Simulating the Departure of Egon Passenger Motorship Using Uncertain Max Plus Linear

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Abstract. Sea transportation is able to build connectivity between islands which can have a positive impact on the Indonesian national economy in various sectors. This transportation mode is much demand by the citizen because of its affordable tickets. Moreover, it's easy and guaranteed distribution of goods from one area to another. However, the arrival and departure schedules of passenger ships often experience delays. This causes ship passengers to pile up at the terminal. Delays are generally caused by bad weather conditions, long periods of time for ships to dock, as well as refilling fuel and fresh water stocks. In order to study the departure of systems, a precise mathematical modelling was typically used in this context. However, this modelling has limitations subjected to bounded noise, disturbances and modeling errors. In this paper, we use uncertain Max Plus Linear (uMPL), Max Plus Linear extension dedicated to model and simulate the departure of The Egon passenger motorship. The Egon passenger motorship is a Roll on-Roll off (Ro-Ro) type ship which is managed by PT. Pelayaran Nasional Indonesia (PT. PELNI). We propose a model for the purpose of studying, analyzing, and simulating the departure of The Egon passenger motorship's behaviour. We analyze the model using Power Algorithm to find eigenvalue and eigenvector. First, we use upper bound matrix of the model and any initial vector. The matrix and initial vector used to iterate linear equation of $\boldsymbol{x}(k+1) = A \otimes \boldsymbol{x}(k)$ for any whole number $k \geq 0$. Our results have used to simulate the departure of The Egon passenger motorship.

Keywords: simulation, departure, passenger motorship, uMPL

1 Introduction

1.1 A Subsection Sample

Max Plus Linear (MPL) system is a Discrete Event System (DES) with a continuous state space to represent sequential discrete events [1]. DES is a dynamic system whose transitions are triggered by events and have discrete states. Several examples of DES are water treatment plant [3], hierarchical structure of the semi-double track railway line [7], and train scheduling system [5]. The system can explain events that occur now

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depending on events that occurred previously. MPL system has a max plus algebra structure which displays synchronization using maximization of several events.

MPL's main characteristic is being able to model a specific class of DES with synchronization. The system assumes the matrix entries are fixed. If the entry is not fixed and expressed in the form of interval of values then the system is called as uncertain Max Plus Linear. It indicates that the system may subject to noise and disturbance.

Several works related to uncertain Max Plus Linear (uMPL) are safety verification of the system that modeled using uMPL [4], verification the train scheduling system's security [5], and conditional reachability of uMPL systems [2]. This work is focused on modeling the departure of passenger ship using uncertain Max Plus Linear (uMPL) to simulate it using eigenvalue and eigenvector of the system.

The remainder of this present paper is organized as follows. After some preliminaries concerning uncertain Max Plus Linear (uMPL) and passenger ship, the next section explains some results and discussions concerning the proposed Egon passenger ship modeling methodology. The last section gives the conclutions and some future works.

2 Preliminaries

This section presents briefly the basic concepts related to uncertain Max Plus Linear (uMPL) and passenger ship.

2.1 Max Plus Algebra

Max Plus Algebra is an idempotent semiring with two arbitrary operations, i.e. maximum and addition. Let $R_\varepsilon := R \cup \{-\infty\}$ be a set equipped with two binary operations: sum ($\oplus$) and product ($\otimes$), defined by:

$$a \oplus b := \max\{a, b\}$$
$$a \otimes b := a + b$$

for all a,b $\in R_\varepsilon$. Note that the neutral element w.r.t maximization and addition operators in R_ε is $\varepsilon := -\infty$ and 0, respectively.

2.2 Uncertain Max Plus Linear

We describe the autonomous Max Plus Linear (MPL) Systems and uncertain MPL systems. The autonomous MPL systems is defined as follows:

$$\boldsymbol{x}(k) = \boldsymbol{A} \otimes \boldsymbol{x}(k-1) \quad (1)$$

where $\boldsymbol{A} \in {R_\varepsilon}^{m\times n}$ is deterministic state matrix, variable k respresents the occurrence index, vector x(k) is the time of k-th occurrence of i-th event.

If some entries in the state matrix in (1) depends on k and the entries belong to an interval, then the MPL systems are called uMPL systems. The uMPL systems are defined by:

$$\boldsymbol{x}(k) = \boldsymbol{A}(k) \otimes \boldsymbol{x}(k-1) \quad (2)$$

where $\boldsymbol{A}(k) \in [\underline{A}, \bar{A}]$ is non-deterministic matrix with $\underline{A}$ dan $\overline{A}$ respectively represent upper bound and lower bound matrix. The interpretation of state vector x in uMPL systems is the same with MPL systems.

