Elementary School Students' Obstacles in Understanding the Concept of Area and Perimeter

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Abstract. Area and perimeter are one of the geometry and measurement concepts that are interconnected and have begun to be studied at the elementary school level. However, in the learning process, students often find difficulties in understanding the concept. This study aims to analyse students 'difficulties in understanding the concept of area and perimeter based on a review of students' conceptual understanding of the definition and its relationship with the quality of problem-solving performance. The qualitative study using 4 stages analysis was conducted by involving a group of grade 4 students (N = 20). The study found that students often do not understand the concept of area and perimeter, causing a decrease in the quality of performance such as difficulty in answering area and perimeter problems in the context of illustrated picture problems or word-problems. In addition, the textbooks used are partial and rigid with formulas and presentation of material that are not in depth. Therefore, it is necessary to improve the quality of the concept definition through the development of innovative learning. It is hoped that the results of the study can be used as material to reflect and evaluate improvements in planning and further learning processes which can anticipate these obstacles.

Keywords: Learning Obstacle, Area and Perimeter, Mathematics for Elementary School.

1 Introduction

The fast-changing times are correlated with the increasing need to understand and use mathematics in life. It is clearer that mathematics plays a role as a human activity which can be specified in four domains, namely mathematics for life, mathematics as a part of cultural heritage, mathematics for the workplace and mathematics for the scientific and technical community [1], [2]. This causes mathematics as a subject that is taught from an early age, including at the elementary school level as the first stage of formal education.

The field of mathematics teaching is continuing to transform by involving the application of the results of educational research. In educational research, mathematical knowledge can be analyzed through two important components, namely the conceptual and procedural components [3], [4]. The two components, which are usually simplified by "knowing that" and "knowing how to", have a position not only in the investigation

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of mathematical knowledge, but also in the development of mathematics learning, including on the topics of area and perimeter.

Area and perimeter are one of the fundamental topics in mathematics learning in primary and secondary schools because they act as the most commonly used geometric and measurement domains [5]. The mathematics curriculum, which is arranged in a spiral from elementary school to higher education, makes the concept of area and perimeter as an important topic because it cannot be separated from other mathematical concepts involving numbers and their place values, geometry and data [6]. Included in the Indonesian curriculum itself, this topic has begun to be formally discussed in grade 4 with a context that involves basic geometric fields such as squares, rectangles and triangles. The purpose of this learning is to have a conceptual understanding of the fundamental area and perimeter which can later be used in learning higher concepts such as volume and others, thus leading to the implementation of comprehensive understanding in student life.

The topic of area and perimeter as part of geometry and measurements content strands is a topic that is not easy for students to understand. We need to define that the area in this study is a two-dimensional size of an area [6], while the perimeter is a measure of length that involves the distance from an area [7]. The area and perimeter are often a source of confusion for students because they both involve the area to be measured or students are taught formulas to solve the concept simultaneously [6], [8].

Several previous studies have found that students' ability in the field of geometry and measurements including area and perimeter is weaker when compared to other topics [6], [9]. A survey study that we have conducted previously found that 46.80% of primary school teachers stated that geometry and measurements is the second difficult topic to teach after the fractions [10]. In addition, there are still common errors in the topic area and perimeter, such as confusion between the concept of perimeter and geometrical area where in solving the perimeter problem the participants use the same method by finding the area of a figure [11], [12]. In another study, it was stated that grade 4 students had a good understanding procedurally because they had good abilities about multiplication, but misunderstood the concept of area and showed general weaknesses in identifying geometric shapes and differentiating between perimeter and area [13].

Some of these problems will certainly not occur if students and teachers really understand the concept of area and perimeter since elementary school. To gain an understanding of the concept of area and perimeter, the teacher needs to provide students with meaningful learning that is able to facilitate the construction of understanding concepts and pay attention to student responses in solving problems and can anticipate some of the difficulties that occur in solving mathematical problems [14]. Thus, the preparation of predictions and a good teaching plan is needed to anticipate student misunderstandings in mathematics learning.

