A Study on the Mahalanobis Universal Portfolio With and Without Transaction Cost for (2K+1)- Bandwidth Toeplitz Matrices

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Abstract. Investing is crucial for wealth accumulation, safeguarding against inflation, and meeting financial objectives. Choosing effective strategies can significantly influence outcomes. Central to this is the Mahalanobis universal portfolio (MUP), characterised by its tendency to yield positive returns over extended durations. This paper delves into its optimisation using $(2k + 1)$-bandwidth Toeplitz matrices associated with positive definite Toeplitz matrices. A novel aspect of our study is examining transaction costs' impact on this portfolio, a territory not previously ventured into. Eight portfolios were formed and obtained from Yahoo! Finance, spanning January 2, 2015, to January 21, 2021, encompassing 1,500 trading days. This paper examines the influence of bandwidth, trading period length, and portfolio sizes on the cumulative wealth investors generate. Alternative investment strategies such as Buy-and-Hold (BH), Constant Rebalanced Portfolio (CRP), and Best Constant Rebalanced Portfolios (BCRP) are scrutinised to gauge their effectiveness in generating wealth. The findings demonstrate that the MUP facilitates notable wealth growth for investors, offering a principal increase and a balance of lower risk with higher returns than other strategies. By incorporating various metrics and transaction costs, the study provides valuable insights into the performance of investment strategies, enabling individuals to make informed decisions to maximise their returns.

Keywords: Universal portfolio, Toeplitz matrix, Transaction cost, Best Constant Rebalanced Portfolio (BCRP)

1 Introduction

1.1 Background and Problem Statement

In recent times, conventional methods of wealth generation, like saving money in banks, are becoming less advantageous due to the surging inflation rate. Inflation is defined as the progressive rise in the prices of goods and services, subsequently leading to the diminished purchasing power of the currency. In an economic environment where inflation rates soar, money's actual value in savings accounts can

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deteriorate rapidly. This devaluation poses significant threats to the financial security of individuals. Historically, parking funds in a bank was considered a reliable method to safeguard one's savings and accrue a consistent return on this investment. Nevertheless, in the face of steep inflation, the interest yielded from these savings does not pace with the escalating prices. It simply means that even though the nominal amount in the savings account might increase due to interest, the real value or purchasing power wanes over time. Given this backdrop, the investment avenue emerges as a compelling alternative, offering an avenue to preserve and increase wealth. By strategically investing, individuals can leverage their capital, making their money work more effectively for them. However, for novice investors, the initial challenge lies in constructing a robust and diversified investment portfolio or strategies that can offer superior returns on their capital.

One promising approach in this study is forming a "Mahalanobis universal portfolio", which uses the positive definite matrix, specifically the Toeplitz matrix. The matrix-based method can be a foundational tool for individuals looking to embark on their investment journey. While this portfolio has been extensively studied, our research offers a fresh perspective. Many universal portfolio strategies explored in past literature often sidestep the intricacies of transaction costs, likely simplifying their models to avoid complexities. However, this exclusion can render such model less reflective of real-world scenarios, reducing their applicability in practical contexts. Recognising this gap, our study introduced transaction costs into the MUP, seeking to present a more holistic and practical view of its performance. This approach raises pressing questions: How does the performance of the MUP fare when transaction costs are factored in? Does it maintain its efficacy as observed in the model without transaction cost?

In this paper, we not only explore the above questions but also delve deeper into the influence of the parameters on the cumulative wealth investors can generate using MUP- the optimal value of the parameter ξ. This parameter plays a pivotal role in shaping the universal portfolio and determining its ideal value can be the key to maximising the investment returns from the said portfolio. Besides, a few scenarios testing were studied in this paper: bandwidth of the Toeplitz matrix, trading period length, and portfolio size. It is important to note that our study focuses on Malaysia's stock market, specifically Bursa Malaysia. This choice has been made strategically since using market indices specific to Malaysia is crucial in accurately calculating market risk. This country's stock market offers many investment opportunities, with easy access to current events and market information. This ensures a level of reliability, in contrast to some foreign markets where uncertainties might arise due to factors such as political, financial, or other external factors, which can bring unpredictability into the analysis.

2 Literature Review

2.1 Development of Portfolio Theory

The world of portfolio investment theory saw its foundations laid by Markowitz [1]. His seminal work revolved around diversification, proposing that investors spread their capital across different industries having unique economic attributes. This approach, he argued would result in portfolios with lower covariances than if the investments were concentrated in a single industry. His expected returns-variance of returns rule (E-V Rule) became a cornerstone, emphasising that efficient diversification arises from a broad spectrum of averages and variances. Investors. Markowitz should make decisions based on probabilistic predictions of stock performance. These predictions, once analysed, would lead to the formulation of portfolios most aligned with an investor's preferences. This initial work by Markowitz was further extended by Sharpe [2]. Sharpe introduced the diagonal model to simplify the often complex calculations associated with portfolio analysis, reducing time and cost.

Kelly's study [3] added a fresh dimension by exploring the interpretation of the information rate. Based on Shannon's pioneering work on information theory, Kelly showcased that a gambler's wealth could grow exponentially. He highlighted that it would align with the rate of information transmission over a channel, contingent upon the odds matching the probabilities of the transmitted results.

Towards the latter half of the 20th century, there was a noticeable shift towards empirical methods. Cover and Gluss [4] applied the empirical Bayes method for portfolio construction. Using the past data combined with prior information, they aimed to estimate parameters vital for portfolio formulation. Interestingly, their portfolios emerged superior when benchmarked against Markowitz's mean-variance portfolios. Cover [5] furthered his work by introducing the theory of universal portfolio. This theory relied on the constant rebalanced portfolio (CRP) technique. The novel approach allowed investors to reallocate their wealth across stocks daily, seeking maximum profitability. The resultant universal portfolios demonstrated impressive performances, outstripping constituent stocks, especially in scenarios devoid of transaction costs.

In the 1990s, a growing emphasis was on adjusting portfolio construction based on market variables. To overcome limitations seen in traditional optimisation methods that rely heavily on assumptions, Jamshidian [6] introduced an optimised portfolio using the stochastic method. Similarly, Cover and Ordentlich [7] explored integrating side information into portfolio construction, defining performance boundaries for worst-case scenarios of universal portfolios incorporating side information. With the new millennium approaching, advanced computational techniques became more prominent. For instance, Helmbold et al. [8] researched portfolio selection using multiplicative updates. They proposed an online investment algorithm that could generate wealth similar to the Best Constant Rebalanced Portfolio (BCRP) without transaction costs, with the added benefits of less implementation time, fewer memory requirements, and simpler execution.

Current Empirical Findings

Universal portfolio theory has steadily evolved, evidenced by many research initiatives targeting unique portfolio optimisation and wealth enhancement dimensions. This journey beginning with Gaivoronski and Stella's [9] pioneering exploration of nonstationary optimisation based on Cover and Ordenlich's foundation [7], expanded the understanding of the Dirichlet-weighted universal portfolio. Subsequent insights from Tan [10] advanced the theory, integrating side information and establishing parameters like the gamma function. Kozat and Singer [11] further bridged computational strategies with foundational concepts, underscoring the value of portfolio switching based on auxiliary data.

As the field advanced, Helmbold et al. [8] and Tan and Lim [12] probed deeper into mathematical nuances, exploring concepts like the Kullback-Leibler information measure and chi-square divergence. While Lim [13] tackled the constraints of the multiplicative-update universal portfolio, emphasising a broader parameter range, Tan [14] steered the discourse towards computational pragmatism, unveiling the potential of finite-order portfolios.

Amidst these advancements, a recurring theme is the frequent absence of transaction costs in many models. Although simplifying assumptions like these facilitate a streamlined theoretical exploration, they often diverge from real-world trading conditions. While contributing significantly to the field, Tan and Phoon [15], Patrick and David [16], and subsequent studies have often leaned towards these idealistic models. This pattern culminates in more recent contributions, such as those by Yang and Phoon [17], Kuang [18], Ling and Phoon [19] and Bhatt et al. [20], which although innovative but overlook the gritty realities of transaction costs.

Universal portfolio theory has been subject to constant innovation and exploration, but upon closer inspection, it appears to have a gap. While many of these models hold theoretical value, they could be further refined to account for the tangible impact of transaction costs. This would help bridge the gap between theoretical maximums and the practical realities of investing.

