# Research on Multi-Product Ordering Strategy Based on Quantity Discounts and Temporary Price Discounts

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**ABSTRACT.** Inventory management is the most important link in the logistics chain of perishable goods, and this paper investigates an inventory control model that takes into account the effects of various costs, quantity discounts offered by the supplier to the retailer, and temporary price discounts offered by the retailer to the consumer on their demand and order quantities. By considering demand, discount promotions, quantity discounts and various types of costs, the paper develops a multi-product nonlinear planning model and solves it using a genetic algorithm. Finally, the optimal multi-product ordering strategy is derived through numerical analysis to enable firms to perform better inventory management activities.

**Keywords:** Inventory Management, Volume Discounts, Sourcing Strategy, Genetic Algorithm

## 1 INTRODUCTION

In order to maximize the retailer's profit, we will use genetic algorithm to optimize the ordering situation. As a retailer, there is less research on inventory modelling that considers both purchasing and selling. In this paper, we will take the retailer's point of view and consider the impact of the two segments on the profit, in order to make a better ordering decision to maximize their profit.

Since the 1960s, many scholars have focused on the study of perishable goods and have assumed that demand is either constant or a function of influences such as time, price, and freshness. Li Shuaipeng [1] studied the profit and various types of costs of retail terminals, and analyzed the various types of costs and optimal order quantity when the profit is maximized, assuming that the demand is known. Zhang Xingong[2]and Zhang Rui[3]assumed that the demand rate depends on the price, and investigated the optimal ordering decision under the influence of deterioration, price discount and shelf life constraints, respectively. Mou jinjin[4] investigates inventory replenishment strategies under the influence of spoilage rate, assuming that demand depends on product freshness and price. Wang Shuyu[5] assuming that demand is jointly affected by price and quality loss rate, we seek to find the optimal price and inventory joint decision under both centralised and decentralised decision-making modes with the


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objective of profit maximization. Mallick Rabin Kumar et al.[6]considered inventory dependence and demand and developed a model for inventory control. Wei Feng[7] constructed an inventory optimisation model based on the premise that demand is a function of price and that stock-outs are not allowed, and derived formulas for the maximum stock level and the optimal order quantity.

In addition to considering whether the demand is certain, many scholars consider the impact of different influences on the ordering decision separately, such as quantity discounts offered by the supplier for the retailer or price discounts offered by the retailer for the customer. Zhao Tianyang[8] investigated the ordering strategy in the context of information sharing in the case of large-volume purchasing with the presence of price discounts. Ye Yong et al.[9] used an iterative algorithm to calculate the retailer's optimal order lot size on the basis of price discounts when assuming that consumer demand for a good satisfies a uniform distribution. Nita H et al.[10] and Reyes A[11] argue that retailers can increase sales by implementing promotions and analyses the examples to conclude that quantity discounts help retailers to reduce the cost of purchases and thus increase sales margins.

## 2 Modeling of ordering considering quantity discounts and temporary price discounts

### 2.1 Title Modeling Assumptions

In order to facilitate the modeling, the following assumptions will be made in this paper:

Assumption 1: Only one ordering cycle of the retailer is considered;

Assumption 2: Orders arrive instantaneously, no stock-outs are allowed, and at the end of the sales cycle, inventory is zero;

Assumption 3: This type of goods using regular ordering hair to the supplier to order, order more products, easy to form a scale effect, so the supplier to provide a certain number of discounts;

Assumption 4: The retailer's ordering decisions for each product are unrelated to each other at the time of purchasing and do not take into account the competitive.

