Applying Ant Colony Optimization for Inventory Routing Problem to Improve the Performance in Distribution Chain: A Case Study of Vietnamese Garment Company

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Abstract

The garment industry holds a paramount position within the Vietnamese economy. However, global industries, due to the aftermath of economic recessions and the impacts of epidemics, have faced imperatives to curtail expenses and optimize operational processes to bolster competitiveness. In this context, a novel trend gaining prominence is the adoption of chain management through the Vendor-Managed Inventory (VMI) method. This advancement has been facilitated by digital technology platforms and the rapid progress of science and technology. Consequently, suppliers can now efficiently oversee the inventory levels of retail units. To address cost-related issues within the supply chain, such as transportation and inventory costs, significant attention has been directed towards the Inventory Routing Problem (IRP), both in terms of research and practical application. The IRP aims to minimize the overall costs within the supply chain by selecting optimal delivery routes for each customer point, while simultaneously satisfying criteria encompassing demand, inventory levels, delivery times, distance considerations, and the number of vehicles deployed. Presently, global research efforts have yielded an array of methodologies for tackling the IRP, encompassing exact algorithms and approximate algorithms. Nevertheless, approximate algorithms, including heuristics and metaheuristics, have gained increasing traction in solving the IRP. These approaches are particularly prized for their ability to address highly complex problems and generate near-optimal solutions within prescribed time constraints. Hence, this research undertakes a focused examination of the application of Ant Colony Optimization (ACO) to resolve the IRP within the context of the garment distribution chain in Vietnam.

Keywords: Vendor Managed Inventory, Inventory Routing Problem, Ant Colony Optimization, Garment Industry

1. INTRODUCTION

In Vietnam, textile and garment industry emerged as a pivotal export sector, playing a crucial role in the country's economic growth. It accounted for a substantial share, about 41 billion USD of the nation's total export turnover, as reported by the Ministry of Industry and Trade of the Socialist Republic of Vietnam. Nevertheless, this industry is grappling with economic crises stemming from factors such as the Covid-19 pandemic and heightened geopolitical tensions. To navigate these challenges, numerous companies have devised and implemented cost-cutting strategies aimed at preserving market shares, reducing operational expenditures, and either increasing or maintaining profitability. Besides, there is a form of chain competition where participants often reliant on one another for survival, instead of competing between companies. However, among the various components contributing to supply chain expenses, logistics costs loom large in the budgets of many companies. In Vietnam logistics report 2022, inventory costs typically constitute 25-30% of the total supply chain expenditure, while transportation costs generally account for 10-12%. Consequently, effective management of both inventory and transportation holds significant importance for companies seeking to minimize their overall supply chain expenses and remain competitive. To address this issue, many organizations have embraced a strategy known as Vendor-Managed Inventory (VMI). VMI represents a significant departure from traditional business models, embodying a critical paradigm shift wherein the vendor (supplier) assumes full responsibility for overseeing the retailer's inventory based on data provided by the retailer. In contrast to conventional inventory management practices where retailers independently determine order size and timing, VMI entrusts the vendor with monitoring the retailer's inventory and making decisions on its behalf. Within this distribution model, determining inventory levels for both the

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supplier and retailer hinges on the timing and quantity of deliveries required by the retailer, which, in turn, is influenced by the capacity of the delivery vehicles used. Simultaneous decision-making is a pivotal element for achieving cost-effectiveness in these systems. The primary advantage of implementing a VMI system is that it enables vendors to enhance service levels while concurrently reducing distribution costs through efficient vehicle utilization. On the retailer's side, resources typically allocated to inventory management can be reallocated for other purposes. In the process of implementing VMI, vendors face the challenge of solving an Inventory Routing Problem (IRP) to formulate a distribution plan aimed at minimizing long-term average distribution and inventory expenses throughout the supply chain, all while preventing shortages.