3 Research Methodology

3.1 Mahalanobis Universal Portfolio (Without Transaction Cost)

The randomness of stock price movement is assumed in a universal portfolio. The main component of Mahalanobis universal portfolio model namely portfolio vector is defined as $\boldsymbol{b} = (\boldsymbol{b_1}, \boldsymbol{b_2}, \dots, \boldsymbol{b_i}, \dots, \boldsymbol{b_m})^{\boldsymbol{T}}$ where $\sum_{i=1}^{m} \boldsymbol{b_i} = \mathbf{1}$ and $\boldsymbol{b_i} \geq 0$. It is formed by allocating the investor's wealth in each stock on n^{th} trading day for i^{th} stock, b_{in}. Another component such as price relative vector is defined as $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_i}, \dots, \boldsymbol{x_m})^{\boldsymbol{T}}$, where $x_{in} \geq 0$. It is formed by dividing the closing price by the opening price of i^{th} stock on n^{th} trading day, x_{in}. The wealth of the universal portfolio on $(n+1)^{th}$ trading day is given by:

$$S_{n+1}(\boldsymbol{b}) = \prod_{j=1}^{n+1} \boldsymbol{b_j^T x_j} \tag{1}$$

where
$S_0(\boldsymbol{b}) = 1$ and
$\boldsymbol{b_j^T x_j} = \sum_{i=1}^{m} b_{ij} x_{ij}$. [5]

The symmetric positive definite matrix $\boldsymbol{A}$, the scalar parameter ξ, and the initial portfolio vector $\boldsymbol{b_1} = (b_i)$ are the three parameters of the Mahalanobis universal portfolio. The expression is given by:

$$\boldsymbol{b_{n+1}} = \boldsymbol{b_n} + \frac{\xi}{\boldsymbol{b_n^T x_n}} \left[\boldsymbol{A^{-1} x_n} - \left(\frac{\boldsymbol{1^T A^{-1} x_n}}{\boldsymbol{1^T A^{-1} 1}}\right)\boldsymbol{A^{-1} 1}\right] \tag{2}$$

where
$\boldsymbol{b_1}$ is given,
$\xi \in \mathbb{R}$, and
$\boldsymbol{b_{n+1}} > 0$ for $n \in [1, \infty]$. [13]

The computational formula of $\boldsymbol{b_{n+1}}$ is simplified by adopting the concept of companion matrix [21]. The entries of companion matrix $\boldsymbol{C}$ of Toeplitz matrix $\boldsymbol{A^{-1}}$ is given by:

$$c_{ij} = a_{ij} - \frac{(a_{.j})(a_{i.})}{a_{..}} \tag{3}$$

where
$i, j = 1, 2, \dots, (2k+1)$,
$a_{.j} = \sum_{i=1}^{k} a_{ij}$,
$a_{i.} = \sum_{j=1}^{k} a_{ij}$ and
$a_{..} = \sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij}$.

The computation formula of $\boldsymbol{b_{n+1}}$ from Eq (2), can be replaced by:

$$\boldsymbol{b_{n+1}} = \boldsymbol{b_n} + \frac{\xi}{\boldsymbol{b_n^T x_n}} [\boldsymbol{C x_n}] \tag{4}$$

where $\boldsymbol{C(n)}$ is a symmetric companion matrix from which the entries are calculated by using Eq (3).

Mahalanobis Universal Portfolio (With Transaction Cost)

In the real world, an additional fee for each transaction is always paid. Thus, the proportional transaction cost model proposed by Blum and Kalai [22] was adopted to increase the empirical result's reliability and truthiness. The transaction cost rate may include various components, such as processing fees, administrative charges, or third-party fees. If there is only one transaction, the platform or service provider may divide the total cost between the buyer and the seller, resulting in each party paying only half of the transaction cost rate. The portfolio vector $\boldsymbol{b_n}$ to the new weight $\boldsymbol{b_{n+1}}$ at the beginning of n^{th} day, producing a transaction cost for the adjustment. The equation of transaction costs incurred can be defined as:

$$\frac{\gamma}{2} \times \sum_i |\boldsymbol{b_{n+1}}(i) - \boldsymbol{b_n}(i)| \tag{5}$$

where $\gamma \in (0,1)$ is the transaction cost rate.

The total investor's wealth generated by Mahalanobis universal portfolio with transaction cost can be denoted as:

$$S_n^c = S_0 \prod_{t=1}^{n} (\boldsymbol{b}_t^T \boldsymbol{x}_t) \cdot \left(1 - \frac{\gamma}{2} \times \sum_i |\boldsymbol{b}_{n+1}(i) - \boldsymbol{b}_n(i)|\right) \quad (6)$$

(2k+1)-Bandwidth Toeplitz Matrix

The general expression of the $(2k+1)$ - bandwidth for an $n \times n$ symmetric Toeplitz matrix can be expressed as follows: [23]

$$\boldsymbol{A} = \begin{pmatrix} a_0 & a_{-1} & \cdots & a_{-(n-2)} & a_{-(n-1)} \\ a_1 & a_0 & & a_{-(n-3)} & a_{-(n-2)} \\ \vdots & & \ddots & & \vdots \\ a_{n-2} & a_{n-3} & & a_0 & a_{-1} \\ a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 \end{pmatrix}$$

where

$a_{ij} = a_{|i-j|}$, for $|i-j| \leq k$ and

$a_{ij} = 0,$ for $|i-j| > k$

This paper focused on utilising the 3-bandwidth Toeplitz and 5-bandwidth Toeplitz matrix on 3 and 5-asset portfolios, respectively. The expression of 3-bandwidth (3×3) Toeplitz matrix is shown below:

$$\boldsymbol{A_1} = \begin{pmatrix} 1 & r & 0 \\ r & 1 & r \\ 0 & r & 1 \end{pmatrix}$$

The companion matrix $(3+4r)^{-1}\boldsymbol{C_1}$ can be generated from the matrix $\boldsymbol{A_1}$, and $\boldsymbol{C_1}$ can be denoted as:

$$\boldsymbol{C_1} = \begin{pmatrix} \emptyset_1 & \emptyset_3 & \emptyset_4 \\ \emptyset_3 & \emptyset_2 & \emptyset_3 \\ \emptyset_4 & \emptyset_3 & \emptyset_1 \end{pmatrix}$$

where

$c_{11} = c_{33} = \emptyset_1 = 2 + 2r - r^2,$

$c_{22} = \emptyset_2 = 2 - 4r^2,$

$c_{12} = c_{21} = c_{23} = c_{32} = \emptyset_3 = 2r^2 - 1,$

$c_{13} = c_{31} = \emptyset_4 = -1 - 2r - r^2$

The sequences of $\boldsymbol{b_{n+1}}$ from Eq (4) generated by the 3-bandwidth Toeplitz matrix can be updated into:

$$\begin{aligned} b_{1,n+1} &= b_{1,n} + \frac{\xi}{\boldsymbol{b_n^T x_n}}[\emptyset_1 x_{1,n} + \emptyset_3 x_{2,n} + \emptyset_4 x_{3,n}] \\ b_{2,n+1} &= b_{2,n} + \frac{\xi}{\boldsymbol{b_n^T x_n}}[\emptyset_3 x_{1,n} + \emptyset_2 x_{2,n} + \emptyset_3 x_{3,n}] \\ b_{3,n+1} &= b_{3,n} + \frac{\xi}{\boldsymbol{b_n^T x_n}}[\emptyset_4 x_{1,n} + \emptyset_3 x_{2,n} + \emptyset_1 x_{3,n}] \end{aligned} \quad (7)$$

where $n \in [1, \infty)$.

Similarly, the expression of the 5-bandwidth (5×5) Toeplitz matrix ($k = 2$) will be:

$$\boldsymbol{A_2} = \begin{pmatrix} 1 & r & s & 0 & 0 \\ r & 1 & r & s & 0 \\ s & r & 1 & r & s \\ 0 & s & r & 1 & r \\ 0 & 0 & s & r & 1 \end{pmatrix}$$

The companion matrix $(5 + 8r + 6s)^{-1}\boldsymbol{C_2}$ can be generated from the matrix $\boldsymbol{A_2}$, where $\boldsymbol{C_2}$ can be denoted as:

$$\boldsymbol{C_2} = \begin{pmatrix} \emptyset_1 & \emptyset_4 & \emptyset_5 & \emptyset_6 & \emptyset_7 \\ \emptyset_4 & \emptyset_2 & \emptyset_8 & \emptyset_9 & \emptyset_6 \\ \emptyset_5 & \emptyset_8 & \emptyset_3 & \emptyset_8 & \emptyset_5 \\ \emptyset_6 & \emptyset_9 & \emptyset_8 & \emptyset_2 & \emptyset_4 \\ \emptyset_7 & \emptyset_6 & \emptyset_5 & \emptyset_4 & \emptyset_1 \end{pmatrix}$$

where

$c_{11} = c_{55} = \emptyset_1 = 4 + 6r + 4s - 2rs - r^2 - s^2$

$c_{22} = c_{44} = \emptyset_2 = 4 + 4r + 4s - 4rs - s^2 - 4r^2$

$c_{33} = \emptyset_3 = 4 + 4r + 2s - 8rs - 4r^2 - 4s^2$

$c_{12} = c_{21} = c_{45} = c_{54} = \emptyset_4 = -1 + 2r - 2s + 3rs + 6r^2 - s^2$

$c_{31} = c_{13} = c_{35} = c_{53} = \emptyset_5 = -1 - 3r + 2s + 4rs - 2r^2 + 4s^2$

$c_{41} = c_{14} = c_{52} = c_{25} = \emptyset_6 = -1 - 3r - 2s - 3rs - 2r^2 - s^2$

$c_{15} = c_{51} = \emptyset_7 = -1 - 2r - 2s - 2rs - r^2 - s^2$

$c_{23} = c_{32} = c_{34} = c_{43} = \emptyset_8 = -1 + r - 3s + 4r^2 - 2s^2$

$c_{24} = c_{42} = \emptyset_9 = -1 - 4r + 3s + 4rs - 4r^2 + 5s^2$

The sequences of b_{n+1} generated by the 5-bandwidth (5×5) Toeplitz matrix can be obtained:

$$\begin{aligned} b_{1,n+1} &= b_{1,n} + \frac{\xi}{\boldsymbol{b_n^T x_n}}[\emptyset_1 x_{1,n} + \emptyset_4 x_{2,n} + \emptyset_5 x_{3,n} + \emptyset_6 x_{4,n} + \emptyset_7 x_{5,n}] \\ b_{2,n+1} &= b_{2,n} + \frac{\xi}{\boldsymbol{b_n^T x_n}}[\emptyset_4 x_{1,n} + \emptyset_2 x_{2,n} + \emptyset_8 x_{3,n} + \emptyset_9 x_{4,n} + \emptyset_6 x_{5,n}] \\ b_{3,n+1} &= b_{3,n} + \frac{\xi}{\boldsymbol{b_n^T x_n}}[\emptyset_5 x_{1,n} + \emptyset_8 x_{2,n} + \emptyset_3 x_{3,n} + \emptyset_8 x_{4,n} + \emptyset_5 x_{5,n}] \end{aligned}$$

$$b_{4,n+1} = b_{4,n} + \frac{\xi}{b_n^T x_n}[\emptyset_6 x_{1,n} + \emptyset_9 x_{2,n} + \emptyset_8 x_{3,n} + \emptyset_2 x_{4,n} + \emptyset_4 x_{5,n}]$$
$$b_{5,n+1} = b_{5,n} + \frac{\xi}{b_n^T x_n}[\emptyset_7 x_{1,n} + \emptyset_6 x_{2,n} + \emptyset_5 x_{3,n} + \emptyset_4 x_{4,n} + \emptyset_1 x_{5,n}]$$

where $n \in [1, \infty)$.

