Students' Obstacles in Solving Non-routine Problems: A Case Study in an Islamic Secondary School

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Abstract In mathematics education, non-routine problems are gaining importance as they require students to use their creative and critical thinking abilities to solve problems. Problem-solving students are in high demand in the twenty-first century. Nonetheless, the process of solving mathematical problems, especially non-routine problems, is becoming increasingly difficult for students. Many of them are hindered by a variety of obstacles that impede their ability to effectively solve such problems. It is essential to identify the obstacles that students encounter when attempting to solve non-routine problems. Thus, the purpose of this study was to determine to what extent the pupils of a secondary Islamic school encounter obstacles when solving non-routine problems. A case study research design was used to collect qualitative data for this investigation. Six secondary school students from an Islamic institution in Indonesia participated in the interviews. They were purposefully selected to answer 16 problem solving questions. A clinical interview was conducted to collect qualitative data. The participants were evaluated on their comprehension of problem-solving questions and their attempts to answer them. In accordance with the data, the students were reported confronting several obstacles when attempting to answer the question. The results revealed that these students lack confidence in employing problem-solving techniques and struggle to comprehend the notation, syntax, and concepts of proportions. This study suggested the need for targeted instructional strategies that address these obstacles and enhance students' mathematical reasoning and problem-solving abilities.

Keyword: Non-routine Problem, Mathematics, Obstacles in Problem Solving

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Introduction

In mathematics education, non-routine problems are essential, but they are difficult for many Islamic residential school students. The difficulties have highlighted the significance of instructors' guidance. However, instructors in schools face obstacles, such as a shortage of teachers to assist students with their coursework (Fadillah, 2015; Zarkasyi, 2020). In schools, teachers are also responsible for instructing all subjects. Moreover, as a result of the new curriculum structure, mathematics is becoming increasingly important (Purnamasari & Afriansyah, 2021). Besides, many parties advocated for solving mathematical word problems to be the primary focus of Islamic institutions (Fuchs et al., 2009). Nonetheless, Islamic secondary schools face a variety of other obstacles. Focusing on a particular investigation may provide instructors with useful information for assisting students, and thus provide opportunities for mathematics development. Consequently, this study focuses on a situation involving mathematics problem-solving in an Islamic secondary school, particularly the obstacles to problem- solving encountered by the students. Specifically, the objective of this study was to determine the extent to which students from an Islamic secondary school encounter obstacles when solving non-routine problems.

Literature Review

Solving Non-routine Problems

Polya (1945) presented five problem-solving processes. It is widely recognised as an essential tool for teaching and learning how to solve mathematical word problems. Understanding the problem, developing a plan, implementing the plan, evaluating the solution, and reflecting on the process comprise the five phases. Later, the five steps were enhanced by incorporating elements of communication, illustrating teachers' roles in assisting pupils via roles of language (Polya, 2004). The effectiveness of Polya's method for solving problems has been demonstrated and acknowledged (Colonnese et al., 2022). It has been proven that Polya's method of problem-solving enhanced the mathematical performance of students (Jonassen & Hung, 2008).

It has been suggested, however, that the success of Polya's method is reliant on students' ability to justify their reasoning and make sense of the problem at hand (Schoenfeld, 1985). Furthermore, nonroutine problems have been shown to be particularly challenging for students as they require more extensive reasoning and justification (Schoenfeld, 1985). In order to effectively solve nonroutine problems, students must have a strong conceptual understanding of the underlying mathematical concepts (Mayer, 2014).

Representation of mathematical ideas has been identified as a critical factor in students' conceptual understanding and problem-solving ability (Jitendra et al., 2016). Notations and symbols used in mathematical problems must be understood in the appropriate context, and syntax errors can significantly hinder students' ability to solve problems (Szabo et al., 2020). Nevertheless, conceptual understanding involves a complicated process. It may need to be integrated between the fundamental knowledge with reasoning via representations and other strategies. Providing visual representations alongside mathematical notation can help students to better understand and solve mathematical problems. Additionally, representations enable learners to self-monitoring within the problem solving stages (Izzati & Mahmudi, 2018).