The Inventory Routing Problem (IRP) constitutes an extension of the well-established Vehicle Routing Problem (VRP) as described by Faulin and Juan in 2008. The IRP is a combinatorial optimization conundrum, holds prominence within the domain of logistics and supply chain management. It entails the determination of optimal delivery schedules and routes for a fleet of vehicles serving a diverse clientele with varying demands for a specific product. The overarching objective is the minimization of total transportation and inventory holding costs. Within the purview of the IRP, the supply chain structure encompasses a supplier and an array of geographically dispersed retailers, each characterized by distinct demand levels. Over a stipulated time window, products traverse from the supplier to retailers employing a fleet of vehicles. Any vehicle may satisfy the demand of each customer within a designated time frame. Within the context of a Vendor-Managed Inventory (VMI) system, the supplier assumes the mantle of formulating the replenishment policy for each retailer, alongside devising vehicle routes to effectuate product delivery while precluding any instances of inventory shortage. The core aspiration of the IRP is the minimization of comprehensive logistics expenses while meeting the demands of retailers within a specified planning horizon. Under this structural framework, three pivotal decisions necessitate resolution: (1) the determination of when to fulfill a retailer's demand, (2) the specification of the quantity of products to dispatch to a retailer, and (3) the optimization of vehicle routes. The historical lineage of the IRP traces its roots back to the 1960s when companies commenced harnessing the computational capabilities of computers to optimize their transportation and inventory management endeavors. The early IRP models exhibited simplicity and operated under the assumptions of static demand and fixed delivery schedules. However, in tandem with the escalating intricacies of contemporary supply chains, IRP models have undergone a transformation to encompass dynamic demand, capacity constraints, multiple products, and the incorporation of variables surrounding uncertain delivery times. Diverse optimization techniques and algorithms have been devised to tackle the IRP conundrum, encompassing methodologies such as linear programming, dynamic programming, and metaheuristic approaches, including genetic algorithms, simulated annealing, and tabu search. The selection of an appropriate optimization technique hinges upon considerations including problem dimensions, complexity, and the availability of data.

In conclusion, the IRP is a critical problem in logistics and supply chain management that has significant potential to improve supply chain performance and reduce costs. This paper proposes an Ant colony optimization for the IRP in a two – echelon supply chain comprised of a single manufacturer and multiple retailers in Vietnamese garment company.

2. LITERATURE REVIEW

2.1 Inventory Routing Problem

In the past researches, many different varieties of IRP have been developed and solved. Bell et al. (1983) first investigated the integration between inventory management and vehicle scheduling. There are many industries which benefit from these studies, as these should present as real-life applications as it is possible, but many of these proposed solutions are often relaxed preventing its applications. There are various versions of IRP that have been studied extensively. IRP indeed can be modelled in different ways depending on its characteristic. In fact, there is no standard version of the problem. In paper by Coelho et al. (2013a), the authors cited that most of the research efforts have been concentrated on 'basic versions' while the study on extended models, denoted as 'extension of the basic version' are relatively new. Generally, the basic version can be classified into seven different criteria including: time horizon, structure, routing, inventory policy, inventory decisions, fleet composition and fleet size.

According to Fig. 1, the time horizon criteria can be categorized into two groups: finite and infinite. The majority of earlier studies have primarily focused on an infinite planning horizon (Anily and Bramel, 2004). The number of customers and suppliers involved can vary, leading to different structural supply chain. One-to-one structuring occurs when a single supplier serves a single customer, while the most prevalent scenario is one-to-many, where one supplier serves multiple customers. Additionally, many-to-many situations arise when several suppliers serve several customers. Notably, recent literature has introduced the many-to-one structure, where several suppliers cater to a single customer. The routing component can be further classified into three categories. "Direct" routing permits only one customer per route. On the other hand, "multiple" routing refers to cases in which several customers share the same route. Lastly, "continuous" routing describes situations without a central depot, akin to certain maritime applications such as ship routing and inventory management problems.

Inventory policies are responsible for defining the guidelines for replenishing customers, and two widely employed policies can be found throughout the literature: Maximum Level (ML) and Order-up-to-level (OU). Under the Maximum

Level (ML) policy, the replenishment level is adaptable but restricted to the available capacity at the customer's location. Conversely, the Order-up-to-level (OU) policy initiates replenishment when the on-hand inventory level drops below a predefined minimum level, and the quantity delivered is precisely what's needed to reach the inventory's full capacity. Meanwhile, inventory decisions determine the specific approach to modeling inventory management. When inventory is allowed to go negative, this results in backorders, with the corresponding demand being fulfilled at a later time. However, if backorders are prohibited, any unmet demand is considered as lost sales. In both cases, penalties may be imposed for stockouts. It's important to note that in most instances, especially in deterministic models, inventory going negative is not permitted.

