Peak Flow Frequency Analysis of the Orkhon River, Mongolia

Battsengel Dashdorj

Erdenet Institute of Technology, Mongolian University of Science and Technology
battsengel@erdenetis.edu.mn

Abstract. At-site peak flow analysis is an important assessment used for estimation of peak flow magnitude in various return periods at a given location. In this paper, peak flow frequency analysis is carried out at Orkhon hydro-station in the Orkhon River of Mongolia based on observed daily flow data between 1945-2014. The analysis evaluates four most commonly used probability distributions namely Lognormal, Gamma, Log Pearson Type III and Gumbel Max for best reflecting historical flow data and estimation of maximum discharges. Kolmogorov-Smirnov and Anderson-Darling goodness of fit tests at 5% significance level were used for selecting best-fitting distribution at this site. Results indicate that Gamma was the best-fitted distribution based on goodness of fit test ranking. Using the Gamma distribution, peak flow magnitudes in 2- to 100-year return periods were estimated and historical peak flow magnitudes was evaluated for the return periods.

Keywords: Peak flow, Best-fitting distributions, Orkhon River

1. Introduction

A flood is excess water that flows from rivers and lakes, that submerges flood prone areas and surrounding land. Floods or peak flows are one of the most common natural disasters in Mongolia that cause loss of lives, damage in engineering infrastructure, agricultural properties and environment - all make destructions to the economy. The estimation and prediction of peak flow magnitudes at various return periods is helpful for safety of local roads, railways, bridges, spillways and culverts that might save humans and livestock during flooding.

Probabilistic distributions are frequently applied to the historical peak flow data to predict future scenarios [1]. The lack of measured streamflow data of rivers makes it difficult to determine maximum flow events through identifying flow magnitude ranges in local watersheds. This problem has inspired decades of worldwide research dedicated to develop methods for estimation and prediction of peak flows at ungauged basins [2].

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The choice of an appropriate probability distribution and parameter estimation method is important to predict peak flows at a specific location in the river watershed [3].

Frequency analysis of peak flows is significant for flood risk assessment and mitigating flood hazard impact in areas [7, 12] select location and designing hydraulic structures [8], and manage local water resources i.e. able to predict potential maximum water availability in water scarce areas [13].

Several previous studies have done by hydrology and peak flow analyses in Mongolia. Even though, the application of probability distribution functions for assessing the quality of fit of statistical and empirical distributions is not well documented in previous studies in the basins of Mongolia.

Recent climate conditions increase the risk of short heavy rainfalls and flash flooding in the Orkhon River watershed in 1990-1994 and 2011-2012 that cause more threats to human and properties in an area [14]. Knowing there are research gaps, demand for practical application to determine peak flows at major rivers and tributary streams in Mongolia. This paper aims to identify best-fitting distribution out of evaluated four distributions and to calculate peak flow magnitudes in 2, 5, 10, 25, 50, and 100 year return periods at particular site.

2. Study Area and Peak Flow Data

The Orkhon valley is unique because of its geology, climate and central location in the country; and historical evidence shows humans settled here from the Bronze Age, during Hunnu time, and Great Mongolian Empire. Geographical coordinates of the origin of the Orkhon River are 101°20’13” E, 40°03’07” N and confluence to the Selenge River is at 106°08’55” E, 50°14’42” N. The Orkhon runs 1124 km to outlet, which makes it the longest river in Mongolia and drains an area of 53,500 square kilometers (km²) exclusive of the Tuul River watershed (**Fig. 1**). The watershed includes all and/or most parts of Arkhangai, Orkhon, Bulgan, Selenge, and Uvurkhangai aimags and their 39 soums in northern part of Mongolia (**Fig. 1**). More than seven percent of Mongolia’s total population (235,600 people) live in this watershed (2010).

