Lie symmetry analysis and exact solutions of the (2+1)-dimensional generalized KdV equation

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ABSTRACT

In this paper, the symmetry group method is used to study a new $(2+1)-$dimensional generalized KdV equation. The Lie point symmetries of $(2+1)-$dimensional generalized KdV equation are obtained. Symmetry reductions and several interesting explicit solutions to the $(2+1)-$dimensional generalized KdV equation are given by the Riccati equation expansion method.

Keywords: *$(2+1)-$dimensional generalized KdV equation, Lie symmetry, Exact solutions, Riccati equation expansion method*

1. INTRODUCTION

The nonlinear partial differential equations (PDEs) arise in different areas of applied mathematics, physics, and engineering, including plasma physics [1], biology [2], fluid mechanics [3], thermodynamics [4], [5], nonlinear optics [6], quantum mechanics [7], condensed matter physics [8], plasma wave [9], etc. Therefore, seeking the exact solutions of the nonlinear PDEs play an important role in the nonlinear field. So far, many effective methods have been developed, such as the Hirota bilinear method [10], simplest equation method [11], auxiliary equation method [12], inverse scattering theory [13], homotopy perturbation transform method [14], Darboux transformation [15], Lie symmetry method [16], [17], [18], [19], and so on. Among the above-mentioned methods, the Lie symmetry method is a very effective method for solving linear and nonlinear PDEs, which enables to derive of the solutions of differential equations in a completely algorithmic way without appealing to special lucky guesses. Lie symmetry method can also be used to determine invariant solutions to initial and boundary value problems and to derive conserved quantities. In recent years, some interesting results for higher-order nonlinear equations have been obtained by combining the symmetry reduction method with other methods, such as the auxiliary function method and the simplest equation method.

In this paper, we consider the $(2+1)-$dimensional generalized KdV equation in the form of

$$6au_xu_{xx} + au_{xxxx} + 3b(u_xu_t)_x + bu_{xxxt} + cu_{xx} + du_{yy} + eu_{xt} = 0, \tag{1}$$

where a, b, c, d and e are arbitrary constants. In [20], the new class of soliton solutions to gKdV (1) were constructed by using the new extended generalized Kudryashov and improved $tan(\phi/2)$-expansion methods. In [21], N-soliton solutions of the equation (1) were investigated by the Hirota method. As far as we know, Lie symmetry analysis of equation (1) have not been studied.

This paper is organized as follows: In Sect. 2, Lie symmetries of equation (1) are given. In Sect. 3, Riccati equation method is introduced. In Sect. 4, we will constructe the exact solutions of the reduced equations by Riccati equation expansion method. In Sect. 5, we give some summaries and discussions.

2. LIE SYMMETRY ANALYSIS

In this section, we will construct Lie symmetries of the $(2+1)-$dimensional generalized Korteweg-de Vries equation.

Let us consider the Lie group of point transformations in x, y, t, u given by

$$\begin{aligned} x^* &= x + \epsilon\xi(x, y, t, u) + O(\epsilon^2), \\ y^* &= y + \epsilon\tau(x, y, t, u) + O(\epsilon^2), \\ t^* &= t + \epsilon\eta(x, y, t, u) + O(\epsilon^2), \\ u^* &= u + \epsilon\phi(x, y, t, u) + O(\epsilon^2), \end{aligned}$$

where $\epsilon \ll 1$ is a group parameter, and ξ, τ, η, ϕ are the infinitesimals. The corresponding Lie algebra is generated by the vector field

$$V = \xi(x, y, t, u)\frac{\partial}{\partial x} + \tau(x, y, t, u)\frac{\partial}{\partial y} + \eta(x, y, t, u)\frac{\partial}{\partial t} + \phi(x, y, t, u)\frac{\partial}{\partial u}. \tag{2}$$

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The above transformation group is admitted by equation (1) if and only if

$$\mathrm{pr}^{(4)}V(\Delta)|_{\Delta=0}=0, \tag{3}$$

where $\Delta = 6au_xu_{xx} + au_{xxxx} + 3b(u_xu_t)_x + bu_{xxxt} + cu_{xx} + du_{yy} + eu_{xt}$ and $\mathrm{pr}^{(4)}$ is the fourth prolongation of the vector field (2).

