Fatigue Strength Estimation Based on the Maximum Likelihood Method

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Abstract. Fatigue strength is one of the core principles for designing mechanical components. It has been a constant concern for engineers, as mechanical failure occurs due to loading exceeding the fatigue strength. This concern has led to a necessity to develop new approaches to estimate the reliability of mechanical components. The conventional method that is used to test fatigue strength is the staircase method. However, the staircase method's ability to calculate fatigue strength is potentially unreliable. The bias and scatter associated with fatigue testing shows the limitations of fatigue strength estimation when the staircase approach is used. The conventional methods of fatigue limit determination also have key flaws, in that they are subjective, time consuming, and costly. This research aims to develop a method that would reliably estimate the fatigue strength of materials, whilst using a lower amount of test results. The present study is intended to formulate and analyze a proposed method of estimating fatigue strength by utilizing a smaller number of tests. The aforementioned method that this paper aims to formulate, mainly focuses on a probabilistic estimation based on the Maximum Likelihood Procedure. The proposed method is applied to existing fatigue test data and its effectiveness is compared with other methods.

Keywords: Fatigue strength, Maximum likelihood estimation, S-N curve.

1 Introduction

The use of ferrous materials allowed for more complex engineering structures in the 1900's. The advancements made in material science allowed for structures and mechanical components such as: train axles, railways, mine shaft hoists, etc. These mechanical components and structures were designed to resist high mean loads, yielding, and environmental conditions. However in the 19th century, structural failures were observed

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occurring in metals subject to cyclic loads far below their yield strength. Later on, Poncelet used the term "fatigue" when referring to these failures [1]. In the 1850s, Wöhler conducted excessive amounts of fatigue tests on smooth and notched railway axles. Wöhler's work introduced the use of S-N diagram which is a plot of failure to a given stress level to number of cycles. His work on the S-N diagram was the first to characterize fatigue in a systematic manner [2].

The use of the S-N diagram led to the concept of the fatigue strength (i.e. fatigue limit); which is a theoretically estimated stress range in that fatigue failure is resisted by the material. After the advancements made by Wöhler, the S-N curve was universally acknowledged as a means of accurately characterizing fatigue. Basquin later propose empirical laws that characterized S-N curves [2]. His work highlighted the fact that if stress and number of fatigue cycles were plotted as a log-log, the result would be a linear relationship. As further research was conducted on fatigue, many advancements in fatigue research were achieved; including, Miner's linear damage hypothesis, Weibull's work on the statistics involved with fatigue [3, 4].

Despite the extensive amount of research and advancements made, the true nature of fatigue remains unknown. The accurate estimation of fatigue strength becomes more important in the modern industries, especially jet engine, in which mechanical components are operated in high frequency cycle. To ensure safety without failure during the life span of mechanical components, the fatigue strength of mechanical material should be accurately determined. However, due to the very high cost and long period to perform fatigue tests with Mega to Giga cyclic loadings, a probabilistic method for the estimation of fatigue strength is inevitable, which utilizes only the small number of test specimen.

In this study, based on the characteristics of fatigue test for fatigue strength, a probabilistic method is proposed based on the maximum likelihood method and applied to simulated fatigue test data sets.

2 Fatigue Strength

2.1 Characterization of Fatigue

The phenomena of fatigue is defined by Fine and Chung as "the progressive, confined to a small area, and continuing structural damage that takes place when a material is under repeated strains at insignificant stresses that are less than the actual yield strength of the material" [5]. Engineering structures are frequently under a myriad of forces and loads. The repeated or cyclic loads that they are subjected to result in microscopic damage to the structure. This microscopic damage eventually accumulates until a macroscopic crack appears. To this extent, fatigue should be viewed as both a material phenomenon and an engineering problem [6]. From a metallurgical standpoint, Fine and Chung [5] specify that the entire process of mechanical failure caused by fatigue is as follows:

i. Cyclic loading causing repeated plastic deformation, initiating the first crack
ii. Initiation of micro cracks
iii. Propagation or growth of microscopic cracks

