Deflection analysis of two-span reinforced concrete continuous beam under load

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Abstract. The deflection of continuous beam is an important problem in engineering application and theoretical calculation. In this paper, the finite difference method is used to calculate the deflection of the two-span reinforced concrete continuous beam under load. The deflection difference equation of the beam element is used to replace the deflection differential equation of the beam to solve the deflection of the beam. Under the assumption that the continuous beam is continuous at the support, the Abaqus finite element calculation program is used to assist the analysis, and the obtained deflection is compared with the analytical solution of the deflection differential equation of the beam. The research shows that the deflection of the beam calculated by the finite difference method has a small error compared with the real value. The more the number of elements, the smaller the error. The finite difference method preliminarily uses the principle of finite element application to analyze the mechanics of the beam, introduces the corresponding relationship, and carries out the discretization analysis. The calculation process is simple and clear, and has certain advantages compared with the traditional calculation method. The finite element theoretical calculation method is one of the more advanced calculation methods.

Keywords: Reinforced Concrete Continuous Beams, Finite Difference Method

1 Introduction

The reinforced concrete beam bridge is the most widely used bridge in the world. It has the advantages of local materials, good durability, good integrity, good moldability and good fire resistance. With the development of bridge engineering, long-span bridges have become the mainstream of construction. The continuous beam has strong stiffness, small beam deformation, which can reduce the stress at the joint, and it has higher bending moment durability, which can completely divide the distributed load on all supports. Reinforced concrete continuous beam has always been a hot topic of research. However, for the deflection calculation of reinforced concrete beams, most of them are based on mechanical analysis. The traditional structural analysis will be inconvenienced by the internal force calculation of the statically indeterminate structure of the bridge, and the use of finite element method to analyze, can quickly and easily solve the problem of deflection calculation. In this paper, the deflection of the left span of the

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double span reinforced concrete continuous beam under the condition of uniform load is analyzed by the finite difference method. The finite difference method divides the structural model with complex shape and complex internal force into finite elements. After analyzing the stress of some elements, the overall situation is obtained. The finite difference method is suitable for a variety of structures and plays an important role in structure and force analysis. In view of the fact that manpower cannot calculate the complex data, this experiment will use Abaqus software to assist the calculation and comparison. As a mature finite element analysis program, it can effectively enhance the calculation efficiency [1].

2 Literature review

The integral method and superposition method described in the university textbook ' Structural Mechanics ' are used to solve the bridge deflection, which is suitable for the statically determinate beam with relatively simple structure, but it is difficult to analyze the statically indeterminate structure of continuous beam [2]. Many scholars have conducted extensive research on the deflection of reinforced concrete continuous beams. Yin Zhaoyun proposed the finite difference method to calculate the deflection of statically determinate beams and statically indeterminate beams [3]. Gu Huiruo regards the reinforced concrete continuous beam as a beam with variable stiffness along the longitudinal axis, and proposes the deflection coefficient tables of the second-order and third-order variable stiffness beams [4]. Zhao Lidu uses ground three-dimensional laser scanning to analyze bridge deflection deformation, which is a bridge deflection deformation analysis method based on point cloud data [5]. Liu Xiaoyao proposed the use of spline function to calculate structural mechanics problems [6]. The quadratic spline finite element method is used to calculate the deflection of the beam. In this paper, the finite difference method is used to calculate the deflection of the two-span reinforced concrete continuous beam. Compared with the traditional integral and superposition method to calculate the deflection of the beam, the finite difference method is not cumbersome and concise. The deflection coefficient table is less applicable, and it is mostly used for the stress situation under special circumstances. For the error problem of finite difference method, dividing the beam element into smaller elements can reduce the error first.

3 Methodology

The finite difference method is a numerical method for solving partial differential equations [7]. The basic idea is to discretize the continuous partial differential equations into discrete algebraic equations, and then obtain the numerical solution by solving the algebraic equations. In the finite difference method, the solution region is divided into a series of grid points or regions, and then the derivative or second derivative of the original equation is approximated at each grid point to obtain a discrete equation set. Usually, the derivative representation methods include the central difference method, the forward difference method and the backward difference method.