To facilitate the construction and analysis of the model, the notation conventions are shown in Table 1 below:

**Table 1.** model assumption.

| notation | hidden meaning |
|---|---|
| T | Length of sales cycle |
| $t_i$ | Time of commencement of temporary price discounts taken by retailers |
| i=1,2,..., n | Product type indicator, with n products |
| j=1,2,..., $k_i$ | Discount interval indicator with ki discount intervals |
| C | Funding budget upper bound |
| $C_{ij}$ | Unit purchase cost of product i under discount j (kg) |
| $p_i$ | Selling price of product i (kg/CNY) |

| | |
|---|---|
| $\alpha$ | Temporary price discount factor ($0 < \alpha < 1$) |
| $\beta$ | Demand amplification factor after temporary price discount (constant) |
| h | Unit inventory holding cost (constant) |
| $Q1_i$ | Demand for product i at regular price |
| $Q2_i$ | i Demand for product promotions |
| $x_{ij}$ | Quantity of product i ordered under discount j (kg) |
| $x_i$ | Total quantity of product i ordered (kg) |
| $x_{ij}^L$ | Lower bound on the quantity of product i ordered under discount j (kg) |
| $x_{ij}^U$ | Upper bound on the quantity of product i ordered under discount j (kg) |
| $y_{ij}$ | 0-1 variable that constrains the choice of product i in discount interval j |
| HR | Sales revenue from retailers |
| HY | Inventory costs for retailers |
| HC | Retailer subscription costs |
| Z | Total profit for retailers |

## 2.2 Modeling

This paper combines demand, quantity discounts offered by the supplier and temporary price discounts offered by the retailer to maximize profits subject to satisfying the resource constraint.

From the model assumptions, it can be known that the product starts the temporary price discount at time $t_i$, then the product has a demand function at time period $0 \le t \le ti$ :

$$D1(t) = a_i - b_i t \tag{1}$$

Starting from the time $t_i$ , the demand rate will be amplified accordingly due to the implementation of the price discount strategy, we assume that the demand amplification factor is β. The demand function of the item in the time period $t_i \le t \le T$ is:

$$D2(t) = \beta \cdot (a_i - b_i t) \tag{2}$$

From the above demand rate function, the demand for each of the two phases can be found:

$$Q1_i = \int_0^{t_i} D1(t)dt = a_i t_i - \frac{1}{2} b_i t_i^2 \tag{3}$$

$$Q2_i = \int_{t_i}^{T} D2(t)dt = \beta(a_i T - \frac{1}{2} b_i T^2 - a_i t_i + \frac{1}{2} b_i t_i^2) \tag{4}$$

Since a retailer's sales revenue is equal to the product of the sales volume and the selling price, when demand is fully satisfied, the retailer's sales revenue can be obtained as:

$$HR = p_i \cdot Q1_i + \alpha p_i \cdot Q2_i \tag{5}$$

Record the inventory holding cost as the storage cost at the moment of arrival, so the inventory holding cost is:

$$HY = x_i \cdot h \tag{6}$$

The retailer purchases a large variety of fruits and has a scale effect, so the supplier offers a certain quantity discount, so its ordering cost can be expressed as:

$$HC = C_{ij} \cdot x_i \tag{7}$$

Based on the assumptions, a multi-product non-linear planning model that considers both quantity discounts and temporary price discounts by the retailer at the time of sale is modeled as follows:

$$maxZ = \sum_{i=1}^{n} HR - \sum_{i=1}^{n} HY - \sum_{i=1}^{n} \sum_{j=1}^{k_i} HC \tag{8}$$

$$C = 1000000 \tag{9}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{k_i} HC < C \tag{10}$$

$$x_{ij}^{L} < x_{ij} \le x_{ij}^{U} \tag{11}$$

$$x_i = \sum_{j=1}^{k_i} x_{ij} \tag{12}$$

$$\sum_{j=1}^{k_i} y_{ij} = 1 \tag{13}$$

$$y_{ij} = \begin{cases} 0, & Product\ i\ does\ not\ choose\ discount\ interval\ j \\ 1, & Product\ i\ chooses\ discount\ interval\ j \end{cases} \tag{14}$$

The meaning of the symbols in the model is expressed as follows: Equation.8 denotes the expression of a function that maximizes the profitability of the retailer; Equations 9 and 10denote that the capital budget for purchasing fruits is limited and must not exceed the maximum ordering cost C; Equation 11 denotes that each fruit is ordered in one of the discount intervals, with certain constraints on the upper limit of the discount interval and the lower limit of the interval; Equation 12 denotes that the fruit is ordered in the j discount interval selected xi; Equation 13 is for each fruit has only one discount interval selected; Equation 14 indicates that there is only one ordering cost per fruit, where 0 means that the quantity discount interval is not selected and 1 means that the quantity discount interval is selected.

