is the water crossing area, m^2; R is the hydraulic radius, m; Parameter s is the process, m; J is for hydraulic slope, $J=\frac{n^2Q^2}{A^2R^{4/3}}$, as a dimensionless quantity; n is roughness; Q is overcurrent flow, m^3 /s; v is the flow rate, m/s; i_0 is the bottom slope ratio of flow section; F_r is Freund's number, $F_r=\frac{v}{\sqrt{gA/B}}$.

Remember dimensionless water depth: $\bar{h}=h/b$, Combined with formula (1) and (2), we can obtain:

$$\frac{\mathrm{d}h}{\mathrm{d}s}=\frac{K_1.f_{(\bar{h})}-i_0.K_2}{g_{(\bar{h})}-K_2} \tag{3}$$

There is no dimensional parameter: $f_{(\bar{h})}=\frac{\left(1+2\bar{h}\right)^{4/3}}{\bar{h}^{10/3}}$, $g_{(\bar{h})}=\frac{1}{\bar{h}^3}$,

$$K_1=\frac{n^2g}{b^{1/3}},\qquad K_2=\frac{gb^5}{Q^2} \tag{4}$$

Using Matlab software to fit analysis no dimensional function numerical $f_{(\bar{h})}$, $g_{(\bar{h})}$, give the high precision fitting function in the scope of 0.05<$\bar{h}$<1. The fitting error is less than 0.5%, the correlation coefficient is greater than 0.99, satisfies the requirement of engineering application.

$$\begin{cases} f_{(\bar{h})} = \dfrac{f_{1(m)}}{\bar{h} - f_{3(m)}} - f_{2(m)} = \dfrac{9.773}{\bar{h} - 0.224} - 9.823 \\ g_{(\bar{h})} = \dfrac{g_{1(m)}}{\bar{h} - g_{3(m)}} - g_{2(m)} = \dfrac{3.160}{\bar{h} - 0.235} - 3.813 \end{cases} \tag{5}$$

Parameters $f_{1(m)}$ =9.773, $f_{2(m)}$=9.823, $f_{3(m)}$=0.224, $g_{1(m)}$=3.160, $g_{2(m)}$=3.813, $g_{3(m)}$=0.235, and the fitting curve is shown in Figure 1.

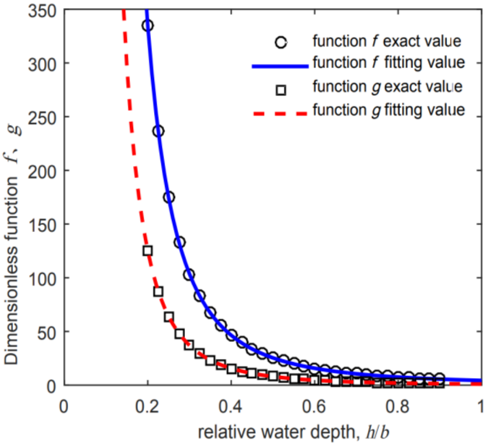

Fig. 1.fitting analysis of dimensionless function $f_{(\bar{h})}$, $g_{(\bar{h})}$

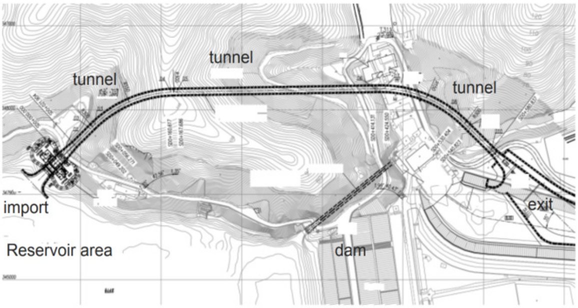

Fig. 2. Thelayout of the project

Equation (5) is substituted into equation (3) to solve the ordinary differential equation, and the analytical solution of water surface line of constant gradient flow is obtained:

$$S=a_2b\left(\bar{h}_2-\bar{h}_1\right)+b\xi.\ln\frac{\left(\bar{h}_2-f_{3(m)}-a_1\right)}{\left(\bar{h}_1-f_{3(m)}-a_1\right)}+b\varsigma.\ln\frac{\left(\bar{h}_2-g_{3(m)}\right)}{\left(\bar{h}_1-g_{3(m)}\right)} \qquad (6)$$

In Equation (6), constant parameters a1 and a2 are given by Equation (7). $\bar{h}_1=h_1/b$, $\bar{h}_2=h_2/b$,h_1 is beginning water depth , h_2 is ending water depth, m; ξ , ς are given in formula (7).

$$a_1=\frac{\left(\frac{n^2g}{b^{1/3}}\right).f_{1(m)}}{M}\ ,\quad a_2=\frac{\left(\frac{gb^5}{Q^2}\right)+g_{2(m)}}{M}\ ,\quad a_3=\frac{g_{1(m)}}{M}\ ,\quad M=\left(\frac{n^2g}{b^{1/3}}\right).f_{2(m)}+i_0.\left(\frac{gb^5}{Q^2}\right),$$

$$\xi=a_1a_2-a_3+\frac{\left(f_{3(m)}-g_{3(m)}\right)a_3}{a_1+\left(f_{3(m)}-g_{3(m)}\right)}\ ,\quad \varsigma=-\frac{\left(f_{3(m)}-g_{3(m)}\right)a_3}{a_1+\left(f_{3(m)}-g_{3(m)}\right)} \qquad (7)$$

The analytical solution is applicable to the calculation of water surface lines in general rectangular open channels (or unpressurized gate tunnels), and it is more simple and convenient than the standard recommended piecewise trial algorithm. The analytical solution of water surface line can be obtained by using the above methods respectively for the change of bottom slope sections.