iv. Propagation of macroscopic cracks
v. Failure of part due to fatigue

The S-N curve plots the number of cycles until a specimen fails (N_f) against the cyclic stress amplitude level (S) at the failure. To draw the S-N curve for a material at least 3 test specimen should be tested at one stress level. The actual line provided in S-N diagrams are generated using the mean of the data from the various tests conducted. The S-N curve shows a certain noticeable stress level that fatigue failure will not occur. This stress level is defined as the fatigue strength. Nelson defines the fatigue limit as "the stress level at which the fatigue life becomes a prescribed long but finite life" [7]. Most ferrous and titanium alloys have a certain fatigue strength, while most nonferrous metals appear to not. In materials that exhibit fatigue limits, the stress level that corresponds to failure at 10^6 or 10^9 cycles on the S-N diagram is regarded as the fatigue strength.

2.2 Scatter in Fatigue Strength

Fatigue lifetimes of similar specimens with the same material can be significantly varied. The test data acquired from fatigue testing is subject to a significant amount of scatter. There are many sources that influence the scatter. Sources such as surface finish, environmental causes, and specimen discrepancy have proven to influence the amount of scatter. Even with extremely controlled conditions when conducting fatigue testing, scatter is still present in testing data.

Scatter has been proven to increase in low stress amplitude testing. At a high stress amplitude, surface conditions and localized conditions of grain boundaries are less important. However, at low stress amplitude values, micro cracks propagate to structural barriers in the material. As a result of this scatter, it is crucial for fatigue test results to be fitted to a probability distribution. In order to predict the fatigue life of materials, it is important to assume a probabilistic distribution type that accurately reflects the uncertainties of the fatigue life.

In Fig. 1, the random variable of the probabilistic distribution at each stress amplitude is the number of cyclic loading while the random variable around fatigue strength is stress amplitude at a specified number of cyclic loading. An average fatigue strength is determined as the stress amplitude with the failure probability of 50%. The fatigue strengths corresponding to the 5% and 95% failure probability are determined according to the probability distribution of fatigue strength.

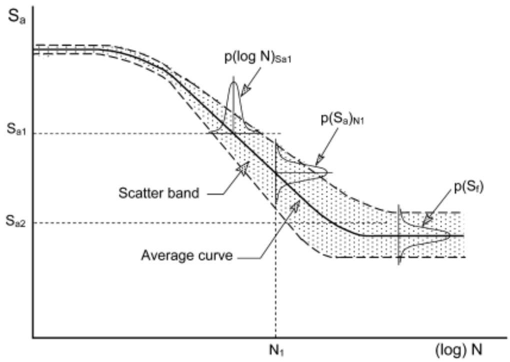

Fig. 1. Variation of scatter band at high and low stress amplitudes [8].

3 Fatigue Strength Estimation Method

3.1 Staircase Method

Generally, for the estimation of fatigue strength estimation the staircase testing method has been used. It was first introduced by Dixon and Mood [8]. In the staircase method, specimens are tested one after the other, starting from an initial stress level estimated to be the median fatigue strength. The stress level is then increased or decreased for the next specimen in the series. Whether the stress level is increased or decreased is determined by the results of the previous specimen test result. When the previous specimen fails, the stress level for the next specimen is decreased; alternatively when the previous specimen survives until the specified number of cycles, the stress level is increased. This test process is carried out until the last specimen. The step size of the test is chosen to be equal for each iteration, as this allows for statistical analysis for determination of the mean and standard deviation [9].

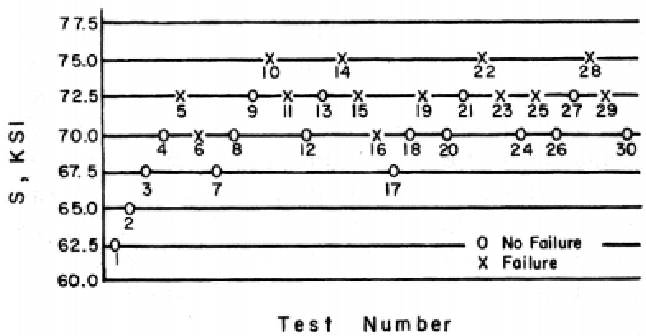

Fig. 2. Staircase method illustration [9].

