



Research on a Nonlinear Dynamic Model Support Vector Machine Based for Rock Mass Evolution

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Abstract. This article is based on time series and applies support vector machine to establish a nonlinear dynamic model of rock mass evolution. The longest predictable time is given based on the Lyapunov index, and a nonlinear dynamic model prediction model based on support vector machine is proposed through function fitting. The nonlinear dynamic model is combined with nonlinear catastrophe theory to timely reflect the evolution direction of rock mass and make predictions and judgments on its stability, Use mutation theory to analyze its stability. The results indicate that the model has ideal prediction performance and good generalization ability.

Keywords: Support Vector Machine, Time series, Nonlinear dynamics, Cusp mutation

1 Introduction

Displacement is one of the important information feedback from rock mass structures during excavation or deformation[1]. The monitored historical displacement values can be used to model and predict its future evolution and development trends, and timely grasp the changes in the rock mass is of great significance in engineering[2]. The slope rock mass system is a nonlinear dynamic system, and its failure does not come from strict periodicity or uniform distribution of randomness, but from the inherent randomness during the landslide incubation process. Therefore, the deterministic models established by classical theory, or the stochastic models (probability models) established assuming that landslides are random events, cannot fully and truly reflect the essence of landslides[3-4].

The study of rock mass stability is a complex and significant topic, and it is also one of the difficulties in rock mechanics research. Many scholars have started to engage in nonlinear theoretical research in rock mechanics, and have proposed overall research objectives and specific technical ideas[5]. In some rock mass engineering, whether artificial or natural, the deformation is dynamic, and the mechanical parameters inside the system are also variable. Therefore, the rock mass system can be re-

garded as a nonlinear dynamic system. The macroscopic stability and unstable behavior of a system are the stability issues of system equilibrium.

Support vector machine (SVM) is a small sample learning method based on statistical learning theory, which adopts the principle of structural risk minimization and has good generalization ability[6]. By changing the number of support vectors, the model structure can be easily continuously changed, and the upper bound of the model generalization error can be obtained and controlled, independent of the distribution of the training and testing sets.

Therefore, this study adopts a chaotic time series prediction method based on support vector machine to establish a nonlinear dynamic model of rock mass, and based on this, conducts stability mutation analysis. The method proposed in this study can timely reflect the evolution direction of rock mass and make predictions and judgments on its stability, which is of great practical significance for establishing monitoring and warning systems for rock mass engineering systems such as slopes or dam foundations.

2 Methodology

2.1 Support Vector Machine

SVM are used to solve regression problems. Firstly, we consider using a linear function factory $f(x) = wx + b$ to fit data $\{x_i, y_i\}, i = 1, 2, \dots, k$. Assuming that all training data is fitted with a linear function without error under ϵ accuracy, i.e.:

$$\begin{cases} y_i - wx_i - b \leq \epsilon \\ wx_i + b - y_i \leq \epsilon (i = 1, 2, \dots, k) \end{cases} \tag{1}$$

The optimal function is $0.5 \|w\|^2$. Considering the allowable error, the relaxation factor is introduced $\xi_i \geq 0, \xi_i' \geq 0$. Thus equation (1) is changed to:

$$\begin{cases} y_i - wx_i - b \leq \epsilon + \xi_i \\ wx_i + b - y_i \leq \epsilon + \xi_i' (i = 1, 2, \dots, k) \end{cases} \tag{2}$$

The optimal function is $0.5 \|w\|^2 + c \sum_{i=1}^k (\xi_i + \xi_i')$. Where the constant $C > 0$ indicates the degree of punishment for samples exceeding the error. By defining the Lagrange function and using optimization methods, the following quadratic problems can be obtained:

$$\begin{cases} \min(0.5 \sum_{i=1}^k (a_i - a_i')(a_j - a_j')(x_i x_j) - \sum_{i=1}^k y_i (a_i - a_i') + \epsilon \sum_{i=1}^k (a_i + a_i')) \\ \sum_{i=1}^k (a_i - a_i') = 0, 0 \leq a_i, a_i' \leq c \end{cases} \tag{3}$$

Thus, the support vector machine fitting function can be obtained as:

$$f(x) = wx + b = \sum_{i=1}^k (a_i - a_i')(x \cdot x_i) + b \tag{4}$$

The sample data corresponding to $a_i - a_i'$ not zero is the support vector.

Kernel functions can be implemented using functions in the original space, without the need to know the specific form of nonlinear mapping, thus cleverly solving the problem of w being unable to be explicitly expressed due to unknowns. Commonly used kernel functions include polynomial kernel functions, radial basis functions (RBF), sigmoid functions, etc.

2.2 Nonlinear Dynamic Model of Rock Mass Evolution

Generally, the longest forecasting duration is defined as $Tm = \frac{1}{\lambda_1}$, λ_1 is Lyapunov index. The Lyapunov exponent is calculated using a small data volume method based on reconstructed phase space. For the nonlinear displacement time series of rock mass evolution, a time series of displacement changes over time can be obtained through monitoring $\{x_i\}$. By using support vector machine to learn the measured displacement, the nonlinear relationship of the time series can be obtained. Based on the learned support vector machine, equation (4) can be used to predict the displacement of rock mass evolution.