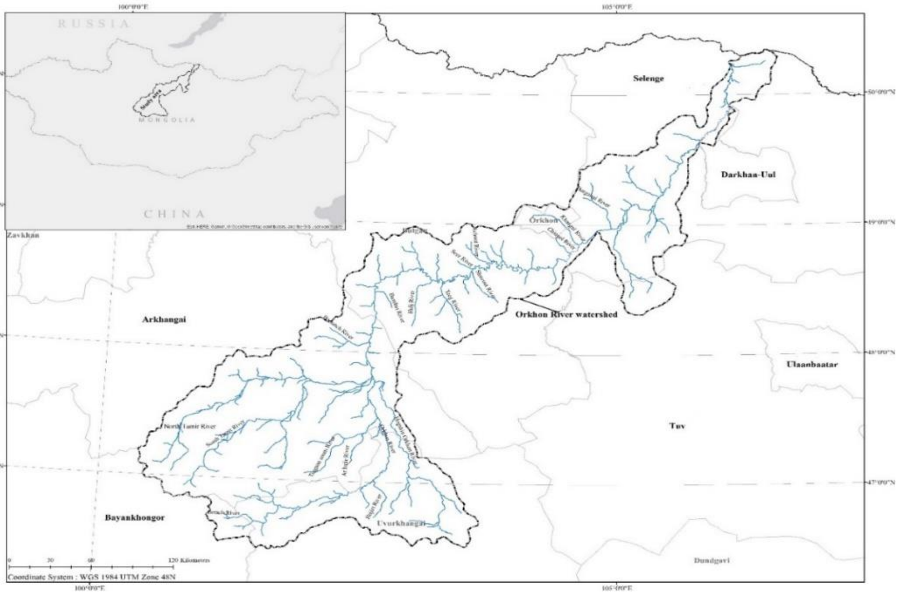

Fig. 1. Study area map. Orkhon River watershed in Mongolia

Annual peak flows were defined based on the existing daily flow data in cubic meter per second (m^3/s) with period of 1945-2014 at midstream hydro-station of the Orkhon River and the data obtained from the Research Institute of Meteorology, Hydrology and Environment of Mongolia. A peak flow frequency analysis and techniques are applicable only to sites where flow data is available at least of 15 years and their future application be enable to use for rural ungauged streams. Frequency analysis on hydrologic data requires that the data needs to be homogeneous and independent. The model's results are directly related to the data quality [15, 16, 17, 18].

3. Methods

One of the primary tasks in hydrology science and application is estimation of the peak flows, i.e., value of river discharge corresponding to a given exceedance of probability or expressed return periods in years. Many distributions used for peak flow frequency and flood probability analyses such as Extreme Value Type I (EVI), Generalized Extreme Value (GEV), two components Extreme Value, Normal, log Normal (LN), Gumbel Maximum, Weibull, Pearson Type III, log Pearson Type III (LPIII), Gamma, Exponential and Generalized Pareto with certain accuracies for the estimation of maximum discharges in different return periods and areas [4, 6]. From the results, Gumbel Maximum, Gamma, log-Pearson Type III (LP III), Lognormal (LN), and Generalized Extreme Value (GEV) are best-fitted distributions to estimate magnitude and frequency of flows in majority of applied locations. Selection of an appropriate probability distribution and associated parameter estimation method is very important at-site flow frequency analysis [3].

Castellarin [5] reviewed flood frequency distributions in Europe that was Gumbel recommended in for Finland while Rahman et al [3] identified Log-Pearson Type III was best-fitting distribution for most applied sites in Australia. Millington et al. [7] and Rahman et al. [3] estimated discharges for different return periods by comparing distributions and found GEV, LPIII, Generalized Pareto, and Gumbel probability distributions were top best-fitted distributions in upper Thames watersheds in UK and watersheds in Australia. Gumbel Maximum Value, Generalized Pareto, and LPIII distributions are effective to calculate the flood intensity in different return periods [8, 9]. Haddad & Rahman [10] evaluated flood risk by estimating probable maximum floods using flood event series in 1942-2008 and concluded Normal, lognormal, and Weibull distributions were fitted best in Indus River. Salarpour et al. [11] used five probability distributions for peak flow, flood volume and flood duration analyses and suggested GEV distribution was the most suitable for peak flows in Malaysia. Singo et al. [9] suggested that the log Pearson Type III is the best-fit probability model for flood prone area of Luvuvhu Basin in South Africa. These distributions are also recommended for flood frequency analysis in various areas of the World [4, 10, 19, 20].

3.1 Continuous Probability Distributions

The selection of an appropriate probability distribution is important to accurate estimation of peak flows at a specific site. Probability distributions namely LPIII, Gumbel Max, Gamma and lognormal will be used to model the distribution of the peak flows. These distributions are recommended in peak flow studies in different countries [4, 3, 19]. The Probability density function (PDF) and Cumulative distribution function (CDF) for the candidate distributions are summarized in **Table 1**.