The staircase method was initially thought to require an upwards of 60 specimen for parameter estimates [9]. It was later proven that sufficient estimates can be made using fewer test samples [10]. Currently the Japanese Society of Mechanical Engineers (JSME) uses the staircase method for the determination of the fatigue limit for S-N curve determination [11].
But the drawbacks of the staircase method for fatigue strength estimation is that scatter is not adequately accounted for. For example, if a faulty specimen fails early, the iteration process will be increased and the results would be flawed. Another disadvantage is the undetermined step size for each test. It is identical for increase and decrease of stress level. This would result in more specimens required in the case of scatter.

3.2 Proposed Method

Probability Density Function. The fatigue test of each specimen at any stress amplitude can be modeled as the Bernoulli sequence. Each test is independent to each other and the output random variable (x) of a trial is either a survival (runout) or a failure at the cyclic number of fatigue lifetime. However, because the failure probability of specimen changes depending on each stress amplitude level, S_i, the probability mass function is expressed as Equation 1.

$$f(x; F(S_i)) = \begin{cases} F(S_i) & if\ x = 1 \\ 1 - F(S_i) & if\ x = 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Maximum Likelihood Function. The principles of the maximum likelihood estimation are based on the maximization of the likelihood function. For a set of random variables $x_1, \cdots, x_n$ from a distribution, assuming the distribution depends on a parameter θ, where θ is either a real-valued unknown parameter or a vector of parameters. For every random sample that was observed $x_1, \cdots, x_n$, a likelihood function is defined such as:

$$f(x_1, \cdots, x_n | \theta) = f(x_1 | \theta) \cdots f(x_n | \theta) \tag{2}$$

The likelihood function above is always dependent on the unknown parameter θ and is denoted as $L(\theta)$. For any given data $x_1, \cdots, x_n$, considerations are made to determine a value of θ for which the likelihood function $L(\theta)$ is a maximum. For simplicity, maximizing the likelihood function $L(\theta)$ is equivalent to maximizing log $L(\theta)$ because it is the monotonic increasing function of the likelihood function. The $log\, L(\theta)$ is defined as the log likelihood function and denoted as:

$$log\, L(\theta) = log \prod_{i=1}^{n} f(x_i|\theta) = \sum_{i=1}^{n} log f(x_i|\theta) \tag{3}$$

The maximization of Equation 3 with respect to the parameter θ results in the maximum likelihood estimation.

Parameter Estimation Using Maximum Likelihood Function. Using the Bernoulli random variable, the probability density function of Equation (1), and the maximum likelihood function (2), the maximum likelihood function for fatigue strength is formulated as Equation (4). The maximum likelihood estimation provides reasonable estimations of the unknown parameter θ. As well as an increase in unbiased minimum variance estimators as sample size is increased.

$$L = \prod_{i=1}^{n}[F(S_i)]^{x_i}\,[1 - F(S_i)]^{1-x_i} \tag{4}$$

where, S_i is stress amplitude for which a specimen i is subjected; $F(S_i)$ represents the failure probability of at stress amplitude of S_i; x_i is a value equal to 1 or 0 depending on the survival or failure of the specimen i; and n is the total number of test results.

Assuming the log-normal distribution for fatigue strength, $F(S)$ take the analytical form:

$$F(S) = \Phi\left(\frac{ln\left(\frac{S}{c}\right)}{\zeta}\right) \tag{5}$$

where, S represents stress amplitude; $\Phi(\cdot)$ is the standardized normal distribution function. The two parameters c and ζ in Equation 5 satisfy the equations to maximize $ln\,L$ and can be determined from Equation 6.

$$\frac{d\ln L}{dc} = \frac{d\ln L}{d\zeta} = 0 \tag{6}$$

4 Results from Proposed Method

4.1 Simulation Cases

The application of the proposed method to determine the fatigue strength using a simulated data set was performed. Though a number of fatigue tests are needed to determine fatigue strength through the staircase method, only six simulated data sets are used in this paper to verify the effectiveness of the proposed method. Table 1 and 2 shows the simulated data of two cases. Each fatigue test data is simulated assuming at the fatigue lifetime N=10^6 cycle numbers. The last column of each table identifies the failure or survival of each test specimen at N=10^6. The only difference between two tables is the result of the test number 6. In Case 1, the test specimen failed at the stress amplitude of σ =400 MPa, while it survived in Case 2. Thus at each stress amplitude the number of failure and survival is the same. But at σ =400 MPa, the number of failure in Case 1 is one and that of survival is two and in Case 2, vice versa.

