Research on Simulation of Convergent Deformation of Highway Tunnel Surrounding Rock by Using GM(1,1), Verhulst and Polynomial Models

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Abstract. The GM(1,1), Verhulst and Polynomial model have been widely applied in geotechnical engineering field. The correct selection of the models is of vital importance when the prediction accuracy and rationality of the construction scheme is considered. In this paper, based on simulation of surrounding rock convergent deformation about Zhuyang highway tunnel, the applicability of GM(1,1), Verhulst and Polynomial models in the field of engineering is studied. The calculation results can show that, based on data characteristic, it is necessary to choose different model. Polynomial model is often used in general tunnel monitoring and measurement, however, under few data conditions, the accuracy of polynomial model is often lower than that of grey model. The data of this paper are close to the saturated S sequence, so the simulation with the Verhulst model has better accuracy. In this paper, the simulation accuracy of Verhulst and GM(1,1) model is level one, and the simulation accuracy of Polynomial model is level two. The conclusion has certain reference significance for highway tunnel information design and construction.

Keywords: grey model; tunnel; applicability

1 Introduction

With the continuous development of highway construction in China, tunnels have been greatly developed as the main structure crossing the mountains. The New Austrian Tunnelling Method is widely used in tunnel design and construction. As one of the three important pillars about New Austrian Tunnelling Method, the tunnel monitoring measurement is not only the basis of parameter design and optimization, but also the information source of determination of final displacement and displacement dynamic trend[1-2]. False prediction of tunnel surrounding rock displacement may cause waste or engineering accident. Therefore, it has been an urgent subject waiting to be solved about searching correct method of simulating tunnel surrounding rock convergence deformation, choosing safe and reliable support method, and determining economical and reasonable support parameters. At present, many scholars at home and abroad put forward a lot of methods to simulate and forecast displacement varia-

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tion rule of rock-soil mass with time change, such as time series method, artificial neural network, ant algorithm, grey system theory, etc. The time series method is a common method to simulate the convergence deformation of tunnel surrounding rock, but it often needs a mass of data when the accuracy is high, and obtaining large quantity of data also needs a lot of money and material resources. The simulation method of artificial neural network and ant algorithm needs large quantity of data too, and the calculation process is complex. In the case of insufficient development of nonlinear science, the grey system theory plays an irreplaceable role in simulating and forecasting. When the grey simulation is carried out, the randomicity of raw data is weakened, and the law of data is explored. By the method of building differential equation, the data may be utilized fully in the case of less data and poor information. It can reduce a lot of tedious measurement work and bring good economic benefits, when few data and poor information system is simulated and predicted by using grey system theory[3-8].

As the main prediction model of grey system theory, the GM(1,1) and Verhulst model have been widely applied in geotechnical engineering field. The GM(1,1) model is suitable for data with obvious exponential law or approximate exponential law. If the data does not have exponential law, and it is close to the saturated S sequence, then the Verhulst model can be used. In data processing of tunnel monitoring measurement, it is influenced by environment, manual operation and other factors, and the detected data is often discrete. In order to judge the trend of surrounding rock deformation, and guide the design and construction, it is necessary to analyze the data comprehensively. Considering higher accuracy, the regression model may not be the best. In this paper, the convergence data of monitoring measurement about Zhuyang tunnel of Shaosan highway is calculated by using three models, and the applicability of models is discussed. The results can provide reference for engineering practice.

2 Research method

2.1 GM(1,1) Model

The GM(1,1) model weakens data randomicity by the method of accumulating, and it is also applicable to sequences with strong discreteness or small sample size. In the modelling process, time series can be transformed into differential equations directly, which makes it possible to establish sufficient information model by using insufficient information. The calculating expression is:

$$\hat{x}^{(1)}(k+1) = \left(x^{0}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \qquad k = 1,2,\dots,n \tag{1}$$

in the equation

$\hat{x}^{(1)}(k)$ is one cumulative value of $x^{(0)}(1)$, and

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{2}$$

K is time, a, b is model parameter residual verification of GM(1,1) model is as follows :