Table 1. Probability density and cumulative density functions of the probability distributions

Distributions	PDF	CDF
Lognormal	$\frac{1}{x\sigma\sqrt{2\pi}} \cdot \exp\left[-\left(\frac{logx-\mu}{\sqrt{2}\sigma}\right)^2\right]$	$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}\frac{1}{z}\exp\left[-\left(\frac{logz-\mu}{\sqrt{2}\sigma}\right)^2\right]dz$
LPIII	$\frac{1}{ax\Gamma(b)}\left(\frac{logx-c}{a}\right)^{b-1} exp\left[-\left(\frac{logx-c}{a}\right)\right]$	$\int_{e^c}^{x}\frac{1}{az\Gamma(b)}\left(\frac{logz-c}{a}\right)^{b-1} exp\left[-\left(\frac{logz-}{a}\right.\right.$
Gumbel Max	$\frac{1}{\sigma}e^{-(x+e^{-z})}$ where $z=\frac{x-\mu}{\sigma}$	$e^{-e^{-(x-\mu)/\sigma}}$
Gamma	$\frac{a^b}{\Gamma(b)}x^{b-1}\exp[-ax]$	$\int_0^x \frac{a^b}{\Gamma(b)}z^{b-1}\exp[-az]\,dz$

3.2 Parameter estimation for the probability distributions

Distribution parameters can be estimated by maximum likelihood estimator, method of moments and/or by method of l-moments [22, 23, 24] for the frequency analysis.

Method of moments is good for limited range of parameters, while l-moments are unbiased and more often used [25]. In this study, we used maximum likelihood (ML) method of parameter estimations for the candidate distributions. ML method is one of the widely used method for estimation of probability distribution parameters. The ML method aims to estimate unknown parameter values using the historical peak flow data. The estimates are obtained by maximizing the likelihood function and/or logarithmic function of probability distribution. The likelihood estimator is derived from the logarithmic function of the likelihood against its parameters. The likelihood (*L(θ)*) of the data set is the product of the probability distribution function evaluated at each of the observed value of the data and can be written as

$$L(\theta) = \prod_{i=1}^{n} f(\widetilde{z_i}|\theta). \quad (1)$$

$l(\theta)$ = The log-likelihood function can be expressed as

$$l(\theta) = \sum_{i=1}^{n} logf(\widetilde{z_i}|\theta) \quad (2)$$

where θ is an unknown parameter vector.

3.3 Goodness of fit distributions

Best fitting distribution can be select based on goodness of fit tests and the tests can reflect how fitting the applied methods can reflect the historical peak flow data. The selection of an inappropriate function for a region may result in underestimating or overestimating the extreme hydro-meteorological events with a specific probability of exceedance [10]. The performance of peak flow frequency models is evaluated by different goodness of fit statistics. Goodness of fits can determine the equation of distributions that has been selected to represent the statistical properties of the used data in analysis. The commonly used model performance evaluation statistics such as F-test, T-test, Kolmogorov-Smirnov, Anderson-Darling, Chi-squared and Cramer-von Mises tests, Akaiki Information Criteria and Bayesian Information Criteria were used for frequency distribution selection in the most literatures. In this study, we used Kolmogorov-Smirnov and Anderson-Darling tests for best fitting distribution selection and they were conducted at 5% level of significance. The mathematical explanation of two goodness of fit tests are shown in the following.

Kolmogorov-Smirnov (KS) statistic is used to decide if sample reflect for a population with a specific distribution and the statistic is defined as follows:

The maximum discrepancy is called the D-statistic (D_n) which is then compared to the critical D-statistic (Tao et al., 1976):

$$D_n = max_i|F(x_i) - F_a(x_i)| \quad (3)$$

Where F is the corresponding to the selected x_i distribution function and F_a is the additive frequency distribution ordinal calculated from the observed sample. The critical D-statistic depends only on the sample size (n).
Anderson-Darling (AD) statistic is used to find out if an observed peak flow data belongs to a specific probability distribution. The test assumes that there are no parameters to be examined in a distribution under control, which means that the test and its critical value sets are distribution-free. AD goodness-of-fit test is more often used to test the family distribution where the parameters need to be estimated. Then, estimated parameters have to be adjusted to the test statistics and/or its critical values.
The AD test statistic (A^2) is given as:

$$A_n^2 = -n - \frac{1}{n}\sum_{j=1}^{n}(2j-1)\left[log(v_j) + log(1 - v_{(n-j+1)})\right] \quad (4)$$

where, $v_j = F_0\,(x_j)$ and $x_1 < x_2 \ldots \ldots \ldots < x_n$ is the ordered sample
Goodness of fit for these tests checked the hypothesis for the candidate distributions:
H0: The peak flow data follow the specified distribution
HA: The peak flow data not follow the specified distribution
For KS and AD goodness of fit test statistics, a hypothesis is rejected if the estimated value is greater than the critical value at a chosen significance level.