From Lie's theory, we have

$$\mathrm{pr}^{(4)}V = \phi^x\frac{\partial}{\partial u_x} + \phi^t\frac{\partial}{\partial u_t} + \phi^{xx}\frac{\partial}{\partial u_{xx}} + \phi^{yy}\frac{\partial}{\partial u_{yy}} + \phi^{xt}\frac{\partial}{\partial u_{xt}} + \phi^{xxxx}\frac{\partial}{\partial u_{xxxx}} + \phi^{xxxt}\frac{\partial}{\partial u_{xxxt}},$$

where

$$\begin{aligned}
\phi^x &= D_x(\phi-\xi u_x-\tau u_y-\eta u_t)+\xi u_{xx} + \tau u_{xy}+\eta u_{xt},\\
\phi^t &= D_t(\phi-\xi u_x-\tau u_y-\eta u_t)+\xi u_{xt} + \tau u_{yt}+\eta u_{tt},\\
\phi^{xx} &= D_{xx}(\phi-\xi u_x-\tau u_y-\eta u_t)+\xi u_{xxx} + \tau u_{xxy}+\eta u_{xxt},\\
\phi^{yy} &= D_{yy}(\phi-\xi u_x-\tau u_y-\eta u_t)+\xi u_{xyy} + \tau u_{yyy}+\eta u_{yyt},\\
\phi^{xt} &= D_{xt}(\phi-\xi u_x-\tau u_y-\eta u_t)+\xi u_{xxt} + \tau u_{xyt}+\eta u_{xtt},\\
\phi^{xxxx} &= D_{xxxx}(\phi-\xi u_x-\tau u_y-\eta u_t)+\xi u_{xxxxx} + \tau u_{xxxxy}+\eta u_{xxxxt},\\
\phi^{xxxt} &= D_{xxxt}(\phi-\xi u_x-\tau u_y-\eta u_t)+\xi u_{xxxxt} + \tau u_{xxxty}+\eta u_{xxxtt}.
\end{aligned} \tag{4}$$

Expanding (3), we obtain

$$\begin{aligned}
&6au_{xx}\phi^x + 3bu_{xt}\phi^x + 3bu_{xx}\phi^t + 6au_x\phi^{xx}\\
&+3bu_t\phi^{xx} + c\phi^{xx} + d\phi^{yy} + 3bu_x\phi^{xt} + e\phi^{xt}\\
&+a\phi^{xxxx} + b\phi^{xxxt} = 0.
\end{aligned} \tag{5}$$

Substituting (4) into (5), we obtain the following determining equations

$$\begin{aligned}
&\eta_u=0,\quad \eta_y=0,\quad \eta_x=0,\quad \xi_u=0,\quad \xi_{tt}=0,\\
&\xi_y=0,\quad \eta_t=\frac{b\xi_t+a\xi_x}{a},\quad \xi_{xt}=0,\quad \xi_{xx}=0,\\
&\tau_u=0,\quad \tau_y=\frac{b\xi_t+4a\xi_x}{2a},\quad \tau_t=0,\quad \tau_x=0,\\
&\phi_u=-\xi_x,\quad \phi_{yy}=0,\quad \phi_x=-\frac{2e\xi_x}{3b},\\
&\phi_t=\frac{1}{3ab^2}(-b^2c\xi_t+abe\xi_t-2abc\xi_x+4a^2e\xi_x).
\end{aligned}$$

Then solving the determining equations, we find that

$$\begin{aligned}
\xi &= a_1t+a_2x+a_3,\\
\tau &= A_1a_1y+2a_2y+a_5,\\
\eta &= 2A_1a_1t+a_2t+a_4,\\
\phi &= -a_2u-A_2a_2x+a_6y+A_3a_1t+A_4a_2t+a_7,
\end{aligned} \tag{6}$$

where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are arbitrary constants and $A_1=\frac{b}{2a}, A_2=\frac{2e}{3b}, A_3=\frac{ae-bc}{3ab}, A_4=\frac{4ae-2bc}{3b^2}$.