### 2.3 Genetic algorithm based model solving

In the following, the solution process will be described by combining the MINLP-MP model and the actual situation of six kinds of fruits:

Coding :In order to facilitate genetic manipulation, we choose floating-point encoding in the modeling solution of the MINLP-MP model, and the values of fruit ordering quantities are the gene values, and the ordering quantities of N fruits are restricted to the closed intervals of LB and UB (where LB is the lower boundary of ordering quantities, and UB is the upper boundary of ordering quantities), and then each gene value can be expressed as a random floating-point number in this range, and N = 6 The full arrangement of floating point numbers of ordering quantities (gene values) is an individual (chromosome), i.e.$x = LB + (UB - LB)\ .*\ rand(1,\ \ dim)$.

Population size: After several experimental calculations to determine a reasonable value for the population size, here NP = 100.

Adaptation function: The optimization problem of MINLP-MP is consistent with finding the maximum value of the objective function, so the fitness function takes the objective function of MINLP-MP.

Selection of operations: This section uses a roulette wheel selection method, where the size of the likelihood of an individual being selected is proportional to the size of its fitness value, i.e., the larger the value of the objective function, the greater the probability of an individual being selected.

Cross-operation: In this part, a single point crossover method is used, first, N individuals in the population are randomly paired two by two, and then crossover is carried out with a probability of PC = 0.9. The two individuals involved in the crossover randomly select a crossover point, and the new next generation chromosome is formed by interchanging some of the genes after the crossover point.

Mutation operation: This section uses Gaussian variation, operating with a probability of variation of Pm = 0.1 in the population.

Elite retention: By sorting, the best individuals in the original population are retained, and the worst individuals in the new population are excluded, with the elite ratio of Pe=0.1, forming an optimal population, which continues to be iterated until the maximum number of iterations reaches the upper limit of the optimal individuals obtained, i.e., the near-optimal solution of MINLP-MP.

# 3 Numerical analysis

## 3.1 Genetic algorithm based model solving

Enterprises in the procurement process, often the existence of the same ordering cycle of the product together, the formation of economies of scale. Therefore, this paper studies the fruits with the same ordering cycle, and selects six fruits with an ordering cycle of five days, namely cantaloupe, Fuji apple, orange, giant grape, golden kiwi and mango, for subsequent operations. During the purchasing process, the shop will be provided with a certain quantity discount by the supplier, which provides different purchase unit prices, i.e., Cij, depending on the ordered quantity; meanwhile, the shop will raise different promotions to stimulate the demand according to the deterioration of the fruits during the sales process. Based on the specific business situation of the shop, the following Table 2 shows the known parameters of each fruit:

**Table 2.** Known values of parameters.

| i | θ | α | β | $P_i$ (kg/CNY) | $C_{ij}$ (kg/CNY) 0-50 | 50-100 | 100-150 | 150-200 | 200-250 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.028 | 0.9 | 1.1 | 9.98 | 5 | 4.94 | 4.88 | 4.8 | 4.75 |
| 2 | 0.018 | 0.9 | 1.1 | 6.58 | 3.5 | 3.38 | 3.26 | 3.1 | 3 |
| 3 | 0.036 | 0.88 | 1.2 | 11.98 | 6.78 | 6.65 | 6.55 | 6.48 | 6.38 |
| 4 | 0.055 | 0.85 | 1.3 | 13.96 | 8.5 | 8.39 | 8.29 | 8.16 | 8.08 |
| 5 | 0.046 | 0.85 | 1.25 | 19.92 | 11.78 | 11 | 10.6 | 10.2 | 10 |
| 6 | 0.042 | 0.88 | 1.2 | 11.96 | 7 | 6.91 | 6.83 | 6.72 | 6.65 |