In addition to the previously mentioned criteria, fleet composition and fleet size are two additional factors that are taken into account. Fleet composition can be categorized into two main types: homogeneous and heterogeneous. In the case of a homogeneous composition, all vehicles in the fleet are of the same type or have similar characteristics. On the other hand, in a heterogeneous composition, the fleet consists of vehicles with varying types or characteristics. Furthermore, the number of vehicles available in the fleet can be classified as either fixed or unconstrained. In scenarios with a fixed number of vehicles, there is a predetermined and unchangeable quantity of vehicles at the disposal of the operation. In contrast, in scenarios with unconstrained fleet size, the number of vehicles available is not limited or predetermined, allowing for flexibility in adjusting the fleet size as needed.

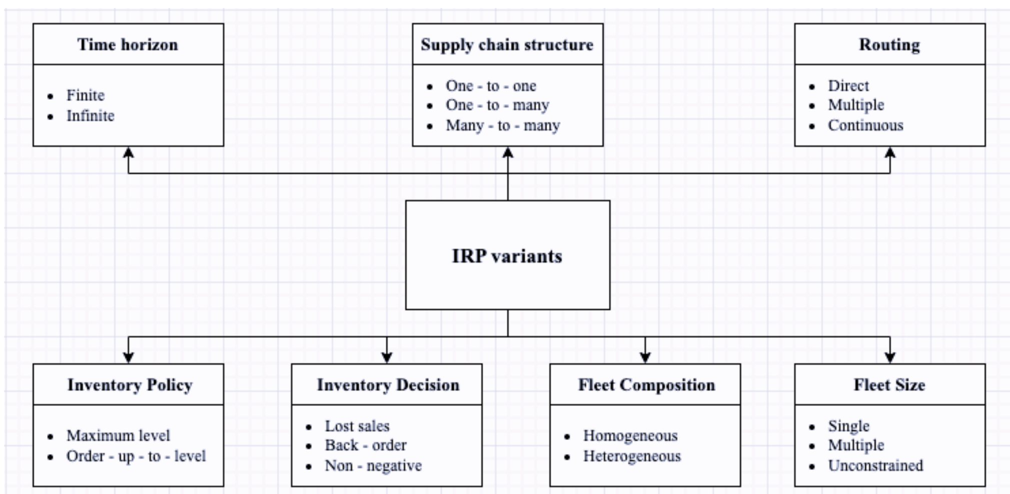

Fig. 1. IRP variants

Source: Adapted from Andersson et al. (2010)

The Inventory Routing Problem (IRP) is widely recognized as NP-hard, mainly due to its close relationship with other well-known NP-hard problems. For instance, the Vehicle Routing Problem (VRP) can be reduced to the Traveling Salesman Problem (TSP) when certain simplifying assumptions are made. These assumptions include a time horizon of one, zero inventory costs, infinite vehicle capacity, and the requirement to serve all retailers. Consequently, it is improbable that a polynomial time algorithm will emerge for its optimal solution. Numerous researchers have proposed various approaches to tackle this complex problem. Archetti et al. (2007) were among the first to introduce branch-and-cut algorithms specifically designed for the Single-Product, Single-Vehicle IRP. They introduced a specialized formulation tailored for maximum order policies and successfully solved instances involving 30 customers with a 6-period horizon and 50 customers with a 3-period horizon. Coelho and Laporte (2013c) expanded the formulation to encompass the Multi-Vehicle IRP (MIRP). They developed a branch-and-cut algorithm capable of providing exact solutions for various IRP subclasses. Their computational experiments covered a range of scenarios, including MIRP solutions under the maximum level (ML) replenishment policy, MIRP solutions with homogeneous and heterogeneous vehicle fleets, IRP incorporating transshipment options, and MIRP with additional consistency features. The computational results from benchmark instances confirmed the effectiveness of their algorithm. Desaulniers et al. (2015) focused on a scenario involving a single supplier producing a single product over a finite horizon to meet the demands of a set of customers. This task was accomplished using a fleet of homogeneous capacitated vehicles. Each customer had distinct inventory capacity and initial inventory levels. The authors introduced an innovative formulation for the IRP and developed an advanced branch-price-and-cut algorithm tailored to solve their proposed IRP model effectively.

