



Study on Mathematical Models of Grid Turbulence

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Abstract. The process of sediment initiation is sometimes caused by high-intensity turbulence. Grid turbulence can effectively simulate the sediment movement under such special flow conditions. In this paper, the governing equations of the turbulence model are determined based on the large eddy simulation model of two-fluid spatial filtration of the level set. The level set model is established, and the fluctuation of the free water surface is tracked by the level set function method. The density and viscosity of the whole computational domain are set as constant values. The single-fluid equation is reconstructed to solve the level set convection equation. The viscous sediment model is integrated into the SCHISM simulation system. The SCB model is used to deal with the fluid-structure interaction problem in the flocculating and settling process of fine sediment. The split-step method is used to solve the fluid equation, which is finally solved, and the numerical simulation results are output. The results show that the mathematical model can simulate the flow velocity field and suspended sediment concentration field well under grid turbulence.

Keywords: Grid turbulence, Suspended sediment concentration, Numerical simulation.

1 Introduction

Grid turbulence is generally known as approximately isotropic turbulence, which is the basis for studying complex turbulence and related material transport ^[1]. Oscillating Grid Turbulence (OGT) refers to the vibration of a single or multi-slice grid in the water tank perpendicular to the plane of the grid. The jet and wake are generated at the opening and connection of the grid, respectively. After mixing with each other, the two produce approximately isotropic turbulence at a certain distance from the grid ^[2]. The characteristics of OGT are easy to change through grid operating parameters, including grid opening size, amplitude, frequency, etc. They can be maintained for a long time without attenuation. They are easy to implement measurements and widely used.

In nature, the start and suspension of sediment are sometimes caused by high-intensity turbulence rather than average flow. The grid turbulence can effectively simulate the sediment movement under such special flow conditions. In addition, grid turbulence can be separated to study the turbulent diffusion of sediment and pollutants and the

adsorption and desorption of inorganic matter by sediment. Therefore, although the formation mechanism of indoor vibration grid turbulence is different from that of natural turbulence, it is widely used to study the movement of sediment and pollutants. Using OGT, Rouse^[3] first studied the characteristics of sediment suspension and obtained the famous Rouse vertical line distribution formula of suspended mass. Other scholars have studied and obtained the key mechanisms of sediment transport, such as the sediment starting criterion and the relationship between sediment diffusion coefficient and momentum exchange coefficient^[4-9]. Therefore, we use the parameters of the adjusted grid to conduct the mathematical model study of the suspended sediment concentration field. We simulate the suspended sediment concentration field under different working conditions.

2 Model fundamentals

Based on the large eddy simulation (LES) model of two-fluid spatial filtration, the governing equation of the turbulence model is determined. The level set model is established, and the fluctuation of the free water surface is tracked by the level set function method. The density and viscosity of the whole computational domain are set as constant values. The above equation is reconstructed as a single fluid equation to solve the level set convection equation. At the same time, the viscous sediment model is integrated into the SCHISM simulation system. The SCB model^[10] is used for the flocculation and settlement process of fine sediment. The immersed boundary method is used to deal with the fluid-structure interaction problem, and the split-step method is used to solve the fluid equation. The calculation efficiency is improved through MPI parallelization. Finally, the turbulence field and suspended sediment concentration field under the disturbance of the output grid are simulated.

Based on the large eddy simulation model of two-fluid spatial filtration with the level set, the governing equation of the turbulence model is constructed. The exchange of sediment particles between the groups of flocculants adopts the two-body interaction mode. It is controlled by the polymerization, shear breakage, and collision breakage of the flocculants, resulting in the capture and loss of sediment particles. The general equation is as follows:

$$\begin{aligned} \frac{dn_k}{dt} = & G_{\text{aggr}}(k) + G_{\text{break-shear}}(k) + G_{\text{break-call}}(k) \\ & - L_{\text{aggr}}(k) - L_{\text{break-shear}}(k) - L_{\text{break-call}}(k) \end{aligned} \quad (1)$$

n_k is the number of sediment particles in the flocculation group (m-3); G_{aggr} and L_{aggr} are the number of sediment particles captured and lost due to flocculation and agglomeration (m-3); $G_{\text{break-shear}}$ and $L_{\text{break-shear}}$ are the number of sediment particles captured and lost due to shear crushing (m-3); $G_{\text{break-call}}$ and $L_{\text{break-call}}$ are the number of sediment particle capture and loss caused by collision and crushing flocculation (m-3), respectively.

The split-step method was used to solve the fluid control equation; the momentum equation was discretized with the second-order precision central difference scheme, in-

cluding viscosity, pressure gradient, and SGS terms; the convection terms were discretized with the third-order precision WENO scheme or the second-order central difference scheme; and the second-order precision Crank-Nicholson scheme was used for time advance:

$$\frac{1}{J} \frac{U^* - U^n}{\Delta t} = P(p^n, \phi^n) + \frac{1}{2} [F(U^*, u^*, \phi^{n+1}) + F(U^*, u^*, \phi^n)] \quad (2)$$

n represents the previous time step, Δt is the time step, F is the right-hand side term except for the pressure term in the control equation of the turbulence model, and P is the pressure term.

