Hypothetical Learning Trajectory Based on Theory of Didactical Situation: An Initial Learning Trajectory Design to Enhance Mathematical Creativity and Resilience

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ABSTRACT

The aim of this study was to formulate an initial Hypothetical Learning Trajectory (HLT) aimed at fostering the growth of mathematical creativity and resilience. During the design process, researchers implement the Theory of Didactical Situations, which includes specific phases including action situations, formulation situations, validation situations, and institutionalization situations. This article shows the results of a prospective analysis conducted as a first step in the implementation of the Didactical Design Research (DDR) approach. The findings of this study suggest that the enhancement of HLT can be achieved by employing the principles of didactic scenario theory, which involves three primary activities. These activities involve involving in solving of open-ended mathematical problems namely solving open-ended math problems with many correct answers (fluency) (activity 1), many correct solutions (flexibility) (activity 2), and correct novelty solutions (originality) (activity 3), as well as predicting student responses and anticipating responses. The researchers expect that the findings of this study can be advanced and substantiated through a pilot experiment conducted in the field.

Keywords: *Hypothetical Learning Trajectory, Theory of Didactical Situation, Mathematical Creativity, Mathematical Resilience.*

1. INTRODUCTIONS

One of the primary objectives of mathematics education is to provide students with the necessary abilities to engage in creative thinking and problem-solving using a scientific methodology [1]. Students possess the capacity to employ diverse approaches and strategies to effectively address difficulties, hence cultivating their ability to confront the exigencies of both the contemporary and forthcoming contexts [2][3]. To attain this objective, the process of mathematics instruction within the classroom must be both enjoyable and meaningful [4].

Nevertheless, a significant number of Indonesian students experience feelings of anxiety when engaging with mathematics or exhibit limited mathematical resilience, particularly when challenged with complex mathematical problems requiring contextual understanding [5][6]. Consequently, there is a notable deficiency in student performance on national evaluations such as the National Examination and the Competency Assessment of Indonesian Students, where several inquiries examine intricate cognitive abilities such as mathematical creativity [7]. The outcomes of evaluations conducted on students' mathematical proficiencies at the worldwide level, such as PISA and TIMSS, are similarly disheartening [8]. According to the findings of the 2018 PISA survey, it was shown that more than 29% of Indonesian students exhibited a growth mindset, and the remaining pupils demonstrated a fixed mindset [9]. Consequently, students retain the belief that their capacities for growth, including their aptitude for mathematical thinking, specifically in terms of their creative problem-solving abilities, remain unchanged.

One of the contributing factors to this situation is the prevailing utilization of a mechanistic learning method in

A. Kusuma Wardana (ed.), *Proceedings of the 2nd UPY International Conference on Education and Social Science (UPINCESS 2023)*,
Advances in Social Science, Education and Humanities Research 812,
https://doi.org/10.2991/978-2-38476-176-0_25

mathematics education, which places significant emphasis on the development of low-level thinking skills [10]. Consequently, students exhibit less mathematical resilience, leading to reduced academic performance and limited growth in mathematical creativity. To facilitate effective learning, it is essential to address the diverse range of obstacles that learners may encounter. This can be achieved through the implementation of a student-centered approach to education, wherein the learning process is actively engaged and enjoyable for students. The theory of didactical circumstances [11] can be employed as a learning theory to facilitate the development of such learning activities. Multiple studies have documented the beneficial outcomes associated with the implementation of the theory of didactical situations in enhancing the overall efficacy of mathematics education [12][13][14].

In the theory of didactical situation, a didactic-adidactic problem or situation is presented in a conducive context so that it allows students to do mathematics and encourages students to solve mathematical problems creatively and independently [15]. The learning process begins with a mathematical task with the rules surrounding it, then students carry out action situations, formulation situations, validation situations, and finally institutionalization situations [11][16]. Teachers and students also have their respective active roles that are balanced and fair in the form of a didactical contract that is adapted to the learning situation [13].

To implement it, teachers can design a learning design based on the theory of didactical situation in the form of a learning trajectory consisting of learning objectives, learning activities, and expected student responses including didactic and pedagogical anticipation (ADP) that may occur when learning takes place [17]. With this learning design, it is hoped that students can develop their potential, such as students' mathematical creativity, and encourage students to become more resilient when learning mathematics.

In this study, mathematical creativity is seen as 1) a process that generates unusual (novel) solutions and/or insights to a given problem or analytical problem, and/or 2) the formulation of new questions and or the possibility of following old problems which are then transformed into a different angle and new perspective [18][19][20]. Mathematical creativity is the highest level of thinking in the cognitive realm. Mathematical creativity can be developed through solving mathematical problems, especially mathematical problems with many possible solutions [21][22][23]. To measure it, mathematical creativity in solving mathematical problems can be indicated from three aspects, namely fluency, flexibility, and novelty [24][25].