The largest instance that can be solved by exact algorithms consist of 50 customers with 7 periods (see Coelho and Laporte (2013b) and most researchers resort to heuristic or metaheuristic algorithm to solve large instances, which represents the real world problems. There are several heuristic / metaheuristic algorithms such as Tabu Search, Genetic Algorithm, Variable Neighborhood Search, Ant Colony Optimization (ACO) et cetera that have been developed widely to solve IRP.

Park et al. (2016) proposes a GA to solve the IRP with lost sales, while Qin et al. (2014) proposes a LS with TS to optimize this problem. Onggo et al. (2019) proposed simheuristic approach which is combined heuristics with simulation to solve the single-period IRP with stochastic demands, finite time horizon, one supplier to many customers, homogeneous vehicle and stock-outs. Their has shown that our proposed method produces results that are better than the best-known solutions published in scientific literature. They have also demonstrated that when making planning decisions for supply chain operations, ignoring stochasticity in the supply chain can significantly underestimate the total cost. Maghfiroh et al. (2022) developed the hybridization of tabu search (TS) and variable neighbourhood descent (VND) for solving the Inventory Routing Problems with Stochastic Demand and Time Windows (IRPSDTW). They minimise total costs from transportation, vehicle failure, stock out cost and penalty cost. Suppliers are liable to meet the demands so that a recourse plan may cut overall costs, including travel, stock out, and failure costs. According to their analysis, adopting large time frames may assist develop a more flexible delivery path. Salim et al. (2017) employed in the resolution of a hybrid metaheuristic that combines between GA and VNS for solving a Multi-Period Inventory Routing Problem which is finite time horizon, single vehicle and order – up – to – level policy They have used to show the effectiveness of their algorithm, most of the results of our hybrid algorithm can provide optimal solutions for small- instances and improving the quality solutions for large-instances.

There have been many different approaches seeking to solve large amount of problems found in operational research. Among these are metaheuristics which start by in a first phase defining an initial solution leading to a second stage of improvement of that same initial solution using the most various methods. In order to find an initial solution one can use many different methods like branch-and-bound method, or its adaptation branch-and-cut, which have been found to be the most used ones in problems as IRP. In a second stage there is a need to improve the initial solution and just like in the first stage, many are the methods used, it is on the second stage that most authors diverge as it represents the most important component of the optimization problems. Exact algorithms, metaheuristics, heuristics and many others can be used to improve this initial solution. This paper will focus on developing ant colony opimization (metaheuristic), even though it does not guarantee to find a global optimal solution it may present a good feasible solution in a shorter time then when compared with other methods.

2.2 Ant Colony Optimization Algorithm

ACO is a metaheuristic method which implements artificial ants to find the solutions to combinatorial optimization. ACO is based on the behavior of ants and possesses enhanced abilities such as storing the memory regarding the past actions and passing the information to other ants. In fact, ants cannot hunt for food effectively if they work individually but in a group, ants possess the ability to solve complex problems and successfully obtain the food for their colony.

ACO algorithms have found applications in numerous combinatorial optimization problems. The initial ACO algorithm, known as the ant system (Dorigo et al. (1996)), was created to tackle the Travelling Salesman Problem (TSP), aiming to find the shortest round-trip connecting a series of cities. ACO has also been employed in solving the Vehicle Routing Problem (VRP), which involves designing routes for a fleet of vehicles to deliver products to a set of customers (Bullnheimer et al. (1999), Bell and McMullen (2004), Chen and Ting (2006), and Yu et al. (2009)). However, more recently, several researchers have adapted ACO to address their proposed Inventory Routing Problem (IRP) models. In fact, these papers served as inspiration for us to modify the traditional ACO approach to address our integrated IRP model, which encompasses both transportation and inventory management. This paper's modification of ACO not only focuses on solving routing challenges but also provides valuable insights to enhance the inventory updating mechanism, ultimately leading to the construction of a more optimal inventory set.