2.3 Passenger Ship

This work discusses about passenger ship, namely motor ship. There are many motor ships that are managed directly by PT. Pelayaran Nasional Indonesia (PELNI), including: KM Bukit Siguntang, Egon, and Lambelu. One of the routes if these ships is Parepare City. Egon passenger ship is a Roll on-Roll off (Ro-Ro) type ship with a maximum capacity of 544 pax. Egon passenger ship is capable of carrying passengers, cargo, and motorized vehicles. Egon currently serves the Waingapu-Batulicin-Parepare-Bontang-Surabaya-Lembar (Lombok) route.

This work describes the scheduling routes for Egon passenger ship using uncertain Max Plus Linear (uMPL). We took the data from Harbormaster's Office and Port Authority and PT. PELNI in Parepare city. The data obtained is used to calculate the length of travel time and the position of the ship at the reference time (in hours). $x(k-1)$ is matrix that describe the $(\text{k-1})^{\text{th}}$ departure of the ship.

3 Results and Discussions

This section presents briefly how to model the departure of Egon passenger ship using uncertain Max Plus Linear (uMPL). This model gives us eigen value and eigen vector to schedule the departure of the ship.

3.1 Modeling The Departure of Egon Passenger Ship

Egon passenger ship consist of 6 ports and 10 routes. The number of ports, travel routes, and length of ship journey are given in Table 1 below.

Table 1. Egon Passenger Ship Travel Route

From	To	Recorded Travel Time (hours)
Surabaya	Lembar	24-25
Lembar	Surabaya	24-25
Lembar	Waingapu	27-29
Waingapu	Lembar	27-29
Surabaya	Batulicin	29-33
Batulicin	Surabaya	29-33
Batulicin	Parepare	23-24
Parepare	Batulicin	23-24
Parepare	Bontang	27-28
Bontang	Parepare	27-28

Based on Table 1 above, Egon's route model is presented as follows:

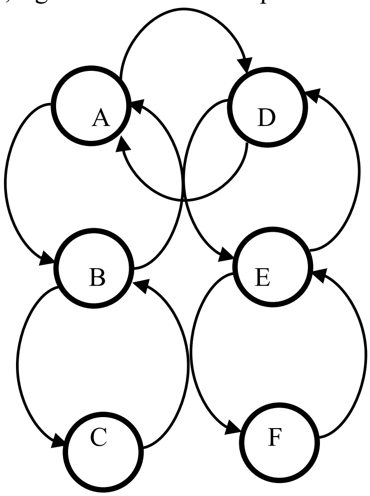

Fig. 1. Egon Passenger Ship's Route

where:

A : Surabaya	D : Batulicin
B : Lembar	E : Parepare
C : Waingapu	F : Bontang

The uMPL model for Egon passenger ship's travel route is as follows:

$$\boldsymbol{x}(k) = B(k) \otimes \boldsymbol{x}(k-1) \tag{3}$$

with $\boldsymbol{x}(k) = \left(x_1(k), x_2(k), \cdots, x_6(k)\right)'$ and

$$B(k) = \begin{bmatrix} \varepsilon & [24,25] & \varepsilon & [29,33] & \varepsilon & \varepsilon \\ [24,25] & \varepsilon & [27,29] & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & [27,29] & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ [29,33] & \varepsilon & \varepsilon & \varepsilon & [23,24] & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & [23,24] & \varepsilon & [27,28] \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & [27,28] & \varepsilon \end{bmatrix}$$

All entries in B(x) above are in the form of intervals as in b_{12}=[24,25]. This indicates that all travel routes on Egon always disturbed which cause delays, such as on the Lembar-Surabaya route, the route takes between 24 and 25 hours.

3.2 Analyzing The Model

This subsection discusses about analysis of uMPL to determine the departure period of Egon passenger ship. The results are used to develop a more optimal and periodic the departure of Egon's schedule. The departure period of passenger ship is obtained by determining the eigenvalue and eigenvector of uMPL model which was discussion in the previous subsection.

We calculate the eigenvalue and eigenvector using Power algorithm [5]. The uMPL model analyzed using an upper bound matrix. The eigenvalue is calculated to find the period of the novel passenger ship's schedule system while the eigenvector is used to find the start time of service so that the new schedule service is more periodic and optimal. The steps of Power Algorithm are explained as follows:

1. Determine any initial state vector $\boldsymbol{x}(0) \neq \varepsilon$
2. Iterate Equation (3) to obtain the any integer $a > b \geq 0$ and real number c which satisfies $\boldsymbol{x}(a) = c \otimes \boldsymbol{x}(b)$
3. Calculate eigenvalue using:

$$\lambda = \frac{c}{a-b}$$

4. Calculate eigenvector using:

$$\boldsymbol{v} = \oplus_{i=1}^{a-b} \left(\lambda^{\otimes(a-b-i)} \otimes \boldsymbol{x}(b+i-1) \right)$$