4 Data Analysis

4.1 General Workflow

The general workflow of this study is illustrated in Fig. 1.

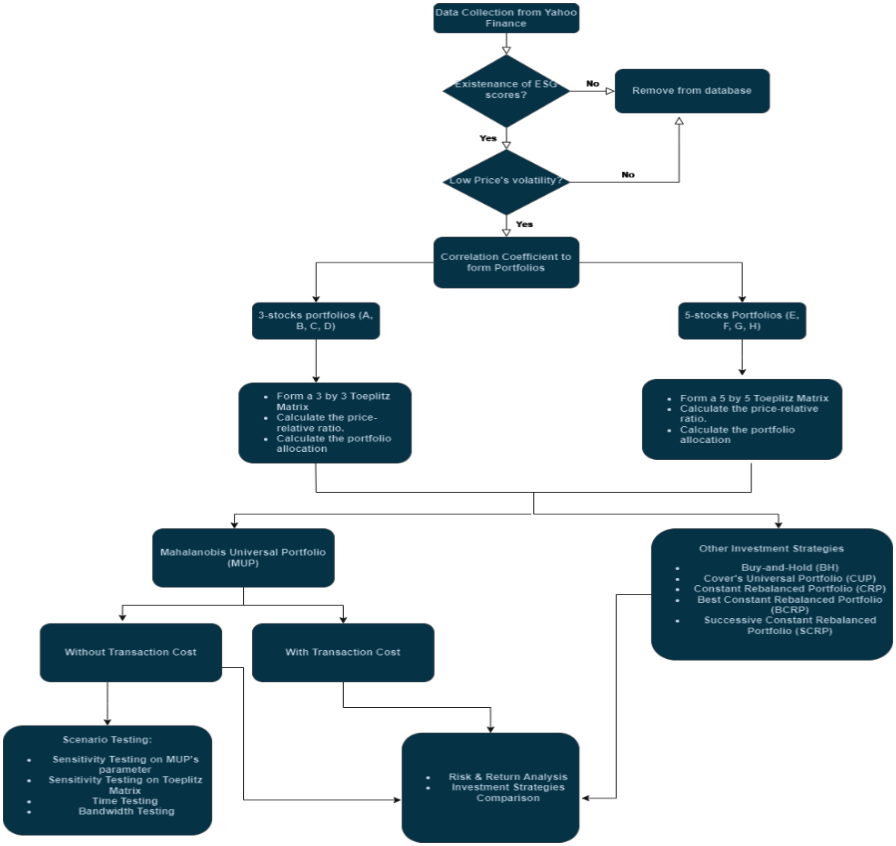

Fig. 1. General workflow of the Mahalanobis universal portfolio and other investment strategies.

The initial stage involved data extraction from Yahoo! Finance using the "getsymbol" function in R-studio. To ensure a robust dataset, this raw data underwent rigorous filtering. Companies were assessed based on their Environmental, Social, and

Goernance (ESG) scores and stock volatility, filtering out those lacking ESG scores or displaying high volatility, hence mitigating potential risks. The next step focused on portfolio diversification by selecting companies with predominantly negative correlations to each other. This led to multiple portfolio configurations, including portfolios with three and five stocks.

These carefully curated portfolios were then subjected to the MUP framework. An appropriate Toeplitz matrix was employed for each portfolio depending on the number of stocks. The study then scrutinised the outcomes with and without accounting for transaction costs to determine the role of side information. A series of sensitivity tests followed, evaluating the resilience and adaptability of the MUP by altering its parameters, time frames, and bandwidth. Finally, the study transitioned into a comparative phase. The MUP was benchmarked against other renowned investment strategies. This comparative analysis assessed each strategy's risk level, ensuring investors have insights to select a strategy aligned with their risk appetites.

Portfolio Construction

The study period is from 2^{nd} January 2015 to 21^{st} January 2021. It was specifically chosen as it encapsulates a time frame with only one major global event, the Covid-19 pandemic. By limiting the period to encompass this singular significant event, the research aims to assess the resilience and adaptability of the strategy in the face of such a significant disruption. This allows for a more concentrated analysis on how the MUP performs and adapts under the stress of a global crisis. 45 companies' stock data were extracted from Yahoo Finance based on several criteria, such as the presence of Environmental, Social and Governance (ESG) scores [24] and stock price volatility. After that, the correlation coefficient between companies is calculated to construct negatively correlated portfolios. Tables 1 to 4 show the heatmap of the correlation Coefficient [25] between companies with a scale from -1 (red) to 1 (green) in portfolios A, B, C and D, respectively. Table 5 shows the list of portfolios formed by three negatively correlated companies.

Table 1. Correlation Coefficient between Companies in Portfolio A.

	Press Metal Aluminium Holdings Bhd	Alliance Bank Malaysia Bhd	MISC Bhd
Press Metal Aluminium Holdings Bhd	1		
Alliance Bank Malaysia Bhd	-0.5234	1	
MISC Bhd	-0.2916	-0.2886	1

Table 2. Correlation Coefficient between Companies in Portfolio B.

	Hartalega Holdings Bhd	UMW Holdings Bhd	Kuala Lumpur Kepong Bhd
Hartalega Holdings Bhd	1		
UMW Holdings Bhd	-0.6716	1	

Kuala Lumpur Kepong Bhd	-0.0589	-0.0867	1

Table 3. Correlation Coefficient between Companies in Portfolio C.

	Parkson Holdings Bhd	Bursa Malaysia Bhd	MISC Bhd
Parkson Holdings Bhd	1		
Bursa Malaysia Bhd	-0.6689	1	
MISC Bhd	-0.0432	-0.3500	1

Table 4. Correlation Coefficient between Companies in Portfolio D.

	Sports Toto Bhd	PPB Group Bhd	Tenaga Nasional Bhd
Sports Toto Bhd	1		
PPB Group Bhd	-0.7304	1	
Tenaga Nasional Bhd	-0.1558	-0.1918	1

Table 5. List of Portfolios which consists of three negatively correlated companies.

Portfolio	Company Name
A	Press Metal Aluminium Holdings Bhd Alliance Bank Malaysia Bhd MISC Bhd
B	Hartalega Holdings Bhd UMW Holdings Bhd Kuala Lumpur Kepong Bhd
C	Parkson Holdings Bhd Bursa Malaysia Bhd MISC Bhd
D	Sports Toto Bhd PPB Group Bhd Tenaga Nasional Bhd

Tables 6 to 9 show the heatmap of the correlation coefficient between companies in portfolios E, F, G and H, respectively. Table 10 shows the list of portfolios formed by five different companies, which are negatively correlated with each other.

Table 6. Correlation Coefficient between Companies in Portfolio E.

	Dialog Group Bhd	Alliance Bank Malaysia Bhd	MISC Bhd	PPB Group Bhd	Genting Bhd
Dialog Group Bhd	1				

Alliance Bank Malaysia Bhd	-0.6129	1			
MISC Bhd	-0.2749	-0.2885	1		
PPB Group Bhd	0.9441	-0.6523	-0.1876	1	
Genting Bhd	-0.702	0.8727	-0.1714	-0.7374	1

Table 7. Correlation Coefficient between Companies in Portfolio F.

	MISC Bhd	Malaysia Airports Holdings Bhd	Petronas Gas Bhd	Hartalega Holdings Bhd	IOI Properties Group Bhd
MISC Bhd	1				
Malaysia Airports Holdings Bhd	-0.4686	1			
Petronas Gas Bhd	0.1392	-0.1807	1		
Hartalega Holdings Bhd	-0.1090	-0.2309	-0.6755	1	
IOI Properties Group Bhd	-0.1233	0.0816	0.2516	-0.2038	1

Table 8. Correlation Coefficient between Companies in Portfolio G.

	Petronas Gas Bhd	Westports holdings Bhd	Bumi Armada Bhd	Kuala Lumpur Kepong Bhd	Media Prima Bhd
Petronas Gas Bhd	1				
Westports holdings Bhd	-0.2165	1			
Bumi Armada Bhd	0.7897	-0.6287	1		
Kuala Lumpur Kepong Bhd	-0.1029	-0.0852	-0.0474	1	
Media Prima Bhd	0.9140	-0.4529	0.9145	-0.1296	1

Table 9. Correlation Coefficient between Companies in Portfolio H.