A three-point central difference scheme is used to apply continuity conditions in the second stage of the splitting step to solve the following Poisson pressure equation:

$$-J \frac{\partial}{\partial \xi^i} \left[\frac{1}{\rho(\phi)} \frac{\xi^i}{J} \frac{\partial}{\partial \xi^j} \left(\frac{\xi^j \Pi}{J} \right) \right] = \frac{1}{\Delta t} J \frac{\partial U^{i*}}{\partial \xi^j} \quad (3)$$

Π is the pressure correction term for the pressure and velocity fields in Equation (2):

$$p^{n+1} = p^n + \Pi \quad (4)$$

$$U^{i,n+1} = U^{i,n} - J \Delta t \frac{1}{\rho(\phi)} \frac{\xi^i}{J} \frac{\partial}{\partial \xi^j} \left(\frac{\xi^j \Pi}{J} \right) \quad (5)$$

3 Model settings

The intrusion boundary method uses the grid as a solid and the water in the tank as a fluid. The pore ratio of the grid is consistent with the condition of producing approximately isotropic turbulent flow. The local encrypted mesh can reflect the shape and spacing of the local grid holes. The LES method is applied to simulate the approximate isotropic turbulent field generated by the grid oscillation. The grid moves in the form of a sinusoidal function in the transverse direction:

$$L = A \cdot \sin(f \cdot 2\pi t) \quad (6)$$

A is the amplitude (cm), f is the vibration frequency (Hz), t is the time (s), and L is the grid position (cm).

By adjusting the vibration frequency and amplitude of the grid, a variety of physical model test conditions can be generated to realize the effective simulation of the turbulent field and the suspended sediment concentration field under the grid disturbance. First, the mathematical model is set up according to the tank test, including initial and boundary conditions. The test conditions of the physical model test are as follows: the configuration concentration is 15 kg/m^3 , the grille amplitude is 5 cm, and the vibration frequency is 5 Hz. In the end, the average measured concentration was 14.921 kg/m^3 . The specific measurement results are shown in Table 1. Therefore, the initial conditions of turbulent flow and suspended sediment concentration field in the numerical simulation are set to 0 m/s and 15.0 kg/m^3 , respectively. They can accelerate the convergence

of turbulent flow and suspended sediment concentration to a full development state. Since it is the test of the cube water tank and transverse vibration grid, the boundary conditions are set according to the periodic boundary.

Table 1. Measurement results of test conditions

Sampler position	Volume of sampling/ml	Filter mass/g	membrane	Total mass/g	Sample mass/g	sand Concentration / $\text{kg}\cdot\text{m}^{-3}$
A ₁	65	0.1106		161.7438	0.9868	15.182
A ₂	64	0.1106		161.8908	0.9524	14.813
A ₃	67	0.1104		162.4816	1.0016	14.949
A ₄	62	0.1115		161.4572	0.913	14.725

4 Model export

After at least one minute of numerical simulation, conditions of turbulent flow and suspended sediment concentration fields tending to stochastic processes will be generated. According to the implementation of the physical model test, the location of one of the observation points A was selected to extract the time series process of the mean value at the time of flow velocity and the mean value at the time of suspended sediment concentration. This is shown in Fig. 1. It can be observed from the numerical simulation results that the longitudinal flow velocity u fluctuates in the range of $0.0\sim 0.10$ m/s, and the transverse flow velocity v fluctuates in the range of $-0.1\sim +0.1$ m/s (Fig. 1(a)). The frequency and amplitude of flow velocity change are consistent with the vibration frequency and amplitude of the transverse grating. However, the concentration change of suspended sediment is completely a random process, which is affected by the approximately isotropic turbulent flow field. The concentration fluctuates around 14.9 kg/m^3 (Fig. 1(b)), which is consistent with the measured average value of the physical model test condition. It is proved that the numerical simulation can reproduce the formation process of turbulent flow and suspended sediment concentration field in the tank test.

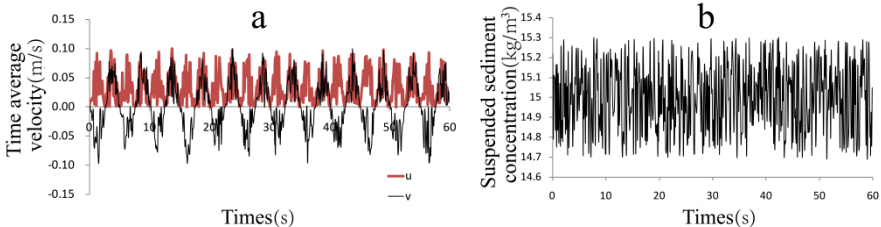


Fig. 1. Change process at measuring point A of the water tank. (a) Average velocity change process. (b) The change of suspended sediment concentration.