The challenges to carry-out mathematical processes has made representation a bigger task by incorporating real-world contexts and authentic problem-solving tasks into mathematics instruction (Hattie et al., 2016). This approach has been supported by several studies which found that using real-world contexts can enhance students' motivation and engagement in mathematical problem-solving (Pierce & Stacey, 2006).

Challenges in Solving Non-routine Problems.

Numerous studies have highlighted the limitations of relying solely on memorising mathematical procedures, as it often hampers students' ability to solve non-routine problems that require a deeper conceptual understanding (Braithwaite & Sprague, 2021). In response to this concern, many educational institutions, including Islamic secondary schools in Indonesia, have recognised the importance of fostering a strong comprehension of mathematical concepts (Sembiring et al., 2008). One approach to enhancing mathematical skills is through the solving of non-routine problems. However, equipping students with the necessary competencies for this task has become increasingly challenging, especially in the context of disruptions caused by the COVID-19 pandemic (Julie et al., 2017; Tanu Wijaya, 2020). Although the aspiration to develop these competencies remains commendable, the current circumstances demand careful consideration of the most effective strategies.

The impact of the pandemic on students' academic performance has garnered significant attention, with educators struggling to fulfill their teaching responsibilities amidst students' reduced focus during distance learning. Mathematics teachers, in particular, face challenges in maintaining students' mathematical competence. It has been reported that students who required more assistance from their math teachers received less support during the pandemic (Gonzalez et al., 2020). Therefore, it may be necessary to increase engagement and exposure to activities in order to enhance mathematics learning. Restricted movement and limited options have also undermined students' confidence. Although efforts are being made to address these issues (Darling-Hammond et al., 2020), less attention has been paid to how students leverage their prior knowledge, especially primary school knowledge, to solve mathematics problems and overcome difficulties in arithmetic learning.

Methodology

A case study research design was used to collect qualitative data for this investigation. Six secondary school students from an Islamic institution in Indonesia participated in clinical interviews. The clinical interviews focused on getting responses on how and why they react as stated and shown in their working.

The interview protocol focused on assessing the participants' problem-solving skills by observing how they approached and solved the given questions. The protocol incorporated Polya's method (Polya, 2004) to examine their ability to implement a systematic approach and utilise cognitive skills in problem solving. The following steps from Polya's method were specifically emphasised: comprehending the problem, planning a strategy, executing the strategy, and reviewing the solution.

To gather a comprehensive understanding of the participants' problem-solving abilities, a series of interview questions were designed. These questions aimed to elicit information regarding the important aspects of the problem, the reasoning behind their choices, the starting point of their solution, the rationale for employing specific steps, alternative approaches considered, and an explanation of their solution. By exploring these aspects, the interviewers could gain insights into the participants' problem- solving processes and their ability to articulate their reasoning. The interview questions included "What is the important information in this question?" "How do you know that?" "Please share from where you start your solution?" "Why do you use this step?" "What are the alternatives?" and "Could you explain your solution?". During the interviews, the participants were asked to verbalise their thoughts while working through the problems, following the "think aloud" approach. This approach enabled the collection of qualitative data by capturing the participants' thoughts, decision-making processes, and any difficulties encountered. The interview scripts were carefully collected and analysed to provide explanations and interpretations of the students' abilities as well as identify any obstacles they faced during the problem-solving process.

Hence, the results presented some working together with the excerpts in the interviews. The participants evaluated their comprehension of problem-solving questions and their attempts to answer them. Before the interview, the students were required to solve 16 non-routine questions. The following questions are part of the questions. The items were written in dual languages, namely English and Indonesian Language.

Item 1: If it takes 6 men to paint a house in 21 days, how many men will be needed to paint the house in 14 days?

(Jika dibutuhkan 6 orang untuk mengecat sebuah rumah dalam 21 hari, berapa banyak orang yang dibutuhkan untuk mengecat rumah tersebut dalam 14 hari?)

Solution:

$$6 \times 21 = 126, then\ y \times 14 = 126, \text{ y denotes the number of men}$$

$$y = \frac{\quad}{14}$$

$$y = 9$$

So, we know that 9 men will be needed to paint the house in 14 days.