	Media Prima Bhd	Public Bank Bhd	Sime Darby Bhd	PPB Group Bhd	Hartalega Holdings Bhd
Media Prima Bhd	1				
Public Bank Bhd	-0.3028	1			
Sime Darby Bhd	0.8099	-0.1727	1		
PPB Group Bhd	-0.8179	0.2871	-0.8319	1	
Hartalega Holdings Bhd	-0.6902	-0.0825	-0.6374	0.7397	1

Table 10. List of Portfolios, which consists of five negatively correlated companies.

Portfolio	Company Name
E	Dialog Group Bhd Alliance Bank Malaysia Bhd MISC Bhd PPB Group Bhd Genting Bhd
F	MISC Bhd Malaysia Airports Holdings Bhd Petronas Gas Bhd Hartalega Holdings Bhd IOI Properties Group Bhd
H	Petronas Gas Bhd Westports Holdings Bhd Bumi Armada Bhd Kuala Lumpur Kepong Bhd Media Prima Bhd
I	Sime Darby Bhd Public Bank Bhd Hartalega Holdings Bhd PPB Group Bhd Media Prima Bhd

Analysis of Mahalanobis Universal Portfolio (Without Transaction Cost)
Initially, the investor's wealth and portfolio vector are assumed to be $S_0 = 1$ and $b_0 = (0.3333, 0.333, 0.3334)$. The valid range of the Toeplitz matrix's entries and parameter ξ were derived to ensure the portfolio vector's positively definite property and nonnegativity respectively. Algorithm 1 shows the process of deriving a valid interval of parameters for the 3-band Toeplitz matrix and Algorithm 2 describes the calculation of the final investor's wealth at day 1,500, S_{1500} which is generated by 3-band Toeplitz matrix Mahalanobis universal portfolio.

Algorithm 1. Derivation of valid entries for the 3-band Toeplitz matrix.

(1) Create a function to generate a 3-band Toeplitz matrix.
(2) Set a b_range between -1 and 1 with increments of 0.1.
(3) For b in b_range
(4) Call the function to generate 3-band Toeplitz matrix
(5) If (all the eigenvalues of the Toeplitz matrix are non-negative)
(6) Valid_intervals=rbind(valid_intervals,data frame(b=b))
(7) End if
(8) Next b

Algorithm 2. Calculate the total investor's wealth at day 1500, S_{1500} which is produced by 3-bandwidth Toeplitz matrix Mahalanobis universal portfolio.

(1) Initialisation: $S_0 = 1, b_1 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
(2) For r in a valid interval of 3-band Toeplitz matrix's entries
(3) Call the function to generate 3-band Toeplitz matrix
(4) Find the valid interval of parameter ξ.
(5) For n in 1:1,500
(6) For ξ in a valid interval of parameter ξ
(7) Generate $b_n^{T+1} x_{n+1}$
(8) Update universal portfolio return: $S_{n+1} = S_n \times (b_{n+1}^T x_{n+1})$
(9) Next ξ
(10) Next n
(11) Next r

The highest wealth generated by the optimal parameter ξ was listed in Tables 11-14 for ten selected Toeplitz matrix entries. Table 11-14 shows that the highest wealth achieved for 1,500 days, S_{1500} is 5.8896, 3.4499, 0.8597 and 2.5724 for portfolios A, B, C and D, respectively. Portfolio A has the highest wealth achieved among the other portfolios. From Table 11, the highest wealth can be achieved by allocating 90.84% to Press Metal Aluminum Holdings Bhd, 8.64% to Alliance Bank Malaysia Bhd, and 0.52% to MISC Bhd.

Table 11. The highest value of S_{1500} and the corresponding allocation of wealth b_{1500} among the available ξ for 10 selected r values generated from the relative price of portfolio A and 3-bandwidth Mahalanobis universal portfolio.

Portfolio A						
r	Range of ξ	Best ξ	S_{1500}	$\boldsymbol{b}_{1500}$		
-0.5	(-0.196, 0.262)	0.262	5.1002	0.7306	0.0810	0.1884
-0.4	(-0.141, 0.193)	0.193	5.1353	0.7390	0.0806	0.1804
-0.3	(-0.112, 0.160)	0.160	5.2103	0.7568	0.0807	0.1625

-0.2	(-0.094, 0.142)	0.142	5.3126	0.7805	0.0818	0.1377
-0.1	(-0.082, 0.134)	0.134	5.4697	0.8168	0.0806	0.1027
0.1	(-0.067, 0.131)	0.131	**5.8896**	0.9084	0.0864	0.0052
0.2	(-0.062, 0.110)	0.110	5.6875	0.8555	0.1388	0.0057
0.3	(-0.058, 0.093)	0.093	5.4994	0.8046	0.1868	0.0086
0.4	(-0.055, 0.080)	0.080	5.3461	0.7613	0.2289	0.0098
0.5	(-0.053, 0.069)	0.069	5.2008	0.7195	0.2671	0.0134

Note: The Green highlighted row indicates the best wealth achieved after 1,500 trading days using the best ξ value.

Table 12. The highest value of S_{1500} and the corresponding allocation of wealth b_{1500} among the available ξ for 10 selected r values generated from the relative price of portfolio B and 3-bandwidth Mahalanobis universal portfolio.

Portfolio B						
r	Range of ξ	Best ξ	S_{1500}		$\boldsymbol{b}_{1500}$	
-0.5	(-0.176, 0.113)	0.113	**3.4499**	0.4437	0.0710	0.4853
-0.4	(-0.128, 0.083)	0.083	3.4468	0.4430	0.0713	0.4857
-0.3	(-0.103, 0.069)	0.069	3.4435	0.4419	0.0706	0.4875
-0.2	(-0.088, 0.061)	0.061	3.4298	0.4392	0.0726	0.4882
-0.1	(-0.077, 0.057)	0.057	3.4164	0.4362	0.0737	0.4901
0.1	(-0.064, 0.057)	0.057	3.3858	0.4283	0.0736	0.4981
0.2	(-0.059, 0.061)	0.061	3.3644	0.4224	0.0724	0.5053
0.3	(-0.056, 0.069)	0.069	3.3331	0.4137	0.0702	0.5161
0.4	(-0.053, 0.068)	0.068	3.1668	0.3887	0.1156	0.4956
0.5	(-0.051, 0.062)	0.062	2.9498	0.3570	0.1848	0.4583

Note: The Green highlighted row indicates the best wealth achieved after 1,500 trading days using the best ξ value.

Table 13. The highest value of S_{1500} and the corresponding allocation of wealth b_{1500} among the available ξ for 10 selected r values generated from the relative price of portfolio C and 3-bandwidth Mahalanobis universal portfolio.

Portfolio C						
r	Range of ξ	Best ξ	S_{1500}		$\boldsymbol{b}_{1500}$	
-0.5	(-0.226, 0.239)	0.127	0.8169	0.0763	0.4274	0.4963
-0.4	(-0.162, 0.172)	0.093	0.8179	0.0757	0.4256	0.4987
-0.3	(-0.125, 0.136)	0.076	0.8200	0.0744	0.4218	0.5039
-0.2	(-0.102, 0.114)	0.066	0.8228	0.0721	0.4168	0.5111
-0.1	(-0.086, 0.098)	0.060	0.8262	0.0715	0.4107	0.5177
0.1	(-0.065, 0.079)	0.053	0.8346	0.0673	0.3968	0.5359
0.2	(-0.058, 0.073)	0.052	0.8398	0.0644	0.3888	0.5468
0.3	(-0.053, 0.068)	0.051	0.8456	0.0622	0.3799	0.5579

0.4	(-0.048, 0.063)	0.050	0.8522	0.0627	0.3699	0.5673
0.5	(-0.044, 0.060)	0.050	**0.8597**	0.0599	0.3593	0.5808

Note: The Green highlighted row indicates the best wealth achieved after 1,500 trading days using the best ξ value.

Table 14. The highest value of S_{1500} and the corresponding allocation of wealth b_{1500} among the available ξ for 10 selected r values generated from the relative price of portfolio D and 3-bandwidth Mahalanobis universal portfolio.

Portfolio D						
r	Range of ξ	Best ξ	S_{1500}		$\boldsymbol{b}_{1500}$	
-0.5	(-0.125, 0.213)	0.213	**2.5724**	0.0572	0.8286	0.1142
-0.4	(-0.093, 0.155)	0.155	2.5576	0.0591	0.8236	0.1173
-0.3	(-0.077, 0.127)	0.127	2.5394	0.0604	0.8179	0.1217
-0.2	(-0.068, 0.110)	0.110	2.4999	0.0653	0.8046	0.1301
-0.1	(-0.064, 0.101)	0.101	2.4686	0.0676	0.7945	0.1379
0.1	(-0.064, 0.093)	0.093	2.3640	0.0782	0.7588	0.1629
0.2	(-0.068, 0.093)	0.093	2.2921	0.0860	0.7334	0.1806
0.3	(-0.077, 0.096)	0.096	2.2059	0.0950	0.7020	0.2029
0.4	(-0.093, 0.102)	0.102	2.0930	0.1078	0.6590	0.2332
0.5	(-0.125, 0.112)	0.112	1.9411	0.1266	0.5972	0.2762

Note: The Green highlighted row indicates the best wealth achieved after 1500 trading days using the best ξ value.

Algorithm 3 shows the process of deriving valid interval of parameters for the 3-band Toeplitz matrix and Algorithm 4 describe the calculation of the final investor's wealth at day 1,500, S_{1500} which is generated by 5-band Toeplitz matrix Mahalanobis universal portfolio.

Algorithm 3. Derivation of valid entries for the 5-band Toeplitz matrix.