Hence, the infinitesimal symmetry of the equation (1) form the seven-dimensional Lie algebra L_7 spanned by the following linearly independent operators

$$\begin{aligned}
V_1 &= t\frac{\partial}{\partial x} + A_1y\frac{\partial}{\partial y} + 2A_1t\frac{\partial}{\partial t} + A_3t\frac{\partial}{\partial u},\\
V_2 &= x\frac{\partial}{\partial x} + 2y\frac{\partial}{\partial y} + t\frac{\partial}{\partial t} + (A_4t-A_2x-u)\frac{\partial}{\partial u},\\
V_3 &= \frac{\partial}{\partial x},\\
V_4 &= \frac{\partial}{\partial t},\\
V_5 &= \frac{\partial}{\partial y},\\
V_6 &= y\frac{\partial}{\partial u},\\
V_7 &= \frac{\partial}{\partial u}.
\end{aligned} \tag{7}$$

Using commutator operator $[V_k, V_j] = V_kV_j - V_jV_k$, we get the commutator table for (1)

3. AN OVERVIEW OF RICCATI EQUATION EXPANSION METHOD

The Riccati equation expansion method is an efficient mathematical analytical tool for solving the nonlinear PDEs and has been successfully applied to many complex nonlinear models.

We consider the following nonlinear PDEs of the form

$$N(F, F_X, F_T, F_{XX}, ...) = 0. \tag{8}$$

Using the wave transformation $F(X,T) = F_1(\theta)$, one can reduce the equation (8) into the following ordinary differential equation

$$N(F_1, F_1', F_1'', ...) = 0, \tag{9}$$

where $\theta = X - \omega T$ is the wave transformation with the arbitrary constants ω.

To simplify (9), we assume the trial solution to be of the following form

$$F_1(\theta) = \sum_{i=0}^{n} m_i f^i(\theta), \tag{10}$$

where m_i are arbitrary constants that will be confirmed later and f satisfies the Riccati equation

$$f' = \lambda + f^2, \tag{11}$$

where λ is arbitrary constant. The well-known solutions of the Riccati Equation (11) is

$$f = \begin{cases}
-\sqrt{-\lambda}\tanh(\sqrt{-\lambda}\theta), & \lambda<0,\\
-\sqrt{-\lambda}\coth(\sqrt{-\lambda}\theta), & \lambda<0,\\
-\frac{1}{\theta}, & \lambda=0,\\
\sqrt{\lambda}\tan(\sqrt{\lambda}\theta), & \lambda>0,\\
-\sqrt{\lambda}\cot(\sqrt{\lambda}\theta), & \lambda>0.
\end{cases} \tag{12}$$

Then applying the homogeneous balancing principal to equation (9), we can determine the value of n.

Table 1. Commutator table for (1)

$[V_k, V_j]$	V_1	V_2	V_3	V_4	V_5	V_6	V_7
V_1	0	0	0	$-V_3-2A_1V_4-A_3V_7$	$-A_1V_5$	A_1V_6	0
V_2	0	0	$-V_3+A_2V_7$	$-V_4-A_4V_7$	$-2V_5$	$3V_6$	V_7
V_3	0	$V_3-A_2V_7$	0	0	0	0	0
V_4	$V_3+2A_1V_4+A_3V_7$	$V_4+A_4V_7$	0	0	0	0	0
V_5	A_1V_5	$2V_5$	0	0	0	V_7	0
V_6	$-A_1V_6$	$-3V_6$	0	0	$-V_7$	0	0
V_7	0	$-V_7$	0	0	0	0	0

Substituting (10) and (11) into (9), and equating the coefficients of the powers of f^i to zero, we can get the system of algebraic equations for m_i and ω. Thus, the transformation turns the solving differential equations into algebraic manipulations.

4. EXACT SOLUTIONS

In this section, we will construct the exact solutions of the $(2+1)-$dimensional generalized KdV equation combining Lie symmetry and Riccati equation expansion method.

4.1. $\mathbf{V_1 = t\frac{\partial}{\partial x} + A_1 y\frac{\partial}{\partial y} + 2A_1 t\frac{\partial}{\partial t} + A_3 t\frac{\partial}{\partial u}}$

For the generator V_1, the characteristic equations are

$$\frac{dx}{t} = \frac{dy}{A_1 y} = \frac{dt}{2A_1 t} = \frac{du}{A_3 t}.$$

The similarity form of the solution of equation (1) is

$$u(x,y,t) = \frac{A_3}{2A_1}t + F(X,Y), \tag{13}$$

with similarity variables $X = 2A_1x - t$ and $Y = \frac{y^2}{t}$, where $A_1 = \frac{b}{2a},\ A_3 = \frac{ae-bc}{3ab}$.