Item 3: A faulty calculator compute 93×134 as 14462. Using this information, find the answer for the following number sentence if the faulty calculator is used: $\frac{14462}{930}$

(Sebuah kalkulator yang salah menghitung 93 × 134 sebagai 14462. Dengan menggunakan informasi ini, temukan jawaban untuk kalimat angka berikut jika kalkulator yang salah digunakan: 14462/930)

Solution 3:

Multiplication is the opposite of division. Then because the calculator calculates 93×134 *as 14462, then* $\frac{14462}{930} = 13.4$

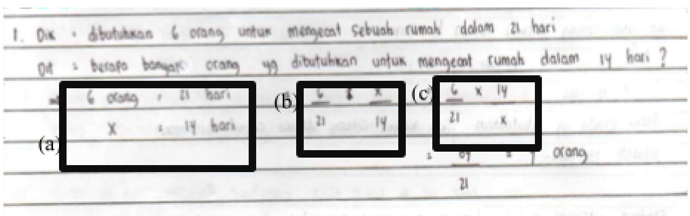
Findings

This study reports a small part of findings from a bigger study on solving nonroutine problems among pupils in an Islamic secondary school. The following analysis refers to "**item 1**". The question (in item1) tested students' ability to handle conceptual understanding in applying proportional concepts. The following results were presented from the selected respondents' work with their excerpts.

Finding 1: Reasoning with concept of proportion

Respondent 4

Figure 1 shows respondent 4's working. Respondent 4 (R4) manage to list the information, namely the question requires the number of men for the completion of the painting in 14 days by listing in (a) as shown in Figure 1.



Item 1: If it takes 6 men to paint a house in 21 days, how many men will be needed to paint the house in 14 days?

Figure 1 Respondent 4's solutions for item 1

Nevertheless, conceptually R4 wrongly constructing the relationship between the given information and information that required to be found (the number of men for completing painting in 14 days). Reasonable justification should show reciprocal relationship since more men is required to complete within a shorter number of days. R4 seemed only know getting a relationship in just a direct way as shown in (b). On the other hand, R4 failed to carry out the plan using a correct procedure for the planned strategy as shown in (c). R4 carried out the direct ratio for $\frac{6}{21} \times \frac{x}{14}$ with the following explanation (excerpts from R4):

Respondent 4 : *My answer is if six people need 21 days, here is a cross multiplication, 6/21 times 14/, which is still a question mark. To paint the house in those 14 days, four people.*

R4 use the phrase "cross multiply" to indicate the procedures which had been carried out. The mistake was also shown that R4 did not the meaning of cross-multiply since he interchanged the sign of ":" (ratio) to "x" (a multiplication). Therefore, respondent 4 failed to find the portion of people needed to paint the house in 14 days.

Some students even realized that for faster house painting work, more people would be needed to do the work. However, they were confused about how to get more people (due to a lack of strategy knowledge). As stated by Respondent 3 (R3) below:

Respondent 3 : *First, I know six people can paint a house in 21 days; then the question is, if it takes 14 days, then how many people are needed? So, 21 days divided by 6 equals 3.5, and how can it be 14? That's 3.5. I multiply by 4, so I conclude that in 14 days, we can take four people.*

R3's working is shown in Figure 2. It illustrates similar but not same interpretation compared to R4. R4 started with the concept of getting number of men for a day. Then, he multiplied with 14 days to get the number of men for 14 days. On the other hand, R3 found number of days a man work, then getting a value of variable to satisfy 14 days, namely 3.5 days a man work × y = 14 days as shown in (a) Figure 2, y is the number of men.

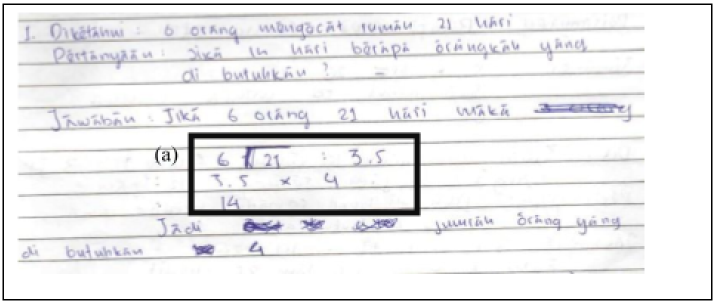


Figure 2 Respondent 3's solutions for item 1

The same mistake was observed from Respondent 5 (R5) who related the number of working days and the number of people. He realized that more people must be needed for faster processing times. But again, students do not know how to get more people. R5 was trying to look for pattern in this problem. He reasoned that because 21 days takes 6 people to paint the house, while 21 days is the number of days in 3 weeks, he assumes that it will take 2 people to paint the house in one week as shown in Table 1 and (a) in Figure 3.