There are two assumptions regarding the causes of the misunderstanding that often occurs in the concept of area and perimeter [5]. First, the misconception is that most of the exercises on area and perimeter in textbooks only deal with two-dimensional geometric shapes based on the Euclidean concept of shapes, so students are less trained to solve problems in real or different contexts. Second, there is a habit of discussing confusing areas and perimeters or in this context the definitions that are difficult for students to understand. Misunderstandings and student constraints in learning should be predictable and their causative factors are known before the learning process is carried out, so that students can gain an understanding of the area and perimeter properly.

In this study we use the terminology of Tall and Vinner [15] where the research questions were adapted from [5] redeveloped by establishing related research questions 1) how the definition of the concept of area and perimeter expressed by students, 2) how the understanding of the two-dimensional area and perimeter expressed in solving the problem of areas and perimeter, 3) the extent to which the quality of the student's concept definition of area and perimeter related to their performance in training. To answer this question, of course, an analysis is needed that can add and update insights about students' understanding of the topic area and perimeter. Thus, this study was conducted to compare the predictions of student responses with the reality that occurred during the learning process in order to find a description of the understanding and procedural strategies of grade 4 students in solving area and perimeter problems. Thus, the final goal can be found a projection of students' obstacle in the topic area and perimeter and analysis of the factors causing it. Therefore, this study aims to analyze students 'difficulties in understanding the concept of area and perimeter based on a review of students' conceptual understanding of the definition and its relationship with the quality of problem-solving performance.

2 Method

This research is a qualitative-research conducted in one of the elementary schools in Bandung, West Java, Indonesia. The study was designed through four stages, including 1) the researcher designed predictions about student responses; 2) researchers collect data related to the identification of obstacles in the topics studied from various sources including the observation process during learning, textbooks, and written tests; 3) the researcher compares the predictions designed in the first stage with the data that has been obtained in the second stage as a data analysis process, 4) constructing the conclusions [14]. As for the participants in this study were 20 fourth graders consisting of 10 male and 10 female students who are 9-10 years old and learning through distance learning with prior mathematical ability are in the moderate category. The test instrument used was adapted from Tossavainen et.al. research questions [5]. The textbook that we analyzed is a public textbook [16] and one of commercially textbooks [17].

3 Result and Discussion

The results of this study are classified into two-section. The two sections include the acquisition of data analysis regarding 1) textbooks and 2) test results given to students.

3.1 Textbooks Analysis

For the first, we found that the two textbooks we analysed (especially Unit 4 with 61 pages [16] & Unit 6 with 27 pages [17]) had partial area and perimeter topics. Partial nature is marked both in the formulation of separate chapters from other geometry and measurement topics as well as in the context of mathematical problems or problems that are partial and abstract. Although in commercial books we find mathematical problems that are contextual in nature, unfortunately they are published at the end of the chapter as a form of application of formulas. Thus, this is the same as the new mathematics paradigm which teaches formulation to solve problems or in other words it is the opposite of realistic mathematics which has the stages of teaching a situation model, schematic model and abstract / symbolic [18].

It is assumed that partial arrangement of textbooks can trigger confusion for students when facing problems in different contexts. To anticipate this confusion, we need a learning that is able to provide a stimulus for students to understand a concept and find various strategies to solve problems [14]. This implies that mathematics knowledge and teacher teaching practices have a considerable function on student mathematics learning outcomes [19]. Especially in completing a conceptual understanding which has 4 indicators including restating the problem, applying problem solving concepts / algorithms, representing mathematical concepts, and relating mathematical concepts internally or externally [20], hence the teaching practice of teachers has an important role in addition to completing textbooks which in terms of content still cannot fulfil all of these indicators.

3.2 Students' Obstacle Analysis

The students 'obstacle that we mean in this case is the identification of students' understanding and procedural abilities in solving mathematical problems based on responses to the 3 questions given to students by adapting the questions developed by [5]. During learning and testing, we make virtual observations to ensure that students answer the test independently and do not influence each other's answers. Based on the test analysis related to the topic and the perimeter, we found several things that match the predictions of student answers that the researcher had previously made. The prediction in question confirms that the mathematical framework as the way of thinking and the way of understanding is very important [4], [21], so that students really understand the topic being studied, not only remembering the formulation. The results of the analysis can simply be seen in Table 1.