Substituting (13) into (1), we obtain reduction form of equation (1)

$$\begin{aligned} &2a^2dF_Y - 3b^3YF_{XX}F_Y + 4a^2dYF_{YY} - bY \\ &[(ae+3b^2F_X)F_{XY} + \frac{b^3}{a}F_{XXXY}] = 0. \end{aligned} \tag{14}$$

4.2. $\mathbf{V_2 = x\frac{\partial}{\partial x} + 2y\frac{\partial}{\partial y} + t\frac{\partial}{\partial t} + (A_4 t - A_2 x - u)\frac{\partial}{\partial u}}$

For the generator V_2, the characteristic equations are

$$\frac{dx}{x} = \frac{dy}{2y} = \frac{dt}{t} = \frac{du}{A_4t - A_2x - u}.$$

The similarity form of the solution of equation (1) is

$$u(x,y,t) = \frac{1}{2}A_4t - A_2x + \frac{1}{t}F(X,Y), \tag{15}$$

with similarity variables $X = \frac{x}{t}$ and $Y = \frac{y}{t^2}$, where $A_2 = \frac{2e}{3b},\ A_4 = \frac{4ae-2bc}{3b^2}$.

Substituting (15) into (1), we obtain reduction form of equation (1)

$$\begin{aligned} &-2eX^2F_X + 6bYF_X^2 - dYF_{YY} - 2eX^2Y^2F_{XY} \\ &- eX^3F_{XX} + 6bY^2F_XF_{XY} + \frac{2ae}{b}X^2F_{XX} \\ &+ 3bYF_{XX} + 6bY^2F_YF_{XX} - 6aYF_XF_{XX} \\ &+ 6bXYF_XF_{XX} + 4bYF_{XXX} + 2bY^2F_{XXXY} \\ &+ bXYF_{XXXX} - aYF_{XXXX} = 0. \end{aligned} \tag{16}$$

4.3. $\mathbf{V_3 = \frac{\partial}{\partial x}}$

For the generator V_3, the characteristic equations are

$$\frac{dx}{1} = \frac{dy}{0} = \frac{dt}{0} = \frac{du}{0}.$$

The similarity form of the solution of equation (1) is

$$u = F(Y,T), \tag{17}$$

with similarity variables $Y = y,\ T = t$.

Substituting (17) into (1), we obtain reduction form of equation (1)

$$dF_{YY} = 0. \tag{18}$$

Solving (18), we have $F(Y,T) = F_1(T)Y + F_2(T)$. Therefore, we get the solution of (1) as

$$u(x,y,t) = F_1(t)y + F_2(t),$$

where $F_1(t)$ and $F_2(t)$ are arbitrary functions.

4.4. $\mathbf{V_4 = \frac{\partial}{\partial t}}$

For the generator V_4, the characteristic equations are

$$\frac{dx}{0} = \frac{dy}{0} = \frac{dt}{1} = \frac{du}{0}.$$

The similarity form of the solution of equation (1) is

$$u(x,y,t) = F(X,Y), \tag{19}$$

with similarity variables $X = x$ and $Y = y$.

Substituting (19) into (1), we obtain reduction form of the equation (1)

$$dF_{YY} + cF_{XX} + 6aF_XF_{XX} + aF_{XXXX} = 0. \tag{20}$$

4.5. $\mathbf{V_5 = \frac{\partial}{\partial y}}$

For the generator V_5, the characteristic equations are

$$\frac{dx}{0} = \frac{dy}{1} = \frac{dt}{0} = \frac{du}{0}.$$

The similarity form of the solution of equation (1) is

$$u(x,y,t) = F(X,T), \tag{21}$$

with similarity variables $X = x$ and $T = t$.

Substituting (21) into (1), we obtain reduction form of the equation (1)

$$\begin{aligned} &eF_{XT}+3bF_XF_{XT}+cF_{XX}+3bF_TF_{XX}\\ &+6aF_XF_{XX}+bF_{XXXT}+aF_{XXXX}=0. \end{aligned} \tag{22}$$

For this case, using the Riccati equation expansion method, we will construct some new exact solution of equation (1).