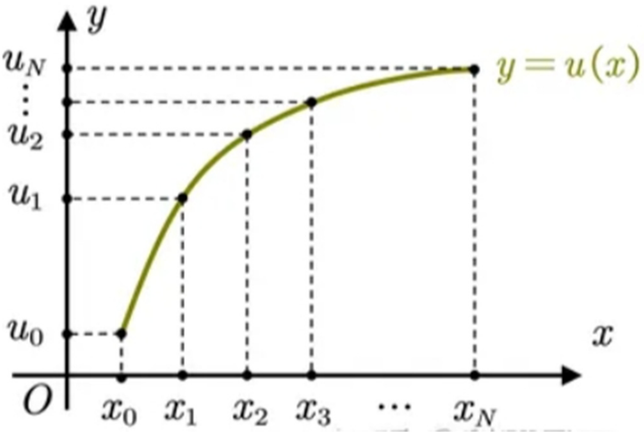

Fig. 1. Discrete representation of a one-dimensional interval

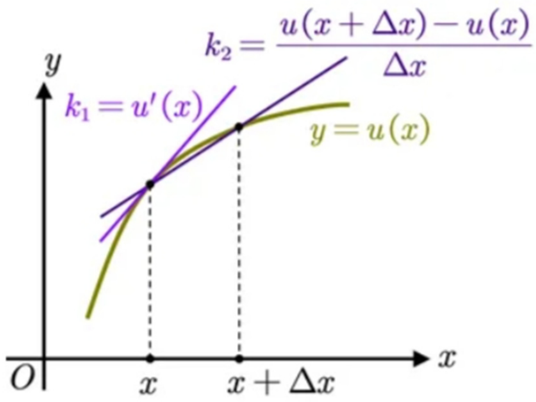

Fig. 2. An approximate representation of differentiation.

Figure 1 shows the discrete representation of a function in one dimensional space of an interval. As shown in Figure 2, representation of first and second derivatives in finite difference method. For the first derivative of u (x) at x2, the central difference method is used. $\frac{dy}{dx} \approx \frac{u_3 - u_1}{2\Delta x}$, Approximately equal to the slope of the secant connecting u2 and u0.

For the second derivative of u (x) at x2, the central difference method is used.

$$\frac{d^2y}{dx^2} \approx \frac{\frac{u_3-u_2}{\Delta x} - \frac{u_2-u_1}{\Delta x}}{\Delta x} = \frac{u_3 - 2u_2 + u_1}{\Delta x^2} \tag{1}$$

its second derivative $\frac{dy}{dx}$ represent the rate of change of the first derivative, so at the x2 point, it can be approximately obtained by subtracting the slope of x1 from the slope of x3, and then dividing it by the length of its section. A function image is divided into finite elements on average. By enumerating the first derivative or second derivative of each element, the coefficients are extracted by matrix method, and each unknown quantity is calculated by Gauss elimination method.

In this experiment, the differential equation of the beam deflection curve is approximately replaced by the difference equation of the beam deflection curve. In mathematics, the calculation formula of curvature is as follows:

$$k=\frac{\frac{d^2\omega}{dx^2}}{\left(1+\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}} \tag{2}$$

When the deformation of the beam is in the elastic range, the deflection ω is small, and the tangent of the deflection is still close to the straight line, so ω' is very small relative to 1 and can be ignored. Therefore, k ≈ ω. The deflection curve differential equation of the beam is

$$\omega'' = -\frac{M(x)}{EI} \tag{3}$$

The differential expression corresponding to the second derivative,

$$\frac{u_{x+1}-2u_x+u_{x-1}}{\Delta x^2} = -\frac{M(x)}{EI} \tag{4}$$

$$\text{Simplify to} \quad u_{x+1} - 2u_x + u_{x-1} = -\Delta x^2 \frac{M(x)}{EI} \tag{5}$$

The beam is divided into a finite number of elements, substituted into equation 5, and a finite number of finite difference equations are written. Using the boundary conditions, the coefficients before u_x are extracted and the result vectors are combined to form an augmented matrix. The Gauss elimination method is used to solve the deflection of each point.

4 Abaqus modeling simulation

In the continuous beam bridge, the bridge span structure and the bearing system of the bridge are not completely hinged, nor are they rigid joints in the strict sense. The bearing in the middle of the continuous beam can transmit the bending moment, and the position of the beam-column joint is only constrained. Vertical displacement does not constrain rotation. It is assumed that the middle support of the continuous beam is hinged, only the vertical displacement constraint, no bending moment release, and the beam is continuous at the support position [8].