3 Engineering case model test

3.1 Project Overview

The flood control capacity improvement project of Qingshan Reservoir is located in Qingshan Reservoir, Lin 'an District, Hangzhou[5]. The total storage capacity of Qingshan Reservoir is 2.13× 10.8 million m^3. The project scale belongs to large (2) type, and the engineering class is second class.

The main construction contents of the project include the new spillway tunnel, the reinforcement of the power station tail channel and the reconstruction of the spillway channel. The spillway tunnel includes the entrance gate, the tunnel and the exit section. The design flood standard is once in 100years.The entrance gate mouth size is 1 hole×12m, the gate bottom elevation is 15.7m, the tunnel length is about 581m, the entrance elevation is 15.7m, the exit elevation is 7.5m, the longitudinal slope of the entrance gradient section is 10%, and the longitudinal slope of the other sections is 1.075%. The tunnel section is city gate shaped, and the tunnel section is 8.0m×9.0m (width×height) after lining. The general layout of the project is shown in Figure 2.

In order to demonstrate the flow capacity of the new spillway tunnel and the energy dissipation of the outlet high speed water flow, the hydraulic calculation and the normal hydraulic physical model of the new spillway tunnel are studied. The water flow of the tunnel meets the requirements of the specification that the clearance height is greater than 0.4m and the clearance ratio is not less than 15%.

3.2 Hydraulic physical model test

This project adopts hydraulic normal global model for experimental study, follows gravity similarity criterion, and designs model according to geometric similarity. According to the test requirements, combined with the test site and model scale, the geometric scale of the model was determined as 1:30.

The model mainly includes water supply system, reservoir area, dam, sluice inlet, flood discharge tunnel, sluice outlet, silting pool, power station tail canal, downstream river, tailgate, water metering and return water system.

4 Comparison of hydraulic calculation and model test in spillway tunnel

When the floodway tunnel of Qingshan Reservoir flood control capacity improvement project[5] in Hangzhou City is 8.0m wide and the designed discharge is 364m^3/s, the calculated critical water depth h_k is 5.95m, and then the critical bottom slope i_k is 0.00365. According to the hydraulic model test, the tunnel can meet the design requirements when the maximum overcurrent capacity is 391m^3/s.

According to the analytical solution calculated by the water surface line, the water depth along the spillway tunnel is shown in Figure 3.

As can be seen from Figure 3, the results of water surface line calculation by analytical method in this paper are basically consistent with the results of hydraulic model test. There is a small bend in the upper reaches and due to the influence of centrifugal force, the water depth of the left bank is slightly higher than the right bank. However, in general, it is basically consistent with the analytical solutions calculated in this paper. In the model test under this condition, the upper, middle and downstream water flow pattern in the spillway tunnel is shown in Figure 4.

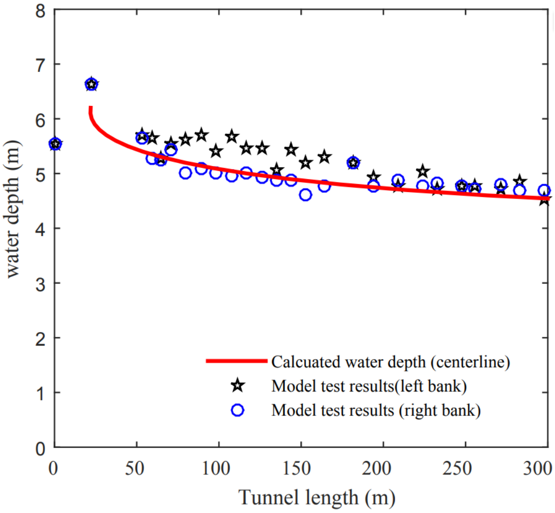

Fig. 3. Water surface line in the spillway tunnel

(a)upstream segment (b) middle segment (c) downstream segment

Fig. 4. Water flow pattern in the spillway tunnel

All water flows are rapids after entering the spillway tunnel through the gate chamber, and there are wavy fluctuations of different degrees along the way (Figure 4).

In summary, the results of analytical solution in this paper are basically consistent, indicating that the analytical solution of water surface line proposed in this paper is convenient and accurate, and it can provide a technical reference for similar engineering design.

5 Conclusions and Suggestions

In view of the complicated water flow pattern of the non-pressure spillway tunnel during operation, the analytical solution of water surface line of constant gradient flow was studied by relying on the reconstruction project of the spillway tunnel of Qingshan Reservoir. The conclusions are as follows:

(1) According to the basic differential equation of constant gradient flow, the analytical solutions between the flow S and the water depth at the beginning and end are given through mathematical derivation; Furthermore, the analytical solution of water surface line of constant gradient flow in rectangular open channel is given.

(2) According to the comparative analysis of 1:30 hydraulic model test results of Qingshan Reservoir spillway tunnel reconstruction project, the calculated results of the new analytical solution are generally consistent with the model test results. Partly due to the effect of centrifugal force and slight vibration fluctuation, the measured water level of the left bank of this section is slightly larger than the calculated value (relative error is less than 2.5%). Therefore, the analytical solution proposed in this paper is convenient and accurate, and provides a technical reference for similar engineering design.

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