relative error sequence

$$\Delta = \left\{ \left| \frac{\varepsilon(1)}{s^{(0)}(t_1)} \right|, \left| \frac{\varepsilon(2)}{s^{(0)}(t_2)} \right|, \dots, \left| \frac{\varepsilon(n)}{s^{(0)}(t_n)} \right| \right\} = \{\Delta_k\}_1^n \tag{3}$$

for $k \leq n$, $\Delta_k = \left| \frac{\varepsilon(k)}{s^{(0)}(t_k)} \right|$ is relative simulation error of k point, $\bar{\Delta} = \frac{1}{n} \sum_{i=1}^{n} \Delta_k$ is average relative error. $1 - \bar{\Delta}$ is average relative accuracy. $1 - \Delta_k$ is simulation precision of k point $k = 1,2, \dots n$, given α, when $\bar{\Delta} < \alpha$ and $\Delta_n < \alpha$, it is called residual qualified model. The accuracy class of checking is shown in Table 1.

2.2 Verhulst Model

The time response function of whitening equation $\frac{dx^{(1)}}{dt} + \alpha x^{(1)} = b(x^{(1)})^2$ about Verhulst model $x^{(0)}(k) + \alpha z^{(1)}(k) = b(z^{(1)}(k))^2$ is

$$\hat{x}^{(1)}(t) = \frac{\alpha x^{(1)}(0)}{bx^{(1)}(0) + \left(a - bx^{(1)}(0)\right) a e^{at}} \tag{4}$$

The time response sequence of Verhulst model is

$$\hat{x}^{(1)}(k) = \frac{\alpha x^{(1)}(0)}{bx^{(1)}(0) + \left(a - bx^{(1)}(0)\right) a e^{at}} \quad k = 1,2, \dots, n \tag{5}$$

The residual verification method of verhulst model is the same as that of GM(1,1) model.

Table 1. Checking table of accuracy class

relative error α	accuracy class
0.01	level one
0.05	level two
0.10	level three
0.20	level four

2.3 Polynomial Fitting Model

Polynomial fitting is completed by using the software Excel 2016. The realization steps are as follows:

1) Select the data to be analyzed, then click the scatter plot in the insert option, and select the first scatter plot with only data markers, a scatter plot appears;

2) Click the scatter with data markers by using right mouse button, then select adding tendency chart of the menu that appears, the linear and logarithmic trend lines in the menu are not suitable obviously;

3) Select polynomial of the menu, and the best fitting curve of polynomial can be got after comparing.

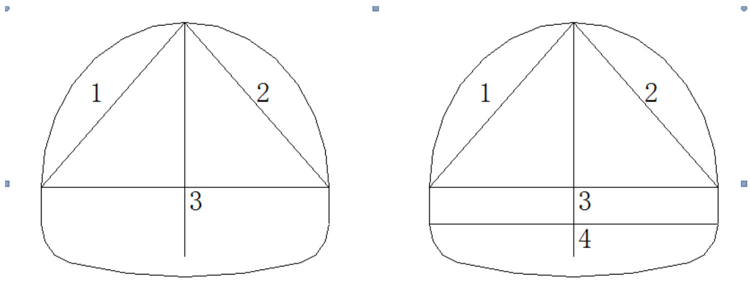
3 Engineering Example

Entrusted by the engineering headquarter about Shaosan expressway of Nanping City, monitoring measurement of Zhuyang tunnel is carried out by the Institute of Geotechnical Engineering of Chang'an University in 2003. The purpose of monitoring measurement is collecting dynamic information of surrounding rock and grasp the stable condition of supporting structure. Based on the result of monitoring measurement, the possible accident and risk of construction may be forecasted, furthermore, the initial support stability and the construction time of secondary lining determination can be determined. The monitoring measurement content includes observation of palm surface, surround rock and support condition, surface subsidence measuring, convergence and arch crown subsidence measuring. This paper mainly studies the convergence data of Zhuyang tunnel.

Zhuyang tunnel is located in the PA1 section of Sanming expressway, and the pile number is K1+930~K2+820; The tunnel is located in the low mountain landscape with a thick overburden layer and vegetation grows well. The maximum elevation of the tunnel is 720m, and the ground elevation of entrance and exit is 439.7 m and 328.0 m respectively. The geology of reveal stratum along the tunnel is: slope accumulation gravel, residual cohesive soil, residual sandy clay, highly weathered granite in sand-soil form, weak weathered granulite and weak weathered granite, etc.