(1) Create function to generate 5-band Toeplitz matrix.
(2) Set a b_range between -1 and 1 with increments of 0.1.
(3) Set a c_range between -1 and 1 with increments of 0.1.
(4) For b in b_range
(5) For c in c_range
(6) Call the function to generate 5-band Toeplitz matrix
(7) If (all the eigenvalues of Toeplitz matrix are non-negative)
(8) Valid_intervals=rbind(valid_intervals,data frame(b=b, c=c))
(9) End if
(10) Next c
(11) Next b

Algorithm 4. Calculate the total investor's wealth at day 1500, S_{1500} which is produced by 5-bandwidth Toeplitz matrix Mahalanobis universal portfolio.

(1) Initialisation: $S_0 = 1, b_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$
(2) For r in a valid interval of 5-band Toeplitz matrix's entries
(3) For s in a valid interval of 5-band Toeplitz matrix's entries
(4) Call the function to generate 5-band Toeplitz matrix
(5) Find the valid interval of parameter ξ.
(6) For n in 1:1,500
(7) For ξ in valid interval of parameter ξ
(8) Generate $b_n^{T+1} x_{n+1}$
(9) Update universal portfolio return: $S_{n+1} = S_n \times (b_{n+1}^T x_{n+1})$
(10) Next ξ
(11) Next n
(12) Next s
(13) Next r

The highest wealth generated by the optimal parameter ξ is listed in Table 15. According to Table 15, the highest wealth achieved for 1,500 days, S_{1500} is 2.7331, 2.1482, 2.0681 and 2.2008 for portfolio E, F, G and H, respectively. Portfolio E has the highest wealth achieved among the other portfolios. From Table 15, the highest wealth can be achieved by allocating 31.70% to Dialog Group Bhd, 14.76% to Alliance Bank Malaysia Bhd, 11.50% to MISC Bhd, 39.65% to PPB Group Bhd, and 2.39% to Genting Bhd.

Table 15. The highest value of S_{1500} and the corresponding allocation of wealth b_{1500} with the best ξ for selected r values generated from the relative price of portfolio E, F, G and H with 5-bandwidth Mahalanobis universal portfolio.

PF	r	s	Range of ξ	Best ξ	S_{1500}	$\boldsymbol{b_{1500}}$				
E	-0.5	-0.1	(-0.219, 0.232)	0.232	**2.7331**	0.3170	0.1476	0.1150	0.3965	0.0239
F	-0.4	0.7	(-0.012, 0.017)	0.017	**2.1482**	0.1492	0.3420	0.0547	0.3827	0.0715
G	-0.1	0.7	(-0.004, 0.005)	0.005	**2.0681**	0.1169	0.4139	0.0186	0.4133	0.0374
H	-0.3	-0.3	(-0.092, 0.073)	0.073	**2.2008**	0.2251	0.1401	0.2740	0.3275	0.0333

Note: PF- Portfolio

Sensitivity Testing on Mahalanobis Universal Portfolio's Parameter (Without Transaction Cost)

The sensitivity test was applied to portfolios A and E since these are the best portfolios among the three and five companies. From the sensitivity testing, we observed that the entries of the Toeplitz matrix played a role in adjusting the magnitude of initial allocation changes in $\boldsymbol{C}$. For example, as r in the 3-band Toeplitz matrix is larger, the changes of allocation $\boldsymbol{b_n}$ for the first and third rows are larger. However, it does not hold for the asset located at the second row of the companion matrix as the equation of entries in terms of r is symmetrical, which indicates that $b(r) = b(-r) = \emptyset$, where $\emptyset$ is the entries of companion matrix in second row. Therefore, as the value of $|r|$ gets smaller, the allocation changes $\boldsymbol{b_n}$ for the second row are higher. Fig.s 2-5 show the relationship between the wealth allocation $\boldsymbol{b_n}$ and the entries of the 3-band Toeplitz matrix at a given level of ξ.

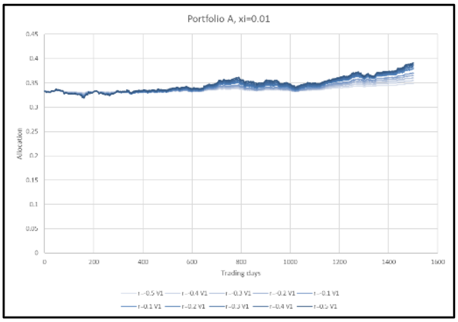

Fig. 2. The allocation of the first asset in portfolio A among 1,500 trading days given $\xi = 0.01, r = -0.5$ to $r = 0.5$.

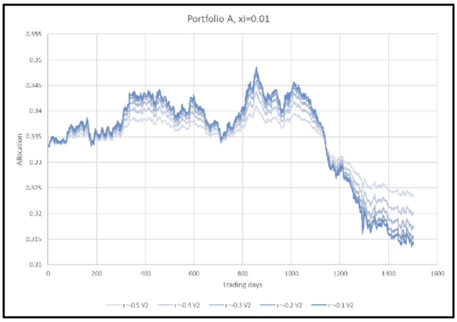

Fig. 3. The allocation of the second asset in portfolio A among 1,500 trading days given $\xi = 0.01, r = -0.5$ to $r = -0.1$.

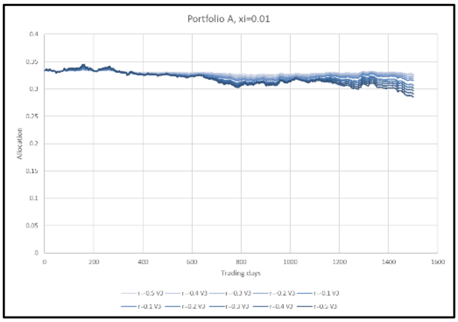

Fig. 4. The allocation of the third asset in portfolio A among 1,500 trading days given $\xi = 0.01, r = -0.5$ to $r = 0.5$.

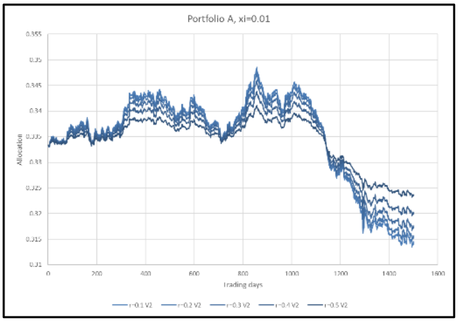

Fig. 5. The allocation of the second asset in portfolio A among 1,500 trading days given $\xi = 0.01, r = 0.1$ to $r = 0.5$.

Additionally, ξ can significantly impact the rate of total wealth changes, but the valid range of ξ is limited due to the portfolio weight constraint. It was also noted that the accumulated wealth on the last trading day is positively correlated with the absolute value of parameter ξ for all possible entries in the Toeplitz matrices within the portfolio. By combining two factors that affect the wealth, such as entries of Toeplitz matrix r and parameter ξ, we observed that for all the tested portfolios, the increasing rate of the wealth would vary based on different values of r from the range of -0.5 to 0.5 as the different value of r will have a different valid range of parameter ξ to ensure the portfolio weight are nonnegative. The sum of weight is equal to one. In certain scenarios, the rate at which overall wealth changes gradually increases with an increment in the value of r starting from -0.5. However, after reaching a specific point, the slope becomes steeper. In other cases, the rate of overall wealth changes initially increases but later decreases after surpassing a certain level of r. Furthermore, a decrease in the value of r leads to an increase in the range of valid ξ. Fig. 6-7 displays the relationship between wealth generated after 1,500 trading day parameter ξ value generated by portfolios A and E, respectively.

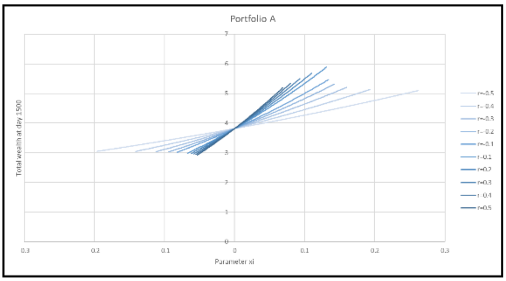

Fig. 6. The wealth generated after 1500 trading day, S_{1500} for each possible value of parameter ξ generated by portfolio A with each selected value of r.

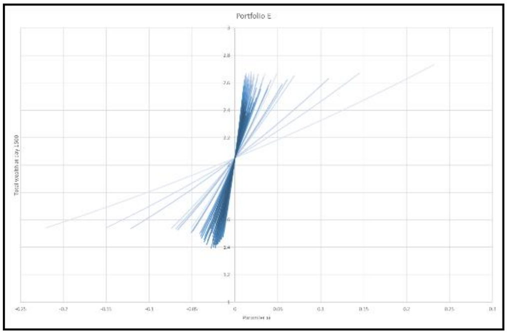

Fig. 7. The wealth generated after 1500 trading day, S_{1500} for each possible value of parameter ξ generated by portfolio E with each selected value of r.

Analysis of Mahalanobis Universal Portfolio (With Transaction Cost)

The total investor's wealth generated by Mahalanobis universal portfolio with transaction cost can be achieved by using Algorithm 5.

Algorithm 5. The calculation of the total investor's wealth at day 1500, S_{1500} produced by 3-bandwidth Toeplitz matrix Mahalanobis universal portfolio given 1% transaction cost rate.

(1) Initialisation: $S_0 = 1, b_1 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right), \gamma = 0.01$

(2) For r in a valid interval of 3-band Toeplitz matrix's entries

(3) Call the function to generate 3-band Toeplitz matrix

(4) Find the valid interval of parameter ξ.