2.3 Catastrophic Analysis of Rock Mass Evolution

Due to the variation of displacement observations over time, the displacement values of the studied rock mass system can be represented by a continuous function $S = f(t)$, where t is the loading time scale and the function is expanded in Taylor series. Therefore, when fitting the rock mass displacement time curve, the fourth order term is taken:

$$S = \sum_{i=1}^4 a_i t^i \tag{5}$$

Where $a_i = \sum_{i=1}^4 \frac{\partial^i f}{\partial t^i}$. Letting $t \rightarrow x - \frac{a_3}{4a_4}$. Then equation (5) can be transformed into a standard potential function form with sharp point mutations:

$$V(x) = x^4 + \mu x^2 + \nu x \tag{6}$$

Where $\mu = \frac{a_2}{a_4} - \frac{3a_3^2}{8a_4^2}$; $v = \frac{a_1}{a_4} - \frac{a_2a_3}{2a_4^2} - \frac{a_3^3}{8a_4^3}$. Equilibrium surface equation M:

$$\frac{\partial v}{\partial x} = 4x_0^3 + 2\mu x_0 + v \quad (7)$$

According to the theory of sharp point bifurcation sets, the bifurcation set equation is obtained as:

$$\Delta = 8\mu^3 + 27v^2 \quad (8)$$

Obviously, only when $\mu \leq 0$ can the system undergo a mutation across the bifurcation set.

The above equation is a necessary and sufficient criterion for sudden instability of the rock mass system. The magnitude of the mutation characteristic value Δ can be used as the distance between the rock mass evolution state and the critical state. When a landslide occurs, Δ will suddenly drop to nearly zero. Therefore, the abrupt characteristic value Δ can be used as a physical indicator to represent the stability of the rock mass system.

3 Experiment test

A certain landslide is a loess landslide on the edge of the plateau. Cracks were discovered in 1980, and its deformation was observed from March 11 of that year until it experienced severe sliding on May 5. This article uses a support vector machine to predict it, with the last 10 data used to test the predictive ability of the support vector machine. Using a small amount of data, the Lyapunov exponent of the time series was calculated to be 0.0083 for the first 51 sets of data in the training set, indicating that the longest predictable time of the time series is 120 days.

After determining the learning sample set, the establishment of displacement prediction models mainly involves selecting the corresponding support vector machine parameters: kernel function and penalty parameter c . Many application experiences have shown that radial basis functions have good learning ability, and this article also chooses radial basis functions, i.e.:

$$K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right) \quad (9)$$

Parameters are determined by cross-validation method, which selects different sets of c, σ , obtaining support vector values from training data and selecting set of c, σ that make the error is minimum. After comparison, $c = 0.9, \sigma = 500$ is determined. In the prediction process, in order to improve the accuracy of the prediction, the latest information should be fully utilized, that is, the latest observation data

should be added to the time series for the next step of prediction. Using this model to predict the next 10 data (see Table 1), the prediction results are shown in Fig.1, indicating that the model has high prediction accuracy and good performance.

Table 1. Displacement observation data for verification.

Monitoring (mm)	Prediction(mm)	Absolute error(%)	Relative error(%)
26	27.1	1.1	4.2
27	26.7	-0.3	-1.1
28	28.4	0.2	0.7
30	29.1	-0.9	-3.0
31	29.4	-1.6	-5.3
32	32.7	0.7	2.2
33	34.1	1.1	3.3
42	40.5	-1.5	-3.6
47	45.4	-1.6	-3.4
61	48.7	-12.3	-20.1

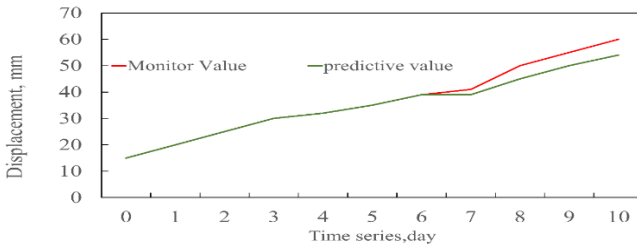


Fig. 1. Relationship curve between displacement prediction value and monitoring value.

From Table 1, it can be seen that the displacement prediction accuracy before the sudden instability of the slope is relatively high, but at the near mutation point, the prediction accuracy is significantly low. This indicates that the displacement change during the landslide instability no longer conforms to the displacement change law determined by previous displacement observation data.

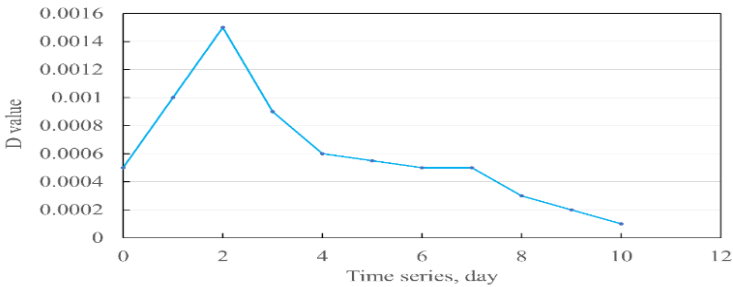


Fig. 2. Mutation characteristic value change curve.

Substitute time series data into the mutation equation to calculate the mutation characteristic value Δ , and its variation curve is shown in Fig.2. We can see that the pre landslide mutation characteristic value Δ significantly decreases and approaches zero. That is, during the slope evolution process, as the control variable changes, the state variable (displacement value) undergoes a sudden change when the slope crosses the bifurcation set, leading to the landslide. This is relatively consistent with the actual situation on site.

4 Conclusions

This article utilizes the relevant research results of modern artificial intelligence to propose a nonlinear dynamic model prediction model based on support vector machine, and combines this nonlinear dynamic model with nonlinear catastrophe theory to timely reflect the evolution direction of rock masses and make predictions and judgments on their stability. The research results indicate that the proposed model has high prediction accuracy, with a relative error of less than 20%; In the process of slope evolution, with the change of control variables, when the slope crosses the bifurcation set, the state variable (displacement value) undergoes a sudden change, leading to landslides.

The prediction accuracy of displacement before sudden instability of the slope is relatively high, but at the near mutation point, the prediction accuracy is significantly low. Therefore, further research is needed to predict the displacement at the mutation point.

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