Table 1. Case 1: Six simulated fatigue test data (N=10^6).

Test No.	Stress amplitude (MPa)	Number of cyclic loading	1-Failure, 0-Survival
1	420	10^6	Failure
2	400	10^6	Survival
3	380	10^6	Failure

4	360	10^6	Survival
5	400	10^6	Failure
6	400	10^6	Survival

Table 2. Case 2: Six simulated fatigue test data (N=10^6).

Test No.	Stress amplitude (MPa)	Number of cyclic loading	1-Failure, 0-Survival
1	420	10^6	Failure
2	400	10^6	Survival
3	380	10^6	Failure
4	360	10^6	Survival
5	400	10^6	Failure
6	400	10^6	Failure

4.2 Results and Discussion

The results from applying the proposed method to determine the fatigue strength are shown in Table 3 and Fig. 3. Table 3 compares the fatigue strength at the failure probability of 50% of each case and standard deviation of each case. Fig. 3 shows the cumulative distribution function from each case. As assumed earlier, the probabilistic distribution of fatigue strength is lognormal distribution.

The fatigue strength with failure probability of 50% of Case 1 is 395.03 MPa, while that of Case 2 is 381.33 MPa. The difference in standard deviation of two cases is 13.35 MPa, almost the same as the difference in the fatigue strength at P_f = 50%. But the difference in the standard deviation is more significant. The standard deviation of Case 2 is 55% greater than that of Case 1. It is caused by the result of the specimen No. 6 in Case 2. The failure at σ =400 MPa in Case 2 make the cumulative distribution function curve steeper than that of Case 1 and results in large standard deviation. This influence caused by the result of one specimen cannot be quantified or barely observed when the staircase method is adopted to determine fatigue strength.

Table 3. Fatigue strength at P_f = 50% and standard deviation.

Case	Fatigue strength (MPa) at P_f = 0.5	Standard deviation (MPa)
1	395.03	37.64
2	381.33	24.29

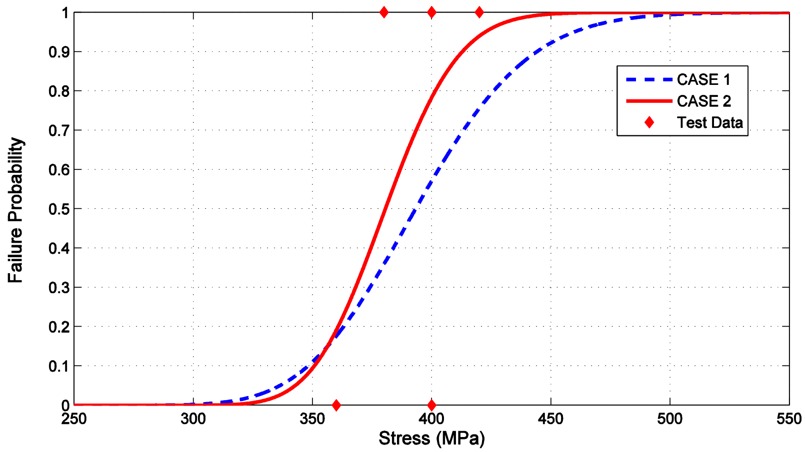

Fig. 3. Results from simulation cases.

5 Conclusion

To avoid a fatigue failure of components under cyclic loading, the fatigue strength of a material or the components should be known. The fatigue strength from fatigue tests shows scatter and it is necessary to adopt probabilistic concept to determine the distribution of fatigue strength. At the fatigue lifetime cycle numbers, the fatigue test results has only two outcomes, either failure or survival. So the Bernoulli sequence can be adopted at each stress amplitude. To estimate the fatigue strength, the Bernoulli random variable is formulated into the maximum likelihood function. The fatigue strength distribution is assumed as the lognormal distribution and determined at the failure probability of 50%. The fatigue strength of different failure probability percentiles, i.e. 5% and 95% can be found easily from the lognormal distribution. The proposed method can be used for the small number of fatigue test data and also it can be used in designing fatigue test scenarios.

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