3.4 Return Period of Peak Flows

One goal of this analysis is to estimate and predict the magnitude of peak flows with a given frequency of occurrence. The frequency of occurrence is expressed in terms of return period T, in years. In other words, there is a probability

$$p = P_r(Q > Q^*) \quad (5)$$

of observing peak flows exceeding Q^*. This exceedance probability p has an inverse relation with T, and is shown mathematically as,

$$T = 1/P \quad (6)$$

In flood frequency analysis, peak flows are assigned a return period. Return period defines the probability that a certain magnitude of flow will be equaled or exceeded in any given year. The commonly used recurrence intervals identified in the water resources research are the 2-year, 5-year, 10-year, 25-year, 50-year and 100-year return periods. All these return periods are chosen as referred periods of peak flow exceedance probabilities of 0.50, 0.20, 0.10, 0.04, 0.02, and 0.01, respectively. For the probabilities, the 2-year return period has the highest probability (50%) of occurrence, whereas the 100-year return period has the smallest probability (1%) of occurrence in any given year. In this study, 2- to 100-year return period began as an initial focus because this is the most frequent flood flow that is equaled or exceeded in any given year and will be practically significant reference for future flow adjustment in the Orkhon River.

4. Methodology

The peak flow data of Orkhon hydro-station with 70 years daily flows were considered peak flow frequency analysis applying candidate distributions: LPIII, Gumbel Max, Gamma and Lognormal. The used procedures of this study to estimate design peak flows in return period, given in below:

1. Annual peak flow data was obtained from long-term daily flow data for the hydro-station.
2. From peak flows data for *n* years, parameters were estimated for each candidate distribution.
3. PDF and CDF were defined for each distribution.
4. Goodness-of-fit tests: Kolmogorov-Smirnov and Anderson-Darling tests were applied to choose best-fitting distribution for the stations.
5. Using best-fitting distribution at each station there were estimated peak flows magnitudes in 2-, 5-, 10-, 25-, 50-, and 100-year return periods.

5. Results

The basic descriptive statistics of the station are presented in **Table 2**. Data series analyzed by initial statistical procedures that makes basic characteristics such as mean, standard deviation, skew and observed peak flows. The observed minimum and maximum of peak flows at the Orkhon hydro-station recorded 33.9 m^3/s in 2000 as minimum and 1690 m^3/s in 1994 as maximum during the studied period.

Table 2. Descriptive statistics for peak flow data of the Orkhon station

Station	**Start and end year**	**Peak flows (m^3/s)**		**Skew**	**Kurtosis**	**Observed peak flows (m^3/s)**	
		mean	sd. dev			min (year)	max (year)
Orkhon	1945 - 2014	262.7	226.2	3.8	22.3	34 (2000)	1690 (1994)

In this study, we have applied Gumbel Max, Log-Pearson Type III, Gamma and Lognormal as candidate distributions and to select best-fitting one based on annual peak flow data at midstream station of Orkhon River. Probability distribution function (PDF) and Cumulative distribution function (CDF) of the Orkhon hydro-station is shown in **Fig. 2**. PDF and CDF for Gamma, Gumbel Max, Lognormal and LPIII distributions are applied to the historical peak flow data on the stations.

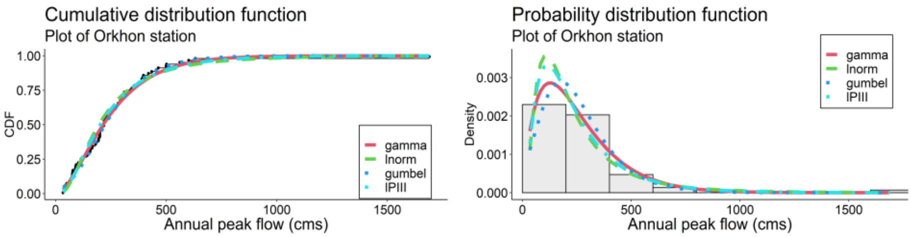

Fig. 2. PDF and CDF for Orkhon station

Since the goodness of fit test statistics indicated the distance between the historical peak flow data and the fitting distributions, it is significant that the distribution with the lowest difference in statistic value is the best to the model fitting. **Fig. 2** presents the main parameters that all distributions were applied at the station. The parameters can be estimated by maximum likelihood method and distributions have been evaluated by Kolmogorov-Smirnov and Anderson-Darling tests for goodness of fits.