Table 1. The students' response classification

Answer	Question 1	Question 2	Question 3
Correct	9	5	4
Incorrect	9	13	15
Skipped	2	2	1

In the first question, we analyzed how the students' understanding of the concept definition of area and perimeter was expressed. In solving it, we predict students' answers according to the limiting aspect [5] includes 1) whether the definition recognizes dimensionality and includes an infinite image, 2) whether the definition is bound by a formula, 3) whether the definition is out of bounds. The things found in this study are that 1) students are able to define concepts, 2) students are able to define but are incomplete or there is still one important element of the missing definition, 3) students define through the formulation of a shape, 4) students define area concepts and perimeter in reverse, 5) students define only by translating the language, 6) students think that area and perimeter are the same thing, 7) students give an explanation out of the previous boundaries. In our view, the categorical order of the boundary aspects and findings represents a degradation in the quality of the definition. Thus, we do not justify a definition in categories 2 and 3. The distribution of student responses can be seen in Table 2 below.

Table 2. The distribution of the students' concept definition of area and perimeter.

The category of definitions	The essential properties of the category	A quotation of the students' definition	Frequency (%)
1-Definition acknowledges dimensionality (an explicit and correct answer)	Able to define the concept (1)	Perimeter is the amount of distance that goes around the figure (2 dimensions). Area is a quantity that states the size of 2 dimensions.	10%
	Only minor vagueness accepted in terminology (2)	Perimeter is the number of sides in a 2-dimensional shape. Area is a quantity that expresses 2-dimensional size.	35%
2-Definition based on the formula or an example	The given definition applies only to a rectangle or square or triangle. (3a)	Perimeter is the side + side + side + side of the shape. Area is the length × the width of the shape.	10%
	The concept and formula are confounded with one another (3b)	Perimeter is a number that goes around the shape. Area is a number that you multiply by the length and width.	10%
3-Several incorrect or vague answer	Misconceptions or meaningless definition of the area and perimeter (4)(5)(6)(7)	Perimeter and area are the total lengths around the figure.	25%

In the second problem, we analyzed how the students understood the concepts expressed in solving the problem area and perimeter. This needs to be done because the focus on using a computational strategy that demands more procedures on the area and perimeter can lead to computational errors, so this indicates a serious leap from procedural skills to conceptual understanding of a formula [6], [13]. Therefore, we try to reveal whether students can calculate something without formulas and numbers. On this occasion we use the illustrated picture problem as follows: "*Explain how you should determine the area of the following figure! In this exercise, you do not have to measure the area but only explain the process!*".

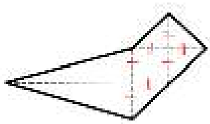

Fig. 1. The 2nd problem to identify the way of mathematical thinking

For the second problem, the researcher made predictions of student problem solving through 6 criteria where the predictions 1-4 were categorized as correct answers and 5-6 were categorized as incorrect answers or did not meet the limitations. Table 3 below shows the distribution of students' problem solving.

Table 3. The distribution of students' procedure to solve illustrated picture problem.

The category of problem-solving procedure description	A quotation of the students' responses	Frequency (%)
1-Constructing a new shape from the figure pieces and make the simple formula.	-	0%
2-Dividing into simple shape/sections and then add them up.	Add up the area of the kite and rhombus.	10%
3-Identify and calculate the equal shape, then add them up (using multiplication concept).	Multiply the area of 4 small triangles, 2 large triangles, and 1 square and then add them up.	5%
4-Counting each shape, then add them up.	First, look at the image carefully, how many flat shapes are there, there are 7 consisting of 6 triangles and 1 square. Then, find the area of the flat shape one by one. After that add them all up.	10%
5-Giving formulation for each shape without add them up.	L for triangle = ½ × b × h and L for square = side^2	20%
6-Several incorrect or vague answer	- You split them into triangles. - The area = Area 1 + Area 2	45%

Next, we analyzed the extent to which the quality of students' understanding of areas and perimeters was related to their performance in the exercise in the third item. The question is a part of illustrated picture problem as follows "*Determine the area of the figure below!*" [5].