1. Establish the model of two-span reinforced concrete beam. The model of two-span reinforced concrete continuous beam is established in Abaqus. The model uses ordinary carbon steel (elastic modulus of 2E + 11 Pascal) and C30 concrete (elastic modulus of 3.4401E + 10 Pascal).

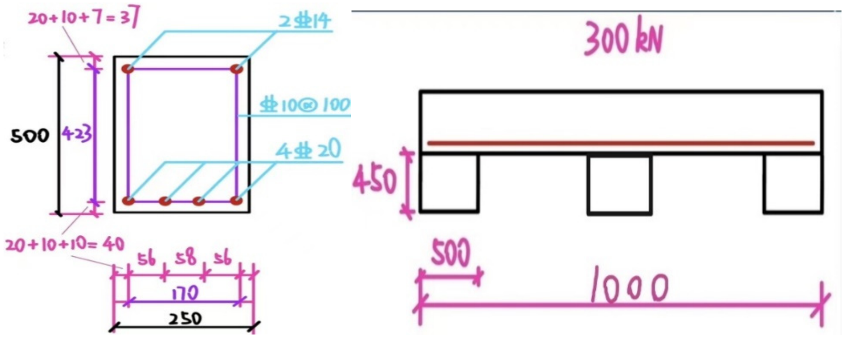

Fig. 3. Basic parameters of bridge model, side view and front view

In the Figure 3, the basic structure and material dimensions of the bridge are shown, as well as the overall front view. Create solid element pads, stirrups, beams, and full-lengths 14 and 20 in the component according to the specified dimensions, and then create the material, concrete, specified, C30, material behavior, selected density, elasticity, and concrete damage plasticity. Then create the material steel, select the density, elasticity and plasticity, and fill in the values. Create different sections for components of different materials and assign sections separately. After that, the component assembly is carried out, the example is created, the three pads and a beam are applied, and the pads are assembled at the two ends and the middle of the beam respectively. After merging into a whole, the concrete is hidden as a whole. Create a reinforcement cage, the upper reinforcement of the beam is two long 14, the lower reinforcement is four usually 20, the length is 9.9 meters, a total of 100 stirrups, evenly distributed on the long reinforcement.

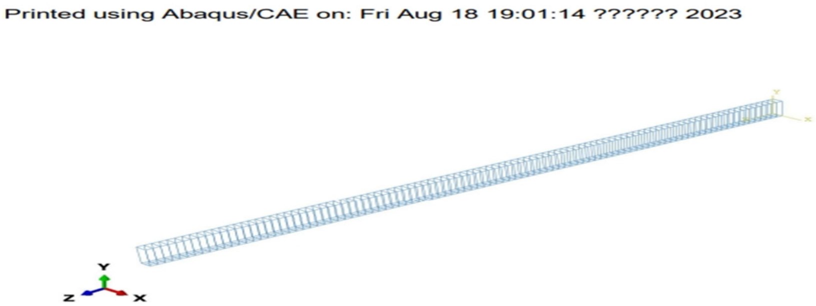

Fig. 4. Steel cage model

Figure 4 shows the constructed steel cage model. The reinforcement cage is placed in a suitable position of the whole concrete, and the reinforcement cage is 50 mm from the edge of the concrete. Create an analysis step, select static, general. The maximum number of incremental steps is changed to 1e + 6, the initial size of incremental steps is 1, and the minimum is 1e-6. The interaction selects the entire steel cage as the built-in area of the concrete. After that, the boundary condition displacement and rotation angle are created. The bottom surface of the three pads is the boundary condition surface. The bottom of the pads on both sides is selected as U1, U2, U3, UR1, and the middle pad is selected as U2. It only provides the displacement constraint in the vertical direction and does not change the continuity of the whole beam.

The boundary conditions are selected to complete the creation of the load, select the resultant force option in the pressure, select the upper surface of one of the spans of the continuous beam, and apply a force of 300000N. The model is divided into grids with an approximate global size of 0.1. Create a job and submit the operation [9].