(5) For n in 1:1500

(6) For ξ in valid interval of parameter ξ

(7) Generate $b_n^{T+1} x_{n+1}$

(8) Update universal portfolio return:

$$S_{n+1} = S_0 \times \prod_{t=1}^{n} (\boldsymbol{b}_t^T \boldsymbol{x}_t) \cdot \left(1 - \frac{\gamma}{2} \times \sum_i |\boldsymbol{b}_{n+1}(i) - \boldsymbol{b}_n(i)|\right)$$

(9) Next ξ

(10) Next n

(11) Next r

The best wealth generation will be determined by using the same parameter for the entries of the Toeplitz matrix. The impact of transaction costs on the Mahalanobis universal portfolio with a transaction cost rate of 1% was studied in this paper. Based on Table 16, it is clear that portfolio A sees a notable reduction in transaction fees. This is demonstrated by the larger allocation adjustments compared to other portfolios that use the 3-band Toeplitz matrix. Investors should be aware that if the transaction cost rate goes above a certain threshold, they should opt for the Constant

Rebalanced Portfolio (CRP) as their investment strategy instead of the 3-band Mahalanobis universal portfolio. The decreasing rate can be denoted as the adjustment rate of each r, implying that $r = 0.5$ is the least adjusted outcome for portfolio A. For example, Table 16 indicates investors should choose CRP for portfolio A if the transaction cost rate surpasses 6%.

Table 16. Comparison between CRP and Mahalanobis Universal Portfolio (MUP) with a 1% transaction cost rate.

	Best S_{1500}				
Portfolio	MUP without Transaction Cost	MUP with Transaction Cost	Decreasing Rate	CRP with transaction cost	(MUP, Particular Transaction Cost Rate)
A	5.8896	5.3949	-8.40%	3.504	(3.479, 6.00%)
B	3.4499	3.3339	-3.36%	2.460	(2.458, 9.90%)
C	0.8597	0.8194	-4.68%	0.718	(0.716, 3.80%)
D	2.5724	2.4716	-3.92%	1.411	(1.407, 15.1%)
E	2.7331	2.6351	-3.59%	1.904	(1.903, 9.90%)
F	2.1482	2.0240	-5.78%	1.752	(1.744, 3.50%)
G	2.0681	2.0168	-2.48%	0.719	(0.712, 42.5%)
H	2.2008	2.1424	-2.65%	1.483	(1.482, 14.7%)

Decreasing Rate

At a given level of r and s, it is discovered that as the parameter ξ increased from zero to the upper bound or decreased from zero to the lower bound, the total wealth fell in a larger amplitude, implying a greater decreasing rate. This is because the larger $|\xi|$ will increase the chances of allocation for each rebalancing process. This is consistent with the finding that $|\xi|$ acts as a converging step of each simulation in both positive and negative directions. Fig.s 8-9 and 10-13 show the relationship between total wealth and the parameter ξ given a selected r and s value for portfolios A and E, respectively.

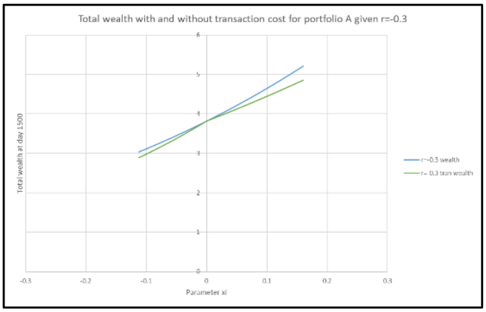

Fig. 8. Given $r = -0.3$, the total wealth achieved by portfolio A with ($\gamma = 0.01$)and without transaction cost at day 1,500.

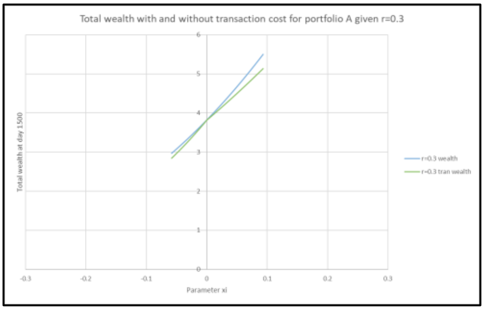

Fig. 9. Given $r = 0.3$, the total wealth achieved by portfolio A with ($\gamma = 0.01$)and without transaction cost at day 1,500.

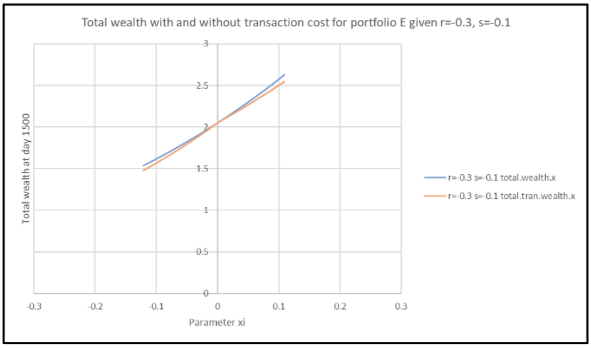

Fig. 10. Given $r = -0.3$, $s = -0.1$ and $\gamma = 0.01$, the total wealth achieved by portfolio E with and without transaction cost at day 1,500.

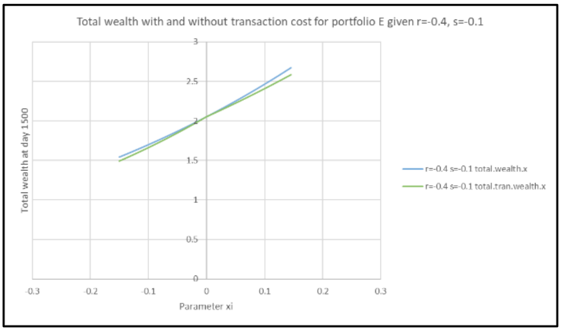

Fig. 11. Given $r = -0.4$, $s = -0.1$ and $\gamma = 0.01$, the total wealth achieved by portfolio E with and without transaction cost at day 1,500.

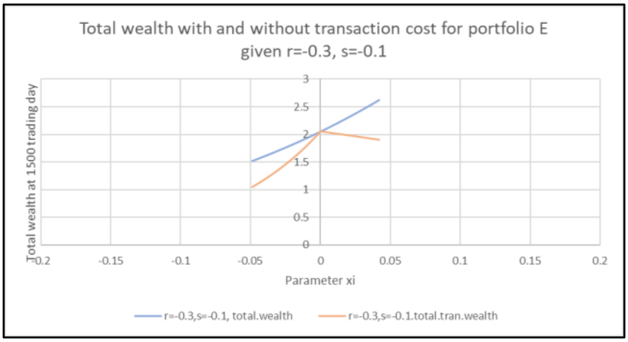

Fig. 12. Given $r = -0.3$, $s = -0.1$ and $\gamma = 0.01$, the total wealth achieved by portfolio E with and without transaction cost at day 1,500.

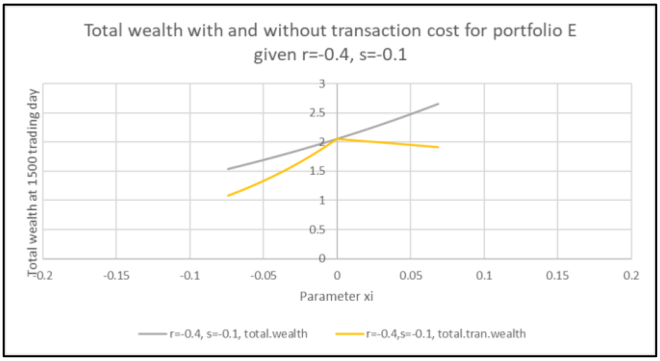

Fig. 13. Given $r = -0.4$, $s = -0.1$ and $\gamma = 0.01$, the total wealth achieved by portfolio E with and without transaction cost at day 1,500.

Time Testing

The following six time frames span lengthy trading periods that began on 2^{nd} January 2015 and encompassed various events and crises. These include the COVID-19 pandemic, the 2020 Malaysia Movement Control Order (MCO) and the Russo-Ukrainian War in 2022. This enabled us to analyse how the Mahalanobis universal portfolio performed in different scenarios. Table 17 displays the total number of trading days in its respective time frame.

Table 17. Number of Trading Days in Different Time Frames

Time frame (YY/MM/DD)	No. of periods (days)
2015/01/02 – 2018/03/15	800
2015/01/02 – 2019/01/10	1,000
2015/01/02 – 2019/11/05	1,200
2015/01/02 – 2021/01/21	1,500
2015/01/02 – 2022/04/12	1,800
2015/01/02 – 2023/02/13	2,000

According to Cover's [5] studies, the universal portfolio's total wealth asymptotically outperformed the market's best-performing stock, implying that the universal portfolio's outperformance may not be immediate. However, it becomes increasingly apparent and significant over time. Therefore, the total wealth should be increased as the longer trading period is adopted. However, from Fig. 14, a downward reversal is observed in the 1,800 trading period that might be attributed to the overall performance of the stock industry. For example, portfolio A was constructed by companies from basic materials, financial services, and industrials. The Russo-Ukrainian War resisted the improvement of the primary material industry due to the higher demand for energy for military purposes [26] [27] [28]. Furthermore, the Covid-19 pandemic led to an economic downturn in Malaysia. It resulted in low profits for financial service providers due to reduced credit demand and a slowdown in economic activities [29]. The pandemic also led to shifts in demand for various goods and commodities, impacting the demand for shipping services classified as industrials.