Fig. 2 shows the flow velocity vector in the tank, the iso-line filling and iso-surface distribution of the mean value of longitudinal flow velocity, and the concentration field distribution of suspended sediment, respectively. As shown in Fig. 2(a), vortices with symmetric distribution will be formed near the grating gap. Due to the use of periodic

boundary conditions, the interaction between the water flow and the side wall of the tank is not reflected at the side wall, as well as the vortex backflow at the corner. Due to the fine suspended sediment particles, about $10\sim 15\ \mu\text{m}$, the sediment particles and the fluid have strong followability. The fine sediment will have flocculation and sedimentation, and the interaction between the flocculation group and the fluid turbulence will lead to the aggregation and fragmentation of the flocculation group. Finally, the random variation of the suspended sediment concentration turbulence field is formed, as shown in Fig. 2(d). The change of suspended sediment concentration is related to the spatial and temporal distribution of turbulent flow. It can result in the shown distribution pattern of size and phase, the average value of which is about $14.9\ \text{kg/m}^3$. It is consistent with the measured average value of the physical model test.

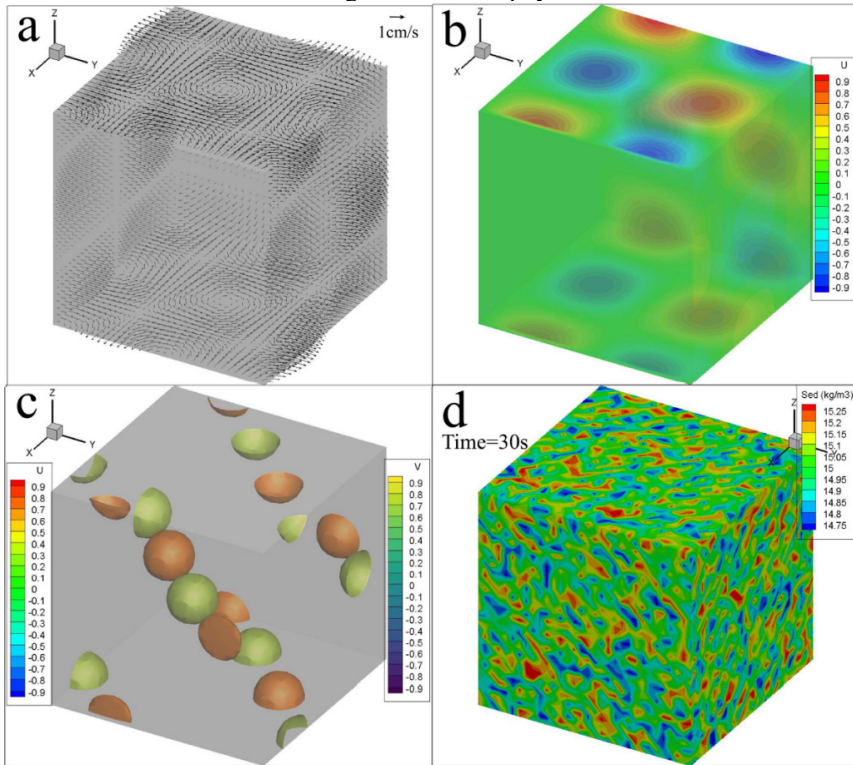


Fig. 2. Results of mathematical model simulation. (a) Velocity vector diagram of the turbulent flow field in the grid. (b) Contour filling diagram of longitudinal velocity U . (c) Contour distribution of longitudinal velocity U . (d) Instantaneous field of suspended sediment concentration ($T=30\ \text{s}$).

5 Conclusions

According to the analysis of the output results of the above model, it can be seen that

the longitudinal flow velocity u fluctuates in the range of 0.0~0.10 m/s, and the transverse flow velocity v fluctuates in the range of -0.1~+0.1 m/s. The frequency and amplitude of flow velocity change are consistent with the vibration frequency and amplitude of the transverse grating. The change of suspended sediment concentration is related to the spatial and temporal distribution of turbulent flow, resulting in the distribution pattern of different sizes, as shown in Fig. 3 (d). The average value is about 14.9 kg/m³, which is consistent with the measured average value of the physical model test. Therefore, the mathematical model can simulate the flow velocity field and suspended sediment concentration field under the grid turbulent flow well. The mathematical model can also be set as a multi-slice grid. The motion of the vibrating grid directly determines the hydrodynamic conditions in the tank. The numerical simulation results can reflect the local details of the turbulent flow and the concentration distribution of suspended sediment in the tank.

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