Meanwhile, mathematical resilience is an attitude of maintaining self-efficacy in facing personal or social threats to the pleasure of learning mathematics [26]. Mathematical resilience is considered a positive adaptive attitude towards mathematics so that it will enable students to continue learning despite obstacles and difficulties [27][28]. To explore students' mathematical resilience, there are at least three dimensions that can serve as indications, including 1) the value dimension, namely an experience that mathematics is a valuable subject and worth studying; 2) the struggle dimension (struggle), namely recognizing that learning mathematics does require struggle or hard work, even for people who have high levels of mathematical ability; 3) the growth dimension, namely the belief that everyone can develop math skills and even learn better through effort and support; 4) the dimension of perseverance, namely an attitude that learning mathematics requires perseverance effort [29][30].

Based on the description that has been presented, it is necessary to develop a mathematics learning design based on didactic situation theory in the form of a learning trajectory to develop mathematical creativity and resilience. The design of the learning trajectory then becomes the basis for constructing a local instructional theory of mathematics learning based on didactic situation theory.

2. RESEARCH METHOD

This research aims to develop a local instructional theory to support the development of mathematics creativity and resilience in junior high school students based on the Theory of Didactical Situation. For this need, this study uses a type of research method, namely Didactical Design Research (DDR). Research using the DDR method [31] includes prospective analysis stages (didactical situation analysis before learning), metapedadidactic analysis during learning, and retrospective analysis after learning.

This research is limited to the first stage, namely designing the initial HLT before conducting a pilot experiment to develop mathematical creativity and resilience based on the Theory of Didactical Situations. HLT consists of three main components, including learning objectives, learning activities, and hypothetical learning processes [10]. In this research, learning objectives are formulated based on the aim of developing mathematical creativity and resilience, while learning activities are formulated based on these learning objectives based on didactic situation theory. Thus, this article presents an initial learning design in the form of HLT before the pilot experiment was carried out.

3. RESULT AND DISCUSSION

To explore students' mathematical resilience there are at least three dimensions that can be indicative, including 1) the value dimension, namely an experience that mathematics is a valuable subject and worth studying; 2) the struggle dimension (struggle), namely recognizing that learning mathematics does require struggle or hard work, even for people who have high levels of mathematical ability; 3) the growth dimension, namely the belief that everyone can develop math skills and even learn better through effort and support, 4) the perseverance dimension, namely an attitude that learning mathematics requires perseverance effort [29][30].

3.1. Hypothetical Learning Trajectory (HLT) to Develop Mathematical Creativity and Resilience

The hypothetical learning trajectory (HLT) is a key aspect in planning mathematics learning that provides pedagogical thinking in teaching mathematics to improve students' understanding of mathematics [17]. HLT is an instrument that is used as an extension of a thought experiment, as well as a link between instructional theory and concrete teaching experiments [32]. An HLT consists of learning objectives that are meaningful to students, mathematical tasks to be used in learning, and hypotheses about student learning processes [17]. Learning objectives that have been developed can be the basis for teachers to choose and design math assignments to be used in learning mathematics, as well as helping teachers develop hypotheses for student learning processes. The selected mathematical assignments also become the basis for developing hypotheses on student learning processes. Thus, the learning objectives, math assignments, and student learning process hypotheses are components that are closely related to each other and form the basis for designing HLT.

In this article, the author will discuss the design of HLT based on the theory of didactical situations to develop students' mathematical creativity and resilience. The proposed learning trajectory design is shown in Figure 1.

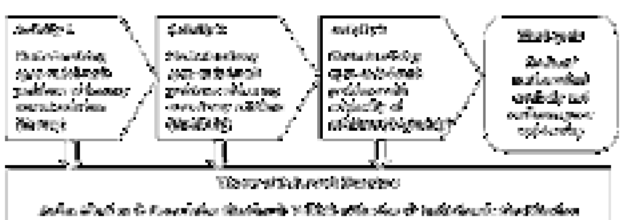

Figure 1 HLT flow scheme for developing mathematical creativity and resilience

3.2. Learning Goals

Researchers formulated learning objectives for this learning design based on the research objectives, namely developing mathematical creativity and resilience (see Figure 1). Therefore, the learning objectives are: 1) Students can develop mathematical creativity based on solving open mathematical problems with indicators of fluency in solving problems with various answers (fluency), solving problems in various ways (flexibility), and novelty in the way of solving and answering (originality); 2) Students have a positive adaptive attitude in the form of mathematical resilience which is shown by appreciation for the usefulness of mathematics, toughness when encountering difficulties, tireless effort, and persistence even though they encounter challenges, difficulties and obstacles in solving open-ended mathematical problems.

3.3. Learning Activities

Based on learning objectives, researchers designed learning activities. In designing learning activities, the researcher uses didactic situation theory, which is based on the stages of an action situation, a formulation situation, a validation situation, and finally an institutionalization situation. In addition, the designs are also based on aspects of mathematical creativity in solving open-ended mathematical problems, namely fluency, flexibility, and originality, as well as the domain of mathematical resilience, including the dimensions of value, struggle, growth, and persistence.