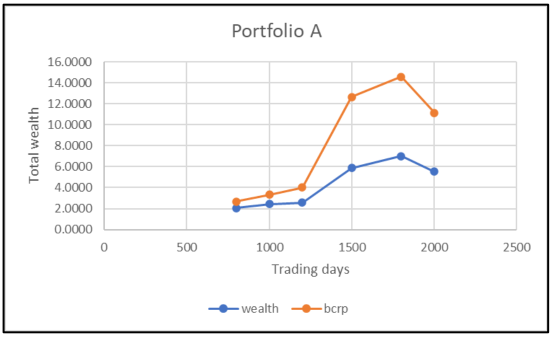

Fig.14. The performance of portfolio A among different trading periods.

Portfolio E consists of Dialog Group Bhd, Alliance Bank Malaysia Bhd, MISC Bhd, PBB Group Bhd, and Genting Bhd, which belong to the energy industry, financial services industry, industrials, consumer defensive, and consumer cyclical, respectively. Dialog Group Berhad is a Malaysian multinational corporation providing integrated technical services to the oil, gas, and petrochemical industries. The company also does the tank terminal business, which involves developing, operating, and managing the facilities that store the oil and gas. However, this pandemic has delayed some incoming projects in Malaysia. According to Abdelrassoul and Rahim's (2021) studies, several factors led to the delay of oil and gas projects: the restriction of moving control order (MCO), the approval from the government, the delay in raw material transmission, and others. Therefore, the project's production decreased significantly due to the frequent disruption, and some other factors further increased the cost of the oil and gas companies. In addition, PPB Group Bhd operates businesses regarding grains and agribusiness, animal feed, consumer food products, and food processing. These products fall within the consumer defensive sector because they are considered essential, meaning they are demanded by people regardless of economic conditions. Based on Zhong's [31] analysis, the consumer defensive sector can still generate a relatively high investment return compared to other poorly affected industries like manufacturing, energy and others due to the low volatility and stable consumer demand. However, the return is insufficient to hedge the loss incurred by other assets in portfolio E, which led to a steep decline in 2021. It can be observed from the Fig. 15.

On the other hand, the consumer cyclical sector is opposite to the consumer defensive sector as it relies on the economic scenario and consumer spending pattern significantly. Genting Bhd is a group operating various businesses like casinos, entertainment, leisure, and hotels. According to Ishak et al. [32] studies, the COVID-19 pandemic negatively impacts companies in the consumer cyclical sector because people are afraid to contact others physically to avoid the spread of COVID-19 and the lockdown measures implemented by the government. So, investors are suggested to hold the non-cyclical companies' stock to earn an abnormal return as the consumer cyclical companies are severely affected by the pandemic.

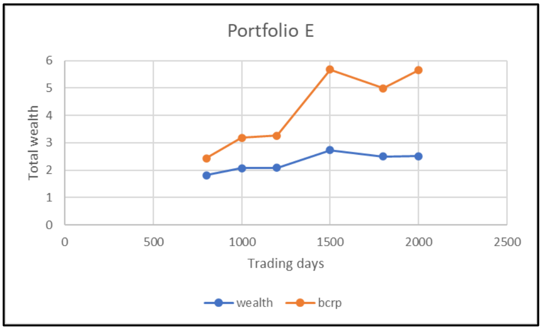

Fig. 15. The performance of portfolio E among different trading periods.

The total wealth generated by $(2k + 1)$-bandwidth Mahalanobis universal portfolio shows a positive relationship to the trading periods due to the rebalancing properties. However, it might be influenced by significant events or global economic downturns, like COVID-19 lockdown measures and the war crisis in Fig.s 14 and 15.

Bandwidth Testing

A study on various bandwidths for Toeplitz matrices to determine how they affect total wealth was conducted in this paper. The bandwidth of a Toeplitz matrix refers to the number of entries used. In this case, four parameters were chosen for a 9-bandwidth Toeplitz matrix when $k = 4$. The portfolio I is adopted in this bandwidth testing, listed in Table 18.

Table 18. The Nine Selected Companies from Various Industries in Portfolio I.

Portfolio I	Industry Field	Company name

	Industrials	Hap Seng Consolidated Bhd
	Energy	Malaysia Marine and Heavy Engineering Holdings Bhd
	Consumer Defensive	FGV Holdings Bhd
	Consumer cyclical	Genting Bhd
	Communication services	Axiata Group Bhd
	Real estate	UEM Sunrise Bhd
	Utilities	YTL Power International Bhd
	Financial services	Malayan Banking Bhd
	Basic materials	Petronas Chemicals Group Bhd

In this analysis process, we will generate the maximum wealth that can be achieved by the Mahalanobis universal portfolio generated by the 3-bandwidth Toeplitz matrix. The optimal value of r obtained from the previous result will be adopted to generate the wealth achieved by the 5-bandwidth Toeplitz matrix for each possible outcome of s. After that, the largest accumulated wealth produced by the 7-band Toeplitz matrix Mahalanobis universal portfolio will be generated from all the possible t values given the optimal value of r and s incurred. Lastly, the optimal value of S_{1500} produced by the 9-band Toeplitz matrix Mahalanobis universal portfolio is obtained given that the value of r, s, and t can generate the maximum S_{1500} at 3-bands, 5-bands, and 7-bands, respectively. Table 19 shows that as more entries of the Toeplitz matrix are adopted, the maximum wealth achieved by Mahalanobis universal portfolio is higher.

Table 19. The highest wealth S_{1500} can be achieved by Mahalanobis universal portfolio using different bandwidths of Toeplitz matrices.

Portfolio	r	s	t	u	Best ξ	S_{1500}
3 band	-0.1	-	-	-	0.006	1.1132
5 band	-0.1	-0.1	-	-	0.007	1.1201
7 band	-0.1	-0.1	-0.1	-	0.009	1.1435
9 band	-0.1	-0.1	-0.1	0.1	0.007	1.1577

Alternative Investment Strategies

Several investment strategies that the other researchers proposed were studied in this paper to evaluate the performance of Mahalanobis' universal portfolio. Table 20 lists all the investment strategies being used in comparison.

Table 20. The Descriptions of various investment strategies

Investment Strategy	**Formulae**
Buy-and-Hold, BH [33]	$S_n(\boldsymbol{b}) = \boldsymbol{b}^{\boldsymbol{T}}\chi_n$

	where $\boldsymbol{\chi_n} = (\chi_{1,n}, \chi_{2,n}, \dots, \chi_{i,n}, \dots, \chi_{m,n})^T$ $\chi_{i,n} = \prod_{j=1}^{n}(1 + x_{i,j})$
Constant Rebalanced Portfolio, CRP [5]	$$S_n(\boldsymbol{b}) = \prod \boldsymbol{b}^{\boldsymbol{T}}\boldsymbol{x}$$ where, $n = 1, 2, 3, \dots,$
Best Constant Rebalanced Portfolio, BCRP [5]	$$S_n^* = \mathbf{max}\ S_n(\boldsymbol{b})$$
Cover's Universal Portfolio, CUP [5]	$$S(\boldsymbol{x}, t) = \prod_{t=1}^{n} \boldsymbol{b}_{\boldsymbol{t}}^{\boldsymbol{T}}\boldsymbol{x}_{\boldsymbol{t}}$$ where $\boldsymbol{x_t}$ is the relative price vector at time t and $\boldsymbol{b_t}$ is the portfolio vector at time t. $\boldsymbol{b_{t+1}} = \left(\frac{\int_X xS(x,t)dx}{\int_X S(x,t)dx}\right)$
Successive Constant Rebalanced Portfolio, SCRP [9]	$$S(\boldsymbol{x}, t) = \prod_{t=1}^{n} \boldsymbol{b}_{\boldsymbol{t}}^{\boldsymbol{T}}\boldsymbol{x}_{\boldsymbol{t}}$$ where $\boldsymbol{x_t}$ is the relative price vector at time t and $\boldsymbol{b_t}$ is the portfolio vector at time t.

The wealth's allocation on the first trading day using the benchmark of $\frac{1}{N}$ portfolio. DeMiguel et al. [34] studies tried 14 optimal stock allocation models, such as the Sharpe ratio and other extensions of the sample-base mean-variance strategy compared to the equal-weight portfolio. As a result, they found that the mean-variance portfolio does not outperform the equal-weight portfolio. Therefore, the portfolio vector $\boldsymbol{b} = \left(\frac{\mathbf{1}}{\boldsymbol{N}}, \frac{\mathbf{1}}{\boldsymbol{N}}, \dots, \frac{\mathbf{1}}{\boldsymbol{N}}\right)^{\boldsymbol{T}}$, where $\boldsymbol{N}$ is the number of stocks in the empirical datasets. were applied in the strategies listed in Table 20. Besides that, we assume that the initial investor's wealth $S_0(\boldsymbol{b}) = 1$. Table 21 shows the best wealth generated by eight different portfolios over 1500 trading days without transaction costs. These portfolios include the Mahalanobis universal portfolio (MUP), BH, CRP, BCRP, CUP and SCUP. According to Fig. 16, the BCRP strategy performed better than all the other investment strategies except for portfolio G. This could be due to the unique allocation measurement used in SCRP. The MUP, with a bandwidth of $(2k + 1)$, is a moderate strategy that can compete with the BH strategy and outperform CUP and CRP. However, portfolio C faced losses due to the economic downturn and other factors discussed in this paper. Therefore, utilising the MUP as an investment strategy involves a certain level of risk.

Table 21. Comparison between different investment strategies among 1500 trading days without transaction cost.