Estimated fitting parameters using maximum likelihood method for the lognormal, LPIII, Gumbel Max and Gamma are given in **Table 3**. As results, the amount of continuous shape parameter (a, μ), continuous scale parameter (b, σ) and continuous location parameter (c) are within limits of the statistics, which mean they are valid. Calculated parameters are within the limits of the statistics.

Table 3. Fitting parameters for Lognormal, LPIII, Gumbel Max, and Gamma distributions on the Orkhon hydro-station

S no	Distribution	Parameters
	Orkhon	
1	Lognormal	σ =5.28, μ = 0.77
2	LPIII	a = 58.3, b = -0.10, c = 11.9
3	Gumbel Max	σ = 179, μ = 131
4	***Gamma***	a = 1.94, b = 0.007

For the selection of best-fitted distribution, this study applied Kolmogorov-Smirnov and Anderson Darling test in R-studio 2022 software and results are presented with critical values for goodness-of-fits in **Table 4**. In the significance level of 0.05, the critical values are obtained from the estimated table for Kolmogorov-Smirnov and Anderson-Darling tests relative to sample numbers on the stations. It can be seen however, that the goodness of fit tests is not rejected when the estimated statistics are smaller than the critical values for all the applied distributions. The results present that the Gamma distribution was more significant for peak flows at the Orkhon hydro-station of the Orkhon River followed by Lognormal, Gumbel Max and LPIII with no rejection of KS and AD goodness of fit tests. Generally, LPIII distribution was not suitable for the station (**Table 4**).

Table 4. Goodness of fit tests results for Orkhon hydro-station of the Orkhon River

No	Distributions	Kolmogorov-Smirnov test			Anderson-Darling test			Rank score
		value	reject	rank	value	reject	rank	
	Orkhon	Critical value at 0.05= 0.2379			Critical value at 0.05= 2.4922			
1	***Gamma***	0.0619	no	*1*	0.3981	no	*1*	2
2	Lognormal	0.0736	no	2	0.4918	no	2	4
3	Gumbel Max	0.0764	no	3	0.4982	no	3	6
4	LPIII	0.1113	no	4	0.8294	no	4	8

Hence, Gamma distribution for the station is used to estimate event magnitudes or peak flows in 2-, 5-, 10-, 25-, 50-, and 100-year return periods with unequal recorded data [21]. Calculated peak flow magnitudes with potential lower and upper limits within 95% confidence intervals using best-fitted distribution at the hydro-station of the Orkhon River presents in **Table 5**. Estimated peak flows in cubic meter per second for 2-year to 100-year return periods and associated exceedance of probability are varied greatly as smallest of 218 m^3/s in 2-year return period and largest of 882 m^3/s in 100-year return period at the Orkhon hydro-station.

Table 5. Estimated peak flows (m^3/s) in return periods at the Orkhon hydro-station

Return period	Exceedance probability	Estimated magnitude, m^3/s	95% confidence interval	
			lower	upper
Orkhon (Gamma distribution)				
2-year	0.50	218	191	264
5-year	0.80	393	340	469
10-year	0.90	514	430	615
25-year	0.96	663	544	807
50-year	0.98	775	627	949
100-year	0.99	882	708	1089