5 Result

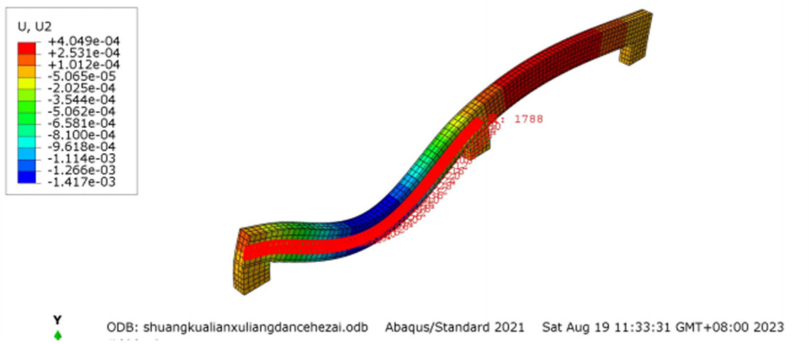

Fig. 5. Model deflection effect diagram

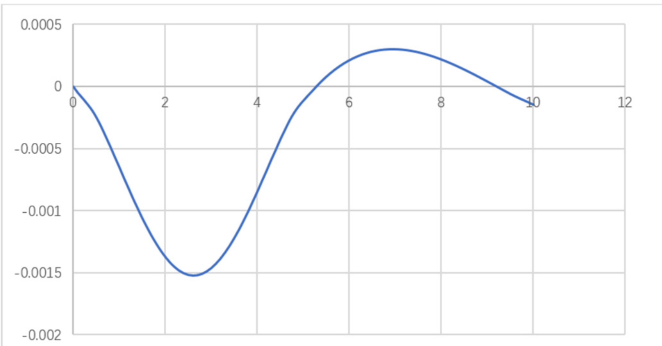

Fig. 6. Abaqus simulation deflection curve

Figure 5 shows the displacement of the continuous beam in the vertical direction after the model is established and calculated by the Abaqus program. The displacement is represented by different colors. Create a path that approximates the midline displacement of the beam to the displacement of the beam in the vertical direction. Figure 6 shows the deflection curve of the beam drawn after extracting the deflection data of the continuous beam.

Table 1 shows the different deflections of continuous beams under this stress state and their differences. They are the analytical solution of deflection, the finite difference decomposition and the software calculation solution respectively.

Table 1. Deflection calculation results and relative error of continuous beam

A	X	ω Analytic Solution Y	ωdifference Decomposition (x_A)	ω Software Simulation Solution ω_A	$Y-\omega(X_A)$	$Y-\omega_A$
1	0.5	-0.0004691	-0.0004153	-0.0002475	-0.0000538	-0.0002216
2	1.0	-0.0008892	-0.0007808	-0.0006486	-0.0001084	-0.0002406
3	1.5	-0.0012210	-0.0010570	-0.0010596	-0.0001640	-0.0001614
4	2.0	-0.0014358	-0.0012153	-0.0013703	-0.0002205	-0.0000655
5	2.5	-0.0015155	-0.0012378	-0.0015155	-0.0002777	0.0000000
6	3.0	-0.0014528	-0.0011168	-0.0014601	-0.0003360	0.0000073
7	3.5	-0.0012507	-0.0010383	-0.0012176	-0.0002124	-0.0000331
8	4.0	-0.0009232	-0.0008557	-0.0008450	-0.0000675	-0.0000782
9	4.5	-0.0004946	-0.0004682	-0.0004251	-0.0000264	-0.0000695
10	5.0	-0.0000000	-0.0000000	-0.0001132	0.0000000	0.0001132

6 Analyzed and discussed

In this experiment, the middle support system of continuous beam is regarded as hinge support, and the beam is regarded as a continuous beam with constant stiffness. According to the Code for Design of Concrete Structures, Article 531 of (GBJ10-89) stipulates that in equal-section members, it can be assumed that the stiffness in the same bending moment section is equal, and the stiffness at the maximum bending moment in the section is taken [10]. That is, the minimum stiffness method. Among them, the side span and the middle span of the continuous beam are regarded as second-order and third-order variable stiffness beams, respectively. This paper has not yet been involved, but the finite difference method may become one of the methods to solve this problem.

7 Conclusion

1. The calculation results show that the finite difference method can be used to calculate the deflection of continuous beams under load. Through comparative analysis, it is found that the beam is divided into more units, and the calculation results are closer to the actual situation. The coordinates of the discrete points are calculated, and the coupled curves are closer to the actual deflection differential equation curve, and the accuracy is higher.

The infinite difference method uses the application principle of finite element to simplify the complex linear partial differential equation into the integral problem of function, which saves the calculation cost and the result is more accurate. The finite element calculation method has certain progressive significance.

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