Let

$$F(X,T)=F_1(\theta), \tag{23}$$

where $\theta=X-\omega T$ is new independent variable and F_1 is new dependent variable.

Substituting (23) into (22), we obtain following ordinary differential equation

$$cF_1^{''}-e\omega F_1^{''}+6aF_1^{'}F_1^{''}-6b\omega F_1^{'}F_1^{''}+aF_1^{(4)}-b\omega F_1^{(4)}=0. \tag{24}$$

The following three cases will be considered for equation (24).

Case $(\mathbf{1})$: When $c=e\omega, a=b\omega, \lambda<0$, the exact solutions of (24) can be determined as

$$F_1(\theta)=m_0-m_1\sqrt{-\lambda}\tanh(\sqrt{-\lambda}\ \theta),$$
$$F_1(\theta)=m_0-m_1\sqrt{-\lambda}\coth(\sqrt{-\lambda}\ \theta).$$

Thus we have the corresponding exact solutions of equation (1)

$$\begin{aligned} &u(x,y,t)=m_0-m_1\sqrt{-\lambda}\tanh(\sqrt{-\lambda}\ (x-\omega t)),\\ &u(x,y,t)=m_0-m_1\sqrt{-\lambda}\coth(\sqrt{-\lambda}\ (x-\omega t)). \end{aligned} \tag{25}$$

Case $(\mathbf{2})$: When $c=e\omega, a=b\omega, \lambda>0$, the exact solutions of (24) can be determined as

$$F_1(\theta)=m_0+m_1\sqrt{\lambda}\tan(\sqrt{\lambda}\ \theta),$$
$$F_1(\theta)=m_0-m_1\sqrt{\lambda}\cot(\sqrt{\lambda}\ \theta).$$

Then we have the corresponding exact solutions of equation (1)

$$\begin{aligned} &u(x,y,t)=m_0+m_1\sqrt{\lambda}\tan(\sqrt{\lambda}\ (x-\omega t)),\\ &u(x,y,t)=m_0-m_1\sqrt{\lambda}\cot(\sqrt{\lambda}\ (x-\omega t)). \end{aligned} \tag{26}$$

Case $(\mathbf{3})$: When $c=e\omega, a=b\omega, \lambda=0$, the exact solutions of (24) can be determined as

$$F_1(\theta)=m_0-\frac{m_1}{\theta}.$$

Thus we have the corresponding exact solutions of equation (1)

$$u(x,y,t)=m_0-\frac{m_1}{x-\omega t}. \tag{27}$$

4.6. $\mathbf{V_4+b_5V_5+b_6V_6=\frac{\partial}{\partial t}+b_5\frac{\partial}{\partial y}+b_6y\frac{\partial}{\partial u}}$

For the generator $V_4+b_5V_5+b_6V_6$, the characteristic equations are

$$\frac{dx}{0}=\frac{dy}{b_5}=\frac{dt}{1}=\frac{du}{b_6y}.$$

The similarity form of the solution of equation (1) is

$$u(x,y,t)=\frac{b_6}{2b_5}y^2+F(X,T), \tag{28}$$

with similarity variables $X=b_5x$ and $T=b_5t-y$.

Substituting (28) into (1), we obtain reduction form of the equation (1)

$$\begin{aligned} &b_6d+b_5dF_{TT}+b_5^3(e+3bb_5F_x)F_{XT}\\ &+b_5^3cF_{XX}+3bb_5^4F_TF_{XX}+6ab_5^4F_XF_{XX}\\ &+bb_5^5F_{XXXT}+ab_5^5F_{XXXX}=0. \end{aligned} \tag{29}$$

For this case, using the Riccati equation expansion method, we will construct some new exact solution of equation (1).

Let

$$F(X,T)=F_2(\theta), \tag{30}$$

where $\theta=X-\omega T$ is new independent variable, and F_2 is new dependent variable.

Substituting (30) into (29), we obtain following ordinary differential equation

$$\begin{aligned} &b_6d+(\ b_5d\omega^2+b_5^3(c-e\omega)+6b_5^4(a-b\omega)F_2^{'}\ )F_2^{''}\\ &+b_5^5(a-b\omega)F_2^{(4)}=0. \end{aligned} \tag{31}$$

The following three cases will be considered for equation (31).