In a study by Huang and Lin (2010), they introduced a modified ACO method for solving multi-item inventory routing problems characterized by stochastic demand. They aimed to minimize total costs by selecting a delivery policy. This algorithm was developed for replenishing vending machines, and the authors adjusted the ACO algorithm to incorporate stock-out costs into the calculation of pheromone values, a feature absent in traditional ACO. Nodes with higher stock-out costs were given greater priority, even when their transportation costs were higher than those of other nodes. The results demonstrated that the modified ACO algorithm yielded significant improvements compared to the conventional ACO approach.

Wong et al. (2014) proposed modified ACO to solve one to many inventory routing problem with a finite horizon where a fleet of capacitated homogeneous vehicles, housed at a depot/warehouse, transport products from the warehouse to meet the demand specified by the customers in each period. They determined the amount on inventory and to construct a delivery schedule that minimizes both the total transportation and inventory holding costs while ensuring each customer's demand is met over the planning horizon. They employed the CPLEX 12.4 optimization software to find both lower bounds and upper bounds (best integer solutions) for each instance of the problem considered. Notably, the results indicated that the ACO algorithm outperformed the upper bounds, particularly on larger instances of the problem. This suggests that their modified ACO approach is effective in solving this specific inventory routing problem with a finite planning horizon.

Lan Teng et al. (2019) modeled a two-echelon model to formulate the integrated inventory-transportation optimization problem under VMI system with soft time window. It is a solution to inventory controlling and transportation scheduling integrated optimization from the perspective of the supplier. The supplier's aim is to explore a best distribution quantity, time, frequency and routing to minimize the total cost in the logistics system. As the problem of vehicle routing is NP-hard problem, we design combining methodologies to solve the two-echelon model. They combined a simulated annealing algorithm and ACO to solve the upper and lower echelon in the two-echelon problem, respectively. Resulting in the model and methodology proposed could help make the whole logistics system operate more effectively and efficiency compared to independent optimization of inventory and transportation.

3. METHODOLOGY

This consider a one to many network where a fleet of hetergeneous vehicle transports, single product from a distribution depot to a set of geographically retailers in a finite planning horizon. The following assumptions are made in this model.

- The fleet of hetergeneous vehicles with limited capacity is available at the warehouse
- Customers can be served by only one vehicle in each period
- Each customer requests the known demand and not vary between different periods. The customers could receive a lot size of ist demand per period, it is not nessessary for customer to receive good each period.
- The holding cost per unit product per unit time is incurrred at the depot and retailers. The holding cost does not vary throughout the planning horizon. Each retailers and supplier (depot) has limitation inventory holding.
- The demand each period must be met on time and backordering and backlogging is not allowed.

The IRP problem is modeled as mixed integer programming and the following notation is used in the model:

The IRP is usually defined as a graph G = (V,A), where V = {0,…, n} is the vertex set, A = {(i,j) : I, j ∈ V, i ≠ j } the arc set and the node {0} represents the supplier

Table 1. Indexes and sets

Parameters	Description
n	Number of vertex in V
i,j = {1,…n}	Retailers
(i,j)	Arc ∈ A
t	Subdivided time periods of T
k	Vehicle ∈ K
V {n}	Vertex set
A {(i,j)}	Arc set
T = {1,…,t}	Set of t time periods of the planning horizon T
K = {1,…,k}	Set of k vehicles of K

Table 2. Parameters

Parameters	Description
h_i	Inventory holding cost incurred by keeping one unit of product at retailer i
h_o	Inventory holding cost incurred by keeping one unit of product at supplier
c_{ij}	Cost incurred when travelling from node i to node j (distance)
C_k	Capacity of vehicle k
Max_i	Maximum capacity of retailer i
d_i^t	Demand of customer i in period t
r_0^t	Quantity of product made available at the supplier in period t
I_0^0	Initial inventory at supplier
I_i^0	Initial inventory at retailer i
I_0^t	Inventory level at supplier in period t
I_i^t	Inventory level at retailer in period t

Table 3. Decision Variables

Variable	Description
q_i^{kt}	Quantity of product is delivered to customer i by vehicle k in period t

x_{ij}^{kt}	Binary variables Equal to the number of times edge (i,j) is usede on the route of vehicle k in period t
y_i^{kt}	Binary variables Equal to one if and only if vertex i (supplier and retailers) is visited by vehicle k in period t