Table 1 R5's ideas

3 weeks	6 men
2 weeks	4 men
1 week	2 men

Below is the answer sheet of respondent 5 in solving item 1.

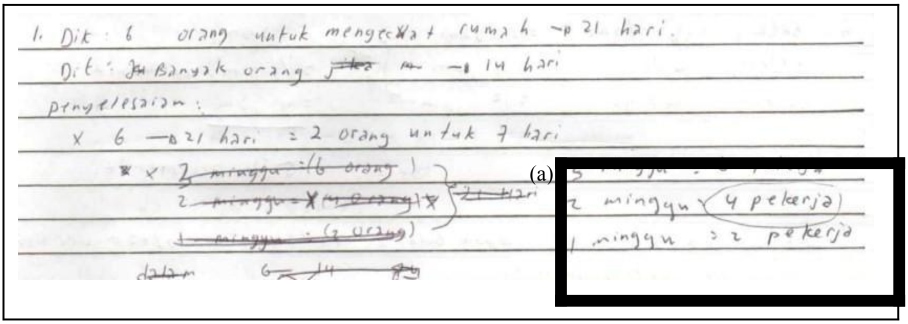


Figure 3 Respondent 5's solutions for item 1

There are also possibilities that pupils do not have any idea of making any connections between days and the number of men. They might simply demonstrate any calculations as presented by one of the respondents (R6) who shown her solution as in Figure 4.

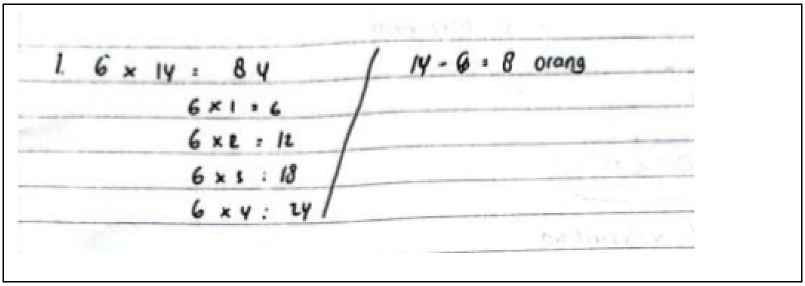


Figure 4 Respondent 6's solutions for item 1

Based on the working in Figure 4, it seems that the pupil attempted to solve the problem using a trial-and-error method instead of using the correct formula for inverse proportionality. R6 started by multiplying the number of people by the number of weeks, which is not the correct approach. This resulted in 6x14=84, which is not a relevant answer to the question being asked. He then attempted to list the products of 6 with different numbers, which is not a helpful strategy in this case. Finally, R6 arrived at an answer of 8 by subtracting the number of days given (21) from the number of days asked (14). R4 struggled with the problem due to a lack of understanding of the correct formula for inverse proportionality.

Table 2 Summary of the analysis

Respondent	Conceptual Mistake	Procedural Mistake
R4	R4 Failed to set a reciprocal relationship for a variable. Fail to carry out the strategy (due to mistakes in the procedure) R4 started with the concept of getting number of men for a day. Then, he multiplied with 14 days to get the number of men for 14 days.	Assume sign of ratio as a procedure of multiplication: $\frac{6}{21} : \frac{x}{14}$ is interpreted as $\frac{6}{21} \times \frac{x}{14}$, then x is assigned to '4'
R3	R3 found number of days a man work, then getting a value of variable to satisfy 14 days, namely 3.5 days a man work × y = 14 days	
R5	R5's demonstrated similar ideas with R4, but he changed the number of days to number of weeks	
R6	R4 struggled with the problem	

Table 2 presents a summary of the above analysis. The above results show the importance of highlighting mathematical syntax in the notation. For this question, the syntax involves the concept of inverse proportionality. The notation can be presented as the product of the number of men and the time they work is constant. That is, if the time is reduced, the number of men required to complete the job will increase proportionally, and vice versa.