Portfolio	S_{1500}					
	MUP	BH	CRP	BCRP	CUP	SCRP
A	**5.8896**	5.505	3.818	12.656	4.230	7.640
B	3.4499	3.673	2.693	6.407	2.956	3.531
C	0.8597	0.926	0.789	1.613	0.836	0.941
D	2.5724	1.498	2.738	6.755	1.745	5.711
E	2.7331	2.896	2.052	5.670	2.211	3.628
F	2.1482	1.926	1.917	3.973	1.915	2.485
G	2.0681	3.896	0.799	8.363	1.203	5.824
H	2.2008	2.790	1.614	6.170	1.823	3.657

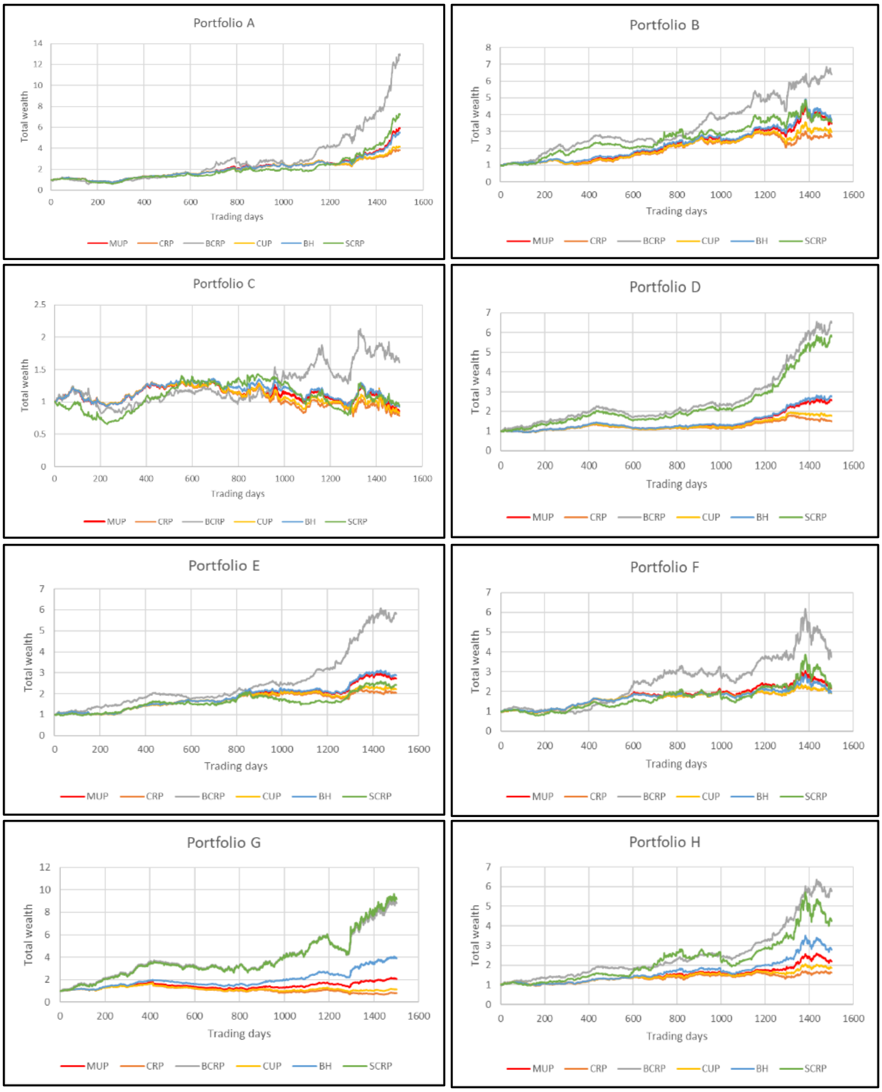

Fig. 16. Comparison of total wealth at day 1,500 between Mahalanobis universal portlio and alternative strategies for portfolios A-H.

Risk and Return Analysis

Other than the above analysis, in our comprehensive assessment of eight distinct investment portfolios, each employing different strategies, we have strategically utilised the Annual Percentage Yield (APY) and Annualised Standard Deviation (ASTDV) to quantify and comprehend these portfolios' return and risk aspects. According to Soldofsky and Biderman's [35] studies, the APY considered the nominal interest rate and compounding periods, presenting a holistic perspective on the annual return potential of these investment strategies. Concurrently, the ASTDV encapsulates the annualised level of risk inherent in these portfolios. Visualising these metrics on a risk and return graph accentuates the dynamic relationship between risk and return. Notably, from Fig. 17 and Fig. 18, our analysis reveals that portfolio A emerges as the most risk-oriented among the eight portfolios, exhibiting the highest ASTDV while concurrently yielding the highest return. Furthermore, delving deeper into portfolio A, our scrutiny identifies the strategy BCRP as the standout performer, attaining the highest return within this high-risk profile. This analysis underscores the crucial role of APY and ASTDV in gauging the potential for returns and the level of risk associated with diverse investment strategies, guiding investors towards informed decisions aligned with their risk tolerance and financial goals. Fig. 16 illustrates the risk and return analysis among eight portfolios, and Fig. 17 demonstrates the risk and return analysis between different investment strategies for portfolio A.

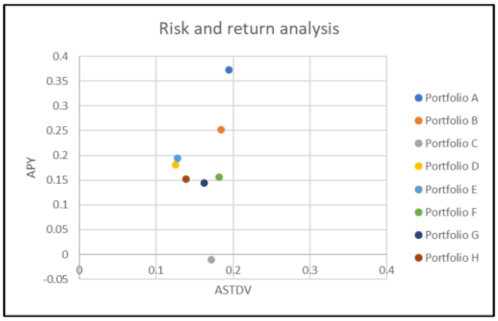

Fig. 17. Risk and Return Analysis for Portfolios A-H.

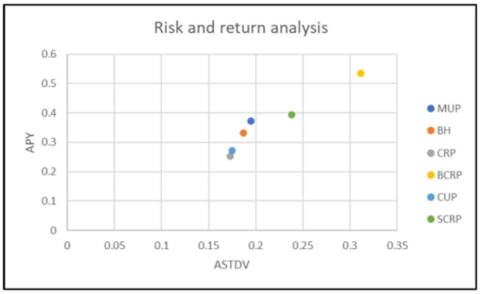

Fig. 18. Risk and Return Analysis between Various Investment Strategies for Portfolio A.

5 Conclusion and Discussion

In this paper, we aimed to create the Mahalanobis universal portfolio by utilising a $(2k + 1)$-bandwidth Toeplitz matrix and based on data gathered from Yahoo Finance. We constructed eight portfolios, with half consisting of three companies and the other half consisting of five companies. To ensure quality, we applied some filtering criteria during the construction process. We then benchmarked the performance of these portfolios against established strategies such as BCRP and CRP and compared them to other investment strategies like BH, SCRP, and CUP. Using a developed mathematical algorithm, we monitored wealth distribution and accumulation over 1,500 trading days. One striking observation was the superior performance of portfolio A over the 1500 days. However, the wealth generated by the $(2k + 1)$-bandwidth Toeplitz matrix Mahalanobis universal portfolio lagged behind strategies like BCRP and SCRP but surpassed CRP across all portfolios. The limited decimal places in the adopted parameters were identified as possible limitations in achieving optimal wealth.

Transaction costs emerged as a significant factor in wealth generation. Beyond a specific transaction cost rate, CRP, with an optimal ξ of zero, proved preferable. Without these costs, the parameter ξ became a vital determinant of wealth. The greater the appropriate value and sign of parameter ξ, the higher the wealth on day 1500. Nevertheless, the volatile stock market, influenced by events like pandemics or economic crises, calls for investor vigilance. In-depth industry analysis and risk diversification are vital before investing.

The bandwidth of the Toeplitz matrix also stood out as a determining factor for end-of-day wealth. The findings revealed a direct relationship between bandwidth and the number of Toeplitz matrix entries. Increasing parameters in the algorithm led to higher allocations to assets with more substantial returns. This suggests that investors could strategically rebalance to potentially high-performing stocks. From a risk-return perspective, the BCRP strategy overshadowed most, except portfolio G, which might be attributed to SCRP's unique allocation metrics. Notably, the MUP with a $(2k + 1)$ bandwidth presented a competitive strategy rivalling BH and outperforming CUP and CRP.

It's crucial to accurately select parameters for the Toeplitz matrix within an optimal range to fully maximise the potential for generating wealth. However, using too few decimal places may negatively impact performance, while increasing them could lead to better results. Integrating fundamental analysis and modern portfolio theory principles is essential to ensure that your stock returns outperform others. Additionally, exploring the pseudo-Mahalanobis universal portfolio, which allows for a wider range of Toeplitz matrix entries, is necessary. Sensitivity testing can also help identify the factors influencing wealth or allocations in higher band Toeplitz matrix Mahalanobis portfolios.

In essence, the universal portfolio strategy, as Cover [5] proposed, is a viable option for long-term investors, suggesting the potential for higher expected returns over prolonged trading durations. However, this study's findings highlight that this strategy's consistent efficacy isn't guaranteed, being susceptible to fluctuations in the broader industry performance. Such variability might make the risk-reward trade-off less appealing for some investors. While transaction costs impact the universal portfolio's performance due to its intrinsic need for ongoing rebalancing, it's noteworthy that the MUP with transaction costs still manages to yield a competitive return. The returns remain commendably similar when comparing this adjusted MUP to its counterpart without transaction costs. Moreover, it's impressive that this version of MUP can surpass the results of several renowned investment strategies, including the CRP, BH and Cover's own universal portfolio model, CUP.

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