Case $(\mathbf{1})$: When $\omega\neq0, d=b_6=0, c=e\omega, a=b\omega, \lambda<0$, the exact solutions of (31) can be determined as

$$F_2(\theta)=m_0-m_1\sqrt{-\lambda}\tanh(\sqrt{-\lambda}\ \theta),$$
$$F_2(\theta)=m_0-m_1\sqrt{-\lambda}\coth(\sqrt{-\lambda}\ \theta).$$

Then we get the corresponding exact solutions of equation (1)

$$\begin{aligned} u(x,y,t)=&\frac{b_7}{b_5}y+\frac{b_6}{2b_5}y^2+m_0-m_1\sqrt{-\lambda}\\ &\tanh[\sqrt{-\lambda}\ (b_5x-\omega(b_5t-y))],\\ u(x,y,t)=&\frac{b_7}{b_5}y+\frac{b_6}{2b_5}y^2+m_0-m_1\sqrt{-\lambda}\\ &\coth[\sqrt{-\lambda}\ (b_5x-\omega(b_5t-y))]. \end{aligned} \tag{32}$$

Case $(\mathbf{2})$: When $\omega\neq0, d=b_6=0, c=e\omega, a=b\omega, \lambda>0$, the exact solutions of (31) can be determined as

$$F_2(\theta)=m_0+m_1\sqrt{\lambda}\tan(\sqrt{\lambda}\ \theta),$$
$$F_2(\theta)=m_0-m_1\sqrt{\lambda}\cot(\sqrt{\lambda}\ \theta).$$

Then we get the corresponding exact solutions of equation (1)

$$\begin{aligned} u(x,y,t)=&\frac{b_7}{b_5}y+\frac{b_6}{2b_5}y^2+m_0+m_1\sqrt{\lambda}\\ &\tan[\sqrt{\lambda}\ (b_5x-\omega(b_5t-y))],\\ u(x,y,t)=&\frac{b_7}{b_5}y+\frac{b_6}{2b_5}y^2+m_0-m_1\sqrt{\lambda}\\ &\cot[\sqrt{\lambda}\ (b_5x-\omega(b_5t-y))]. \end{aligned} \tag{33}$$

Case $(\mathbf{3})$: When $\omega \neq 0, d = b_6 = 0, c = e\omega, a = b\omega, \lambda = 0$, the exact solutions of (31) can be determined as

$$F_2(\theta) = m_0 - \frac{m_1}{\theta},$$

Thus we get the corresponding exact solutions of equation (1)

$$u(x, y, t) = \frac{b_7}{b_5}y + \frac{b_6}{2b_5}y^2 + m_0 - \frac{m_1}{b_5 x - \omega(b_5 t - y)}. \tag{34}$$

4.7. The generator $\mathbf{V_5 + b_6 V_6 = \frac{\partial}{\partial y} + b_6 y \frac{\partial}{\partial u}}$

For the generator $V_5 + b_6 V_6$, the characteristic equations are

$$\frac{dx}{0} = \frac{dy}{1} = \frac{dt}{0} = \frac{du}{b_6 y}.$$

The similarity form of the solution of equation (1) is

$$u(x, y, t) = \frac{b_6}{2}y^2 + F(X, T), \tag{35}$$

with similarity variables $X = x$ and $T = t$.

Substituting (35) into (1), we obtain reduction form of equation (1)

$$\begin{aligned} &b_6 d + (e + 3bF_x)F_{XT} + (c + 3bF_T + 6aF_X)F_{XX} \\ &+ bF_{XXXT} + aF_{XXXX} = 0. \end{aligned} \tag{36}$$

5. CONCLUSION

In this paper, the Lie group analysis is used to carry out the similarity reductions of the $(2+1)-$dimensional modified KdV equation. We have obtained the infinitesimal generators, commutator table of Lie algebra, and similarity reduction for the modified KdV equation. The modified KdV equation has been reduced into a new partial differential equation with less number of independent variables. Using Riccati equation expansion method, the new partial differential equation is reduced into an ordinary differential equation and further some new exact solution of the modified KdV are constructed. It is hoped that all the results obtained in this paper can be used to enrich the applications of the nonlinear PDEs in mathematical physics.

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