Discussion

The findings of this study indicate that the students involved in the research would benefit from additional support. One prominent difficulty they faced was a lack of confidence in implementing problem-solving strategies effectively. This lack of confidence hindered their ability to approach and solve the given problems.

When presented with a real-life question related to completing a painting project with a specified amount of manpower, the students encountered challenges in visualising and understanding the practical aspects of the problem. For instance, even though some students, like R5, demonstrated an awareness that reducing the number of days would necessitate an increase in manpower, they struggled to grasp this conceptually and ultimately failed to arrive at the correct solution. This highlights the importance of effectively connecting proportional reasoning to specific contexts, as failure to do so can impede the acquisition of knowledge in related areas. This challenge is not unique to this study but has been observed in various other mathematical fields as well, such as probability (Begolli et al., 2021).

Despite the challenges faced by students in effectively solving real-life problems, it remains crucial to encourage their engagement and attempts to tackle such tasks. The incorporation of real-life situations in mathematics education serves as a powerful motivator for students, providing them with opportunities to develop mathematical thinking and behaviours (Hattie et al., 2016). By presenting problems that have practical relevance to their lives, students are more likely to feel a sense of connection and ownership over the learning process. They can ask questions, seek additional solutions, and actively engage with the content within their own environment.

However, it is important to acknowledge that students often encounter difficulties in fully solving real-life problems (Magdas & Glava, 2016). Research indicates that a significant portion of high school students, approximately 10%, struggle to demonstrate competence in solving real-life mathematical problems. Additionally, approximately 57% of students were identified as partially competent in their ability to cope with the challenges posed by mathematical problem-solving. These findings highlight the prevalence of students who fall short of achieving the required levels of mathematical knowledge, particularly when it comes to solving problems in real-life contexts.

Also, this finding shared that even though students may find it difficult, the obstacles do not totally hurdle their efforts to proceed. Their engagement provides opportunities for educators to further incorporate STEM-based mathematical tasks. In addition, it was envisaged that mathematics tasks are fundamentals for the development of STEM education (Kohen & Orenstein, 2021).

In accordance with the data, the students reported confronting several obstacles when attempting to answer the question. They failed to provide correct notations. Their understanding of notations showed syntax errors since they failed to demonstrate a conceptual understanding of proportion. They have shown an intention to present a direct proportion in the number of days and men. Nevertheless, they also failed to present a correct notation for the direct proportion since they were struggling to use the symbols ":," which indicates proportion, and "= ", which indicates equality. Emphasising the use of notation with syntax understanding is crucial in conceptualising mathematical ideas (Oyoo, 2022). Thus, the findings enlightened that communication needs to take place in mathematics discourse and classroom activities. It supports the organised guidance of conducting meaningful and effective classroom mathematics discussions with the ease of language (Pimm, 1987).

The difficulties encountered by the students in this study underscore the need for targeted interventions and instructional strategies that address their lack of confidence and support their development of problem-solving skills. Providing students with explicit guidance, practise opportunities, and scaffolding in understanding and applying proportional reasoning within real-life contexts can enhance their mathematical understanding and problem-solving abilities. By addressing these challenges, educators can help students overcome their difficulties and foster a more robust acquisition of mathematical knowledge and skills.

Conclusion

The findings of this study indicate that the secondary school students from the specific Islamic school lacked confidence in employing problem-solving techniques and faced difficulties in interpreting and conceptualising the real-world scenarios presented in the questions. Despite their engagement and persistence, they struggled to comprehend the concept of proportion. The study also emphasised the significance of fostering communication and meaningful classroom discussions to enhance mathematical discourse and improve students' understanding of notation and syntax. These findings highlight the importance of implementing targeted instructional strategies that address these obstacles and promote students' mathematical reasoning and problem-solving skills. However, it is important to acknowledge that the study's generalizability is limited due to its focus on a single secondary Islamic school with unique characteristics and instructional approaches. To enhance the applicability of the findings, future research should incorporate a diverse range of educational contexts.

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