Mathematical formulation

The IRP can be formulated as a mixed integer programming model as follow:

$$\text{MIN } (\sum_{i\in V} \sum_{t\in T} h_i.I_i^t \; + \; \sum_{t\in T} h_o.I_0^t \; + \; \sum_{i\in V} \sum_{j\in V, i<j} \sum_{t\in T} \sum_{k\in K} c_{ij}.x_{ij}^{kt}) \tag{1}$$

Subject to:

Inventory constraints:

$$I_0^t = I_0^{t-1} + r_0^t - \sum_{k\in K} \sum_{i\in V\setminus\{0\}} q_i^{kt}, \forall t \in T, \text{ supplier} \tag{2}$$

$$I_i^t = I_i^{t-1} + \sum_{k\in K} q_i^{kt} - d_i^t, \forall i \in V \setminus \{0\}, \forall t \in T, \text{ retailers} \tag{3}$$

Capacity constraints:

$$0 \le I_i^t \le Max_i, \forall i \in V, \forall t \in T, \text{ retailers} \tag{4}$$

$$\sum_{k\in K} q_i^{kt} \le Max_i - I_i^t, \forall i \in V \setminus \{0\}, \forall t \in T, \text{ retailers} \tag{5}$$

$$q_i^{kt} \le Max_i.y_i^{kt}, \forall i \in V \setminus \{0\}, \forall k \in K, \forall t \in T, \text{ retailers} \tag{6}$$

$$\sum_{i\in V\setminus\{0\}} q_i^{kt} \le C_k.\, y_0^{kt}, \forall k \in K, \forall t \in T, \text{ vehicle capacity} \tag{7}$$

$$q_i^{kt} / d_i^t = \alpha, \alpha \in \{0,1,2,3, \dots N\} \tag{8}$$

Routing constraints:

$$\sum_{j\in V, i<j} x_{ij}^{kt} + \sum_{j\in V, j<i} x_{ji}^{kt} = 2y_i^{kt}, \forall i \in V, \forall k \in K, \forall t \in T \tag{9}$$

$$\sum_{i\in S} \sum_{j\in S, i<j} x_{ij}^{kt} \le \sum_{i\in S} y_i^{kt} - y_m^{kt}, \text{S} \subseteq V \setminus \{0\}, \forall k \in K, \forall t \in T, \text{m} \subseteq \text{S} \tag{10}$$

$$\sum_{k\in K} y_i^{kt} \le 1, \forall i \in V, \forall t \in T \tag{11}$$

Variable constraints:

$$I_i^t, q_i^{kt} \ge 0, \forall i \in V, \forall j \in V \setminus \{0\}, \forall k \in K, \forall t \in T \tag{12}$$

$$x_{i0}^{kt} \in \{0,1,2\}, \forall i \in V, \forall k \in K, \forall t \in T \tag{13}$$

$$x_{ij}^{kt} \in \{0,1\}, \forall i,j \in V \setminus \{0\}, \forall k \in K, \forall t \in T \tag{14}$$

$$y_i^{kt} \in \{0,1\}, \forall i \in V, \forall k \in K, \forall t \in T \tag{15}$$

$$I_i^t \ge 0, \forall i \in V, \forall t \in T \tag{16}$$

$$Max_i = M, \forall i \in V \tag{17}$$

$$C_i \ge 0, \forall i \in V \tag{18}$$

Objective function (1) minimize total cost in the supply chain including holding cost at retailers, holding cost at manufacuture and transportation cost. Constraints (2) and (3) define the inventory constraint at the retailers and manufacture. Constraint (4) represents inventory in each retailers which are not exceeded the maximum capacity. Constraint (5) and (6) illustrate the quantities delivered to retailer i are not exceeded the empty space inventory and the maximum inventory respectively. Constraint (7) ensure the total quantities delivered to retailer by vehicle k are not exceeded the capacity of this vehicle. Constraint (8) show that the amount of goods received at each retailer must be a multiple of its demand each period. Constraint (9) ensure that if a vertex i is visted by vehicle k in period t, then this vehicle k must leave this vertex to go to other vertex. Constraint (10) are subtour elimation. Constraint (11) ensure that the retailers i is visited by vehicle k in period t only 1 time. Constraint (12) – (18) ensure that the parameters are non negative.