In this article, a summary of the learning activities in activities 1, 2, and 3 is presented, namely students can solve open-ended math questions with many correct answers (learning activity 1), many correct solutions (learning activity 2), and answers that contain elements of novelty or original (learning activity 3)

The learning objectives of learning activities 1, 2, and 3 are 1) Students can solve open-ended math problems with many correct answers (fluency), many correct solutions (flexibility), and new correct answers (originality), 2) Students have a mindset growth, attitude of persistence and toughness in the form of efforts to solve open math problems creatively and have an appreciation for mathematics.

In the early stages of learning, students are given a problem situation in the form of a solution-oriented open-ended math problem that gives rise to many correct answers (activity 1), many correct solutions (activity 2), and new correct answers (originality) (activity 3). The situation of open mathematics problems can also be a means for students to develop aspects of mathematical resilience such as aspects of value, growth (growth mindset), perseverance, and struggle.

In the **Action Situation Stage**, students try to understand the problem by identifying what is known from the problem independently. Then, students try to understand the problem by identifying what is being asked of the question independently. Students try to identify the mathematical concepts used to solve open-ended math problems independently so that various answers are obtained (planning to solve math problems). Students solve the math problem independently based on the mathematical concepts that have been identified as a settlement plan. In these open-ended math problem-solving activities, students must have a growth mindset, an attitude of persistence, and resilience to the obstacles and difficulties they face.

Next, the **Formulation Situation** stage where students in one group discuss the results of solving problems that have been done independently. The discussion was carried out to obtain the results of solving various answers to these questions according to each group. Next, the results of the problem-solving discussion in groups were presented in front of the class with confidence and not afraid of being wrong.

At the **Validation Situation** stage, students who present ask for responses from other groups of students. The answers presented by the presentation group can be accepted or rejected by other students. Criticism or suggestions given by other students can be used as input for improving the answers that have been prepared. The results of group discussions were given verification regarding the correctness of the answers by the teacher.

The final stage is the I**nstitutionalization Situation** stage. At this stage, students pay attention to the explanations and reviews conducted by the teacher regarding the correctness of the variety of answers given by students, including the concepts used to answer the questions. Students answer reflective questions from the teacher about the mathematics material they have studied related to the didactic situation given.

3.4. Prediction of Responses and Anticipation of Student Responses in the Learning Process

In the learning design of this research, there are predictions of responses and anticipation of student responses that will occur. In summary, examples of predicting responses and anticipating student responses based on the stages of didactic situation theory are as follows in Table 1.

Table 1. Examples of Predicted Responses and Anticipated Student Responses to Learning Activities in the Action Situation Stage Short cut keys for the template

Learning Activies	Student Response Prediction	Anticipate Student Responses
Action situation	Students try to understand math problems by identifying what is known from the problem independently, but there may be students who have difficulty	• The teacher provides assistance in the form of clarification questions using the Socratic questioning technique, such as "What is known from the problem?", "What is being asked from the question?" • The teacher provides encouragement to students to try to overcome difficulties in solving problems with mindset intervention. For example, with the sentence, "Come on, solve the problem, don't be afraid of making a mistake!".

Table 1 shows that the teaching and learning process based on the HLT design which was developed based on didactic situation theory contains predictions of student responses and anticipation of student responses from the teacher. An example of predicting student responses in the action situation stage, for example, when students try to understand a mathematical problem by identifying what is known and asking from an open mathematics problem that is presented independently, even though the student may encounter difficulties. An example of anticipating a student's response from the teacher to this matter, for example, the teacher assists in the form of clarifying questions using Socratic questioning techniques, such as "What is known from the question?", "What is asked about the question?". Apart from that, teachers can encourage (and motivate) students to try to overcome difficulties in solving problems with mindset intervention. For example, with the sentence, "Come on, solve the problem, don't be afraid of making a mistake!".

4. CONCLUSION

HLT is a key aspect in planning mathematics learning which presents pedagogical thinking in teaching mathematics to improve students' mathematical understanding. The HLT in this article was developed based on didactic situation theory to develop mathematical creativity and resilience. In the initial design of this HLT, there was a sequence of three main activities, namely students solving open mathematics problems with many correct answers (fluency) (activity 1), many correct solutions (flexibility) (activity 2), and new correct solutions (originality) (activity 3). These learning activities are based on the theory of didactic situations with the stages of action situations, formulation situations, validation situations, and finally institutionalization situations. In addition, the HLT design is also equipped with response predictions and student response anticipation to optimize the achievement of learning objectives.

The results of this research are still preliminary HLT designs that need to be developed further. Apart from that, to prove its validity, it is necessary to carry out a pilot experiment in the field.

AUTHORS' CONTRIBUTIONS

AAS: study conception and design; data collection; analysis and interpretation of results; draft manuscript preparation; AA: study conception and design; analysis and interpretation of results; draft manuscript preparation; SIAD: study conception and design; analysis and interpretation of results; draft manuscript preparation. All authors reviewed the results and approved the final version of the manuscript.

ACKNOWLEDGMENTS

Researchers would like to thank the Directorate General of Higher Education, Research and Technology, Ministry of Education, Culture, Research and Technology of the Republic of Indonesia for funding this research through a Doctoral Dissertation Research Grant, Contract Number 042/E5/PG.02.00.PL/2023.

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