## 3.2 Analysis of results

The algorithms covered in this paper are written and solved via MATLAB 2019a, and to illustrate the usefulness of the model, application examples are given below. After optimization, the results of the solution for the six fruits were obtained as shown in the Table 3:

**Table 3.** Optimized results.

| i | $t_i$ (days) | $X_i$ (kg) | HR(CNY) | HY(CNY) | HC(CNY) | Z(CNY) |
|---|---|---|---|---|---|---|
| 1 | 3.9 | 119.22 | 8538.42 | 365.02 | 4934.17 | 3239.19 |
| 2 | 2.6 | 117.84 | | | | |
| 3 | 1.9 | 124.48 | | | | |
| 4 | 2.7 | 124.94 | | | | |
| 5 | 3.2 | 120.32 | | | | |
| 6 | 2.2 | 123.24 | | | | |

From the above table, we can see the optimal time to start discounting and the optimal ordering quantity of the 6 fruits and calculate to get their sales revenue and total ordering cost, inventory cost and maximum profit, in which the original ordering quantity of fruit 1 is 141.66kg while the optimized ordering quantity is 119.22kg, which reduces the ordering quantity by 22.44kg compared to the original ordering scheme and saves the ordering cost of 109.51.Through the data collection, the operations before and after the optimization were integrated as shown in the Table 4:

**Table 4.** Optimization comparison before and after.

| | $x_i$(kg) | | HR(CNY) | | HY(CNY) | | HC(CNY) | | Z(CNY) | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ahead | empress | ahead | empress | ahead | empress | ahead | empress | ahead | empress |
| 1 | 141.66 | 119.22 | 1141.21 | 1120.94 | 70.83 | 59.61 | 691.30 | 581.79 | 2832.84 | 3239.19 |
| 2 | 136.35 | 117.84 | 768.47 | 737.44 | 68.17 | 58.92 | 444.50 | 384.15 | | |
| 3 | 142.24 | 124.48 | 1490.91 | 1387.91 | 71.12 | 62.24 | 931.67 | 815.34 | | |
| 4 | 145.11 | 124.94 | 1743.74 | 1631.12 | 72.55 | 62.47 | 1202.96 | 1035.75 | | |
| 5 | 143.68 | 120.32 | 2396.17 | 2279.67 | 71.84 | 60.16 | 1523.00 | 1275.39 | | |
| 6 | 140.92 | 123.24 | 1473.23 | 1381.27 | 70.46 | 61.62 | 962.483 | 841.72 | | |
| | add up the total | | 9013.75 | 8538.38 | 424.98 | 365.02 | 5755.92 | 4934.16 | 406.35 | |

By comparing the data before and after the optimization of the ordering strategy, it can be seen that the ordering volume is higher before the optimization, and although its sales revenue is higher than that after the optimization, its ordering cost and inventory holding cost have also increased a lot. Through detailed comparative analysis, it can be found that the ordering strategy before and after optimization effectively reduces its ordering cost and inventory holding cost, and makes its total profit increase by 406.35.

## 4 Conclusion

The profitability of each node in the logistics chain affects the profitability of the entire supply chain, and this paper analyses inventory management costs, ordering costs, etc. Discount promotions will stimulate demand to a certain extent during the down-selling process is considered, as well as quantity discounts offered by suppliers during the retailer's purchasing process. Comprehensively analyzing the various types of costs existing in the retailer's purchasing and selling process, establishing a non-linear function with profit maximization as the objective, and solving it using genetic algorithm to obtain the optimal ordering quantity and maximum profit. The optimal order quantity and maximum profit are obtained, which also provide reference for purchasing similar products.

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