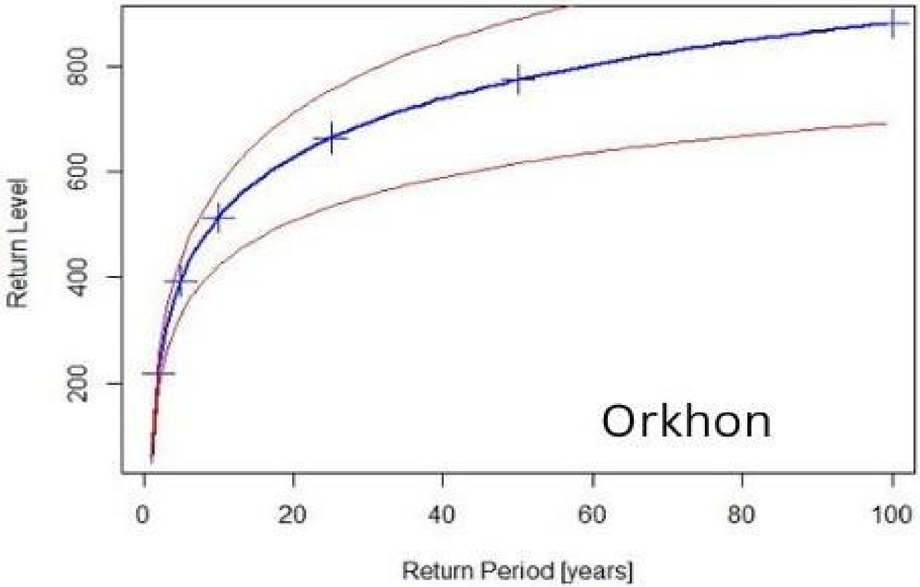

Fig. 3. Estimated peak flows by quantiles

In addition, estimated peak flows in the Orkhon hydro-station illustrated by graphs that is shown in **Fig. 3**. Return periods are placed on *x-axis* while their corresponding peak flows magnitude in cubic meter per second is placed as return level on *y-axis* in the figure. In these graphs, historical maximum peak flows of 1690 m^3/s in 1994 on the station (**Table 5**) is above the 100-year return period (**Fig. 3**).

6. Discussions

Through the site peak flow analysis, this work applies simple and widely employed methods to estimate peak flows at selected return periods in specific site and large streams using potential longest daily flow data in mountainous areas of northern Mongolia. Hence, this study uses Gumbel Maximum, Log-Pearson Type III (LPIII), Lognormal and Gamma distributions for identifying best fitted one on the Orkhon hydro-station of the Orkhon River.

The Gamma distribution was more suitable for midstream large station having highest positive skewness and kurtosis. The results maybe shown an important geographic and watershed insights such as headwater station in higher elevation area in Khangai Mountains [14] and large amount of evapotranspiration in the Selenge River Basin [26]. The poor ranking of LPIII distribution fitted results might be perhaps due to seasonal flow pattern (ice regime, low flow domination), climate change (continuous drought years after 2000) and uneven distributions of annual precipitation (up to 70% of the precipitation falls in summer) in Mongolia.

When we analyze peak flow frequencies, it is difficult to develop accurate model especially estimating peak flows in years featuring drought which are common in last decades of the study period [27] and the lower peak flows are recorded after 1995 in the Selenge River watershed [28].

However, peak flows or annual large floods occur in July and August in the Orkhon River watershed and spring snowmelt flood should contribute some portion to the annual flood series in the Arctic Ocean Basin, Mongolia [29]. Because spring flood magnitude can be depending on previous autumn precipitation and flows, snow depth and temperature in winter, that will also be impacted to soil moisture and water availability in late spring and early summer. Annual rainfall and snowfall work together to generate summer peak flows and they might have similar distributions. This would tell us accounting mixture of annual snowmelt and rainfall processes can describe peak flows in this watershed [30].

These frequency analyses involve only annual peak flows that might be not perfect to represent total peak flow series in the watershed. In addition, to generate more accurate and complete models not ignoring the watershed parameters that can be augmented with historical climatic information and various watershed parameters: soil properties, land cover and human influences particularly, recent livestock number increase and land use change in the Orkhon River watershed [14].

The result of peak flow analysis of the Orkhon River could be used for the flood hazard assessment and mitigation, water resources management i.e., for maximum water availability estimations and designing water-engineering structure. Moreover, it will be referencing guide for the ungauged small reaches as well. The difficulty to these frequency analyses arises when it involves climate change impacts and physical variables for the watershed not found in remote areas.

7. Summary

The frequency analysis for peak flows on Orkhon hydro-station in the Orkhon River watershed is carried out by using Gamma, lognormal, Gumbel Max and LPIII distributions with maximum likelihood method of parameter estimation. Based on the Kolmogorov-Smirnov and Anderson-Darling tests of goodness-of-fit statistics we have ranked to candidate distributions at the stations in 5% significance level. For Orkhon hydro-station, Gamma was significant and selected as best-fitted distribution. LPIII was poorest fitting distribution in the Orkhon River. Using the best-fitting Gamma distribution at the hydro-station, the peak flow magnitudes are estimated as 218, 393, 514, 663, 775, and 882 m^3/s in 2-, 5-, 10-, 25-, 50-, and 100-year return periods, respectively. For further study, Gamma, lognormal and Gumbel Max distributions with different parameter estimation methods will be recommended to evaluate by the performance into the Arctic Ocean Basin Rivers in Mongolia.

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