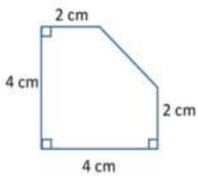

Fig. 2. The 3rd problem to identify the way of procedural fluency

The researcher made 9 prediction categories (6 correct and 3 incorrect answers) in solving these math problems. Predictions of problem solving are categorized based on the complexity in linking mathematical concepts internally and externally through identification of shapes and formulating them in simpler solutions. In this case, we totally assume that the connectivity of internal and external concepts is an indicator of a good understanding of the concept as expressed in [20]. We present the predictions and responses of students in detail in Table 4.

Table 4. The distribution of students' illustrated picture problem responses.

The category of mathematical problem-solving	Frequency
1-Using 2 steps problem solving (subtracting the area large square by little triangle)	5%
2-Using 2 steps problem solving (adding the rectangle area and the trapezoid area)	5%
3-Using 3 steps (multiplies 3 area of squares and then adds with the area of a triangle)	10%
4-Using 3 steps problem solving (add up the areas of rectangle, square, & triangle)	0%
5-Using 4 steps problem solving (add up the areas of 3 square and 1 triangle)	0%
6-Using pictorial method (draw imaginary lines & calculate them by 1 square units)	0%
7-Miss or error in writing the unit	5%
8-Error in using formula	5%
9-Several incorrect or vague answer	65%

The findings indicate that a good conceptual understanding of the area and perimeter formula requires acquisition or knowledge regarding the basic forms and properties of the figures which can help logical thinking about the relationships that exist in the area and perimeter formulas [13], [22]. Therefore, the ability to identify the figures and explain how to measure the area and perimeter of these figures are the main requirements in understanding the concept of the area and perimeter of the figures-formula. We found evidence of how students responded to answer number 3 where the majority of errors occurred with the same result, namely the 12 cm2 answer obtained from an incorrect formula due to failure to identify the figure. Based on this evidence, we can learn that a simple way of learning the topic area and perimeter is by enriching the variety of figures discussed [5], so that students' conceptual and procedural understanding can be built solidly even though they are faced with different contexts.

Furthermore, several sources of error can be attributed to the tendency to direct the thought to size rather than concept [8]. This study reverts to the previous conclusion that 4th graders students have good experience measuring fields and memorizing formulas well but not necessarily having an accurate regional conception [13]. Thus, because the mathematical ability of elementary students' have a relationship with the ability of teachers [23], [24], it is hoped that the teacher can anticipate this to develop meaningful understanding through the sequences of experiences: 1) identified the figures, 2) comparing and ordering, 3) using informal units, 4) using formal units, and then 5) seeing the formulas and their applications [6], [8].

4 Conclusion

Based on the students' responses in the area and perimeter, it is clear that students who have a good understanding of the concept will be more successful in solving the problem area and perimeter. However, 4th grade students still experience learning obstacle in learning and understanding the concept of area and perimeter. Most students still experience confusion in defining the concept of area and perimeter. The emergence of poor-quality definitions has an effect on the quality of the problem-solving procedures performed by students. In addition, the use of a single strategy provided by the teacher and textbooks presents difficulties in resolving different contexts. Presentation of material that is partial and abstract in textbooks can also have an influence on the quality of students' conceptual understanding. Therefore, it is necessary to improve the quality of the concept definition through the development of innovative learning in order to be able to improve student performance in practicing the concept of area and perimeter. Thus, the ability of teachers to develop learning and presentation of mathematical problems is assumed to be the front line in overcoming and anticipating these learning barriers. We realize that this research is still limited to investigating the connectivity between understanding the definition and the procedures performed by students in the problem area and perimeter. This study has not revealed more deeply related to problem solving in the context of the area and perimeter with a review of procedural fluency in solving mathematical problems. Therefore, we recommend an in-depth analysis of the epistemological obstacles that occur in the related topic.

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