In this work, we use initial state vector $\boldsymbol{x}(0) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$ and upper bound matrix of Egon motorboat system obtained in Equation (1), namely:

$$B^* = \begin{bmatrix} \varepsilon & 25 & \varepsilon & 33 & \varepsilon & \varepsilon \\ 25 & \varepsilon & 29 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 29 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 33 & \varepsilon & \varepsilon & \varepsilon & 24 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 24 & \varepsilon & 28 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 28 & \varepsilon \end{bmatrix}$$

The initial state vector $\boldsymbol{x}(0)$ indicates that the respective ship's initial departure route is available at minute 0. The system is assumed to start scheduled according to the given $\boldsymbol{x}(0)$ value. We substitute the initial state vector value into Equation (1) for k=1, we have:

$$\boldsymbol{x}(1) = \begin{bmatrix} 33 \\ 29 \\ 29 \\ 33 \\ 28 \\ 28 \end{bmatrix}$$

We iterate Equation (3) for k=0, 1, 2, ⋯. We have the following change of state as follows:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 33 \\ 29 \\ 29 \\ 33 \\ 28 \\ 28 \end{bmatrix}, \begin{bmatrix} 66 \\ 58 \\ 58 \\ 66 \\ 57 \\ 56 \end{bmatrix}, \begin{bmatrix} 99 \\ 91 \\ 87 \\ 99 \\ 90 \\ 85 \end{bmatrix}, \begin{bmatrix} 132 \\ 124 \\ 120 \\ 132 \\ 123 \\ 118 \end{bmatrix}$$

The change of state obtained shows that we found integers $a = 4$ and $b = 3$ and real number $c = 33$ that satisfy:

$$\boldsymbol{x}(4) = c \otimes \boldsymbol{x}(3)$$

$$\begin{bmatrix}132\\124\\120\\132\\123\\118\end{bmatrix} = 33 \otimes \begin{bmatrix}99\\91\\87\\99\\90\\85\end{bmatrix}$$

According to a, b, and c, we have:

$$\lambda = \frac{c}{a-b} = \frac{33}{4-3} = 33$$

The eigen value $\lambda = 33$ indicates that the departure period for each ship is 33 hours. We use this eigenvalue to obtain eigenvector, namely:

$$\boldsymbol{v} = \bigoplus_{i=1}^{a-b} \left(\lambda^{\otimes(a-b-i)} \otimes \boldsymbol{x}(3+i-1)\right)$$

$$\boldsymbol{v} = \bigoplus_{i=1}^{1} \left(\lambda^{\otimes(1-i)} \otimes \boldsymbol{x}(3+i-1)\right)$$

$$\boldsymbol{v} = [99 \quad 91 \quad 87 \quad 99 \quad 90 \quad 85]^T$$

Based on the eigenvalue and eigenvector obtained, we assumed that Egon passenger ship will start departing on Oktober 06th 2023 at 07.00. The design of Egon passenger ship's schedule is presented in the following Table 2.

Table 2. The New Schedule Egon Passenger Ship

From	Period 1	Period 2	Period 3
Surabaya	21.00 October 06th, 2023	06.00 October 08th, 2023	15.00 October 09th, 2023
Lembar	13.00 October 06th, 2023	22.00 October 07th, 2023	07.00 October 09th, 2023
Waingapu	09.00 October 06th, 2023	18.00 October 07th, 2023	03.00 October 09th, 2023
Batulicin	21.00 October 06th, 2023	06.00 October 08th, 2023	15.00 October 09th, 2023
Parepare	12.00 October 06th, 2023	21.00 October 07th, 2023	06.00 October 09th, 2023
Bontang	07.00 October 06th, 2023	16.00 October 07th, 2023	01.00 October 09th, 2023

4 Conclussions

We have described a method for modeling the departure of Egon passenger ship using uncertain Max Plus Linear (uMPL). Our model is $\boldsymbol{x}(k) = B(k) \otimes \boldsymbol{x}(k-1)$ where $B(k)$ is an 6×6 matrix and $\boldsymbol{x}(k) = \left(x_1(k), x_2(k), \cdots, x_6(k)\right)^T$. This system has an eigen value $\lambda = 33$ and eigen vector

$\boldsymbol{v} = [99 \quad 91 \quad 87 \quad 99 \quad 90 \quad 85]^T$. We use eigen value and eigen vector to develop a more optimal and periodic the departure of Egon passenger ship's schedule. Further research issues related to this work include estimation bounded error state and reachability or conditional reachability analysis of the system.

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