The structural form of Zhuyang tunnel is double-tube and separate type. Based on design specification of highway tunnel(JTJ026-90), technical specifications for construction of highway tunnel(JTJ042-94) and other related specification and rule, the monitoring measurement is performed. Mechanical or numerical display convergence instrument is used in the process of monitoring measurement, and the instrument accuracy is 0.01mm. The mechanical convergence instrument is made by Chinese People 's Liberation Army Unit 8902. The numerical display convergence instrument is developed by Beijing Zhongmei Coal Mine Engineering Limited Company of China Coal Research Institute, and the model of which is JSS30A-type.

The convergence of tunnel surrounding rock about type I~III is measured per 10~20m one section, and one section per 20~50m about tunnel surrounding rock of type IV~V. For some special position, the testing section can be densified, and a testing section can be added at the changing position of surrounding rock classification. The arrangement of measuring line is shown in Figure1. The evaluation criterion is based on the examination of convergence rate. When convergence rate >5mm/d, the surrounding rock is in a rapidly changing state; When convergence rate < 0.1~0.2mm/d, the surrounding rock can reach a stable state. The measurement was completed after two weeks of stable convergence deformation. As shown in Table 2, consider convergence data of Zhuyang right tunnel in YK2+860, as an example to investigate.



Construction of whole section method Construction of bench method

Fig. 1. Arrangement drawing of tunnel convergence

Table 2. Original convergence data of Zhuyang right tunnel in YK2+860

Number of monitoring periods	Distance from palm surface (m)	Temperature in tunnel (℃)	Convergence value (mm)
1	55	11.6	5.925
2	60	12.9	6.043
3	65	12.6	6.247
4	70	13.8	6.428
5	70	12.9	6.629
6	70	12.1	6.723
7	75	11.5	6.825
8	78	12.1	6.924
9	80	11.9	6.995
10	80	15.9	7.045
11	80	17.5	7.149
12	80	18.3	7.200

The top nine data of Table 2 is calculated by using GM(1,1), Verhulst and Polynomial models. The simulated result and relative error is shown in Table 3. In this case, time response function of GM(1,1) model is:

$$\hat{x}^{(1)}(k+1) = (5.925 + 293.6207)e^{0.0203k} - 293.6207, k = 1,2,3,\ldots,n$$

Table 3. The top nine data simulation analysis of Zhuyang right tunnel in YK2+860

Observed data	5.925	6.043	6.247	6.428	6.629	6.723	6.825	6.924	6.995
Simulation data of GM (1,1)model	5.925 0	6.1427	6.268 5	6.397 0	6.5280	6.6617	6.798 2	6.9374	7.0795,
Simulation data of Verhulst model	5.925 0	6.1065	6.273 7	6.426 8	6.5662	6.6924	6.806 2	6.9085	6.9999,

Simulation Data of polynomial model	5.8759	6.0764	6.2591	6.4240	6.5711	6.7004	6.8119	6.9056	6.9815
Relative error of GM（1,1）model	0.000	0.0165	0.0034	0.0048	0.0152	0.0091	0.0039	0.0019	0.0121
Relative error of Verhulst model	0.000	0.0105	0.0043	0.0002	0.0095	0.0046	0.0028	0.0022	0.0007
Relative error of polynomial model	0.0491	0.0334	0.0121	0.0040	0.0579	0.0226	0.0131	0.0184	0.0135

Time response function of Verhulst model is:

$$\hat{x}^{(1)}(\mathrm{k}+1)=\frac{0.1394x^{(1)}(0)}{0.0182x^{(1)}(0)+(0.1394-0.0182x^{(1)}(0))e^{-0.1394k}}$$

$\mathrm{k}=1,2,3,\dots,\mathrm{n}$. Polynomial fitting function of the top nine data is: $\mathrm{y}=-0.0089\mathrm{x}^2+0.5298\mathrm{x}-0.7769$, $\mathrm{R}^2=0.9949$. In the function, x is time of observation, and y is convergence value.

From Table 3, it can be seen that the simulation with Verhulst model has the highest accuracy. Average relative error $\bar{\Delta}=\frac{1}{9}\sum_{\mathrm{i}=1}^{9}\Delta_{\mathrm{i}}=0.4337\%$, and the simulation accuracy is level one. Simulation accuracy of single data is close to level one. Average relative error of GM(1,1) simulation $\bar{\Delta}=\frac{1}{9}\sum_{\mathrm{i}=1}^{9}\Delta_{\mathrm{i}}=0.8378\%$, and the simulation precision of GM(1,1) model is only second to that of Verhulst model. Average relative error of polynomial simulation $\bar{\Delta}=\frac{1}{9}\sum_{\mathrm{i}=1}^{9}\Delta_{\mathrm{i}}=2.49\%$,and the simulation accuracy is level two. Because the data exploring is insufficient enough by using polynomial model, the accuracy of polynomial fitting is the worst. The data are close to the saturated S sequence, so the simulation with Verhulst model has better accuracy than that of GM(1,1) model. The top nine data are analyzed to forecast the twelfth data, and the forecasting result of GM(1,1), Verhulst and polynomial model is respectively 7.5236,7.2750 and 7.1024. This is because the simulation precision of Verhulst model is the highest, and thus causing the highest forecasting precision. Based on the calculation of top nine data, the tenth data is forecasted, and the forecasting result of GM(1,1), Verhulst and polynomial model is respectively 7.2246,7.1539 and 7.045. The polynomial model prediction has the highest accuracy obviously, and the relative error of prediction is 0.0007. This shows that the forecasting value of Verhulst model is not always the optimal for a single data. This is because the discreteness of data can bring error.

4 Discussion

The two grey prediction models selected in this paper are representative. Under the condition of few data and poor information, it is necessary to simulate data by using grey models firstly. Based on grey theory, the raw data should meet the quasi-smooth condition, and the 1-AGO sequence should satisfy the pre-exponential law. The grey theory is a kind of method of discovering data from data, and it is widely used in practice. Like any other mathematical model, the grey theory has the scope of application. Beyond the scope of application, it is difficult to acquire ideal results. Take the GM(1,1) model as an example, if the development coefficient is less than 0.3, the simulation precision can reach 98%. If the development coefficient is less than 0.5, the simulation precision can reach 95%. If the development coefficient is larger than 1, the simulation precision is less than 80%. In scope of application, in order to raise forecast accuracy, it is a better way to select the appropriate model. The top nine data of Table 2 is calculated by using discrete GM(1,1) model. The average relative error is $\bar{\Delta}=\frac{1}{9}\sum_{i=1}^{9}\Delta_i = 0.8377\%$. The simulated result and relative error of discrete GM(1,1) model is close to that of GM(1,1) model. This is because the property of discrete GM(1,1) model is similar to that of GM(1,1) model. Furthermore, the development coefficient and raw data does not have the nonnegative restriction. When the raw data is close to index sequence, the simulation result is more precise by using discrete GM(1,1) model. As the time span of simulation is bigger and bigger, it is necessary to metabolize the grey model and give priority to the use of new information to improve the prediction accuracy. The Excel software has characteristics of easily learning and wide popularization range. Despite this, the Excel software is not suitable for large-scale calculation. Compared to the calculation method of Excel, the programming computation is more convenient. In data processing of tunnel monitoring measurement, it is necessary to obtain the optimum predicting accuracy by using different methods, and to select the best method.

5 Conclusion

Main merit of deformation prediction about surrounding rock by using grey model is：Modeling doesn't need a lot of data and treatment method is simple. In general case, when swinging sequences or saturated S-shape sequences are simulated, GM(2,1) model, Verhulst model or discrete Verhulst model can be used. Discrete GM(1,1) model can be considered, when sequence of approximate exponential growth is analyzed. In order to improve precision, residual correction model or other grey optimization model can also be used in practice.

Polynomial order is not the higher the better when convergence data is fitted by using polynomial model. Second degree, three degree or four degree polynomial is more applied in practice of polynomial fitting. If the polynomial order is more than 6, and fitting accuracy is still not enough. At this time, it is necessary to consider other method to simulate. Due to uncertainty and complexity of tunnel deformation simulation, so it is necessary to carry out simulating calculation by many methods, and to avoid error caused by single models, furthermore, field test, finite element simulation or experiment is also needed to verify the simulated results.

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