*** Ant Colony Optimization:**

Ant Colony Optimization (ACO) is inspired by the nature behavior of ants finding the shortest path between their colony and a source of food. The information collected by ants during the searching process is stored in pheromone trails. The higher density of pheromones on an arc leads to attract more ants to the arc. Therefore, an appropriate formulation associated to the model for updating pheromones trail is very crucial.

The procedure for ACO can be divided into three main steps: the route construction, a local pheromone-update rule and a global pheromone-update rule. These steps are described in detail in the following subsections and Figure 1 outlines the algorithm and the following definitions are required:

N_GL: The routes can be further improved by adding route improvement strategies in the route construction procedure

MaxITER: Total iterations

N_DEM: After predefined number of iterations, the inventory level is updated

m: the number of ant

iter: the iterations

Step 1: Initial solution
Step 2: Set the value of parameters Set initial pheromones for all arcs
Step 3: Start with zero inventory and iteration with *iter = 1*
Step 4: Set *ant = 1*
Step 5: ACO- route construction:
    While (*ant <= m*)
    then
        Construct route by starting from depot each ant utilizes pheromones equation to select the next customer to be visited.
        $ant^{+} = 1$
Step 6: Obtain best solution
Step 7: Local pheromones updating
Step 8: If (*iter modulo N_GL = 0*)
    Then
        Choose the best solution
        Route improvement strategies
        Global pheromones updating
        $iter^{+} = 1$
    Else
        $iter^{+} = 1$
Step 9: If (*iter > MaxiITER*)
    Then
        Break
Step 10: If (*iter modulo N_DEM = 0*)
    Then
        Updating inventory level
    Retrun to step 4

4. EXPERIMENTS

The data used in this research was collected based on a list of retail stores of the M10 fashion chain in Northern Vietnam. The list currently includes 16 retail stores, each with weekly demand for goods, current inventory levels, and storage costs per unit of goods per week, as shown in the table below... The names of each customer location are encoded as ABCDx (where x is a natural number from 1 to 16, and x = 0 represents the manufacturing plant's central warehouse). M10's clothing products are standardized into a single type, and demand is predetermined and fixed for each time cycle. Since the factory produces and distributes its own products, the central warehouse receives 500 units of goods in each period with holding cost 500 VND/unit, and initial inventory of central warehouse is 1500 units. In M10's distribution chain, there are three main types of transport vehicles with capacities of 100, 200, and 500 units, respectively. A total of 5 time periods are applied.

The current distribution method in the M10 fashion retail chain will be based on the inventory levels and weekly demand at each location to place orders from suppliers. Each week, there will be transportation vehicles moving goods from the distribution warehouse with code ABCD0 to the retail points with code from ABCD1 to ABCD16 which have demand exceeding their current inventory levels. Multiple retail points with demand will be consolidated onto a single vehicle for transportation. By default, the vehicle will be delivered once a week to customers, each customer can only receive goods once from a single vehicle.

Table 4. Input data

Central warehouse	Longitude	Latitude	Receiving good from factory	Holding cost	Initial inventory

ABCD0	105.91975919683962	21.028718702485918	500	500	1500

Retailers	Longitude	Latitude	Weekly demand	Holding cost	Initial inventory
ABCD1	105.77767549540415	21.015229671814765	50	4000	80
ABCD2	105.85031716589764	20.997059835970447	40	5000	100
ABCD3	105.84224965307592	21.042735221129046	70	3000	120
ABCD4	105.8563099223916	21.030378979280506	20	3000	70
ABCD5	105.8495214242395	21.01122329730831	130	3000	90
ABCD6	105.85555825122691	21.021971539574317	30	5000	150
ABCD7	105.81320131711492	21.01725151823375	40	4000	200
ABCD8	105.85740022423977	21.02279411584225	80	5000	50
ABCD9	105.89348889697617	21.050413550857193	110	3000	100
ABCD10	105.77772008606304	21.01524809944646	80	5000	150
ABCD11	105.8495214242395	21.011193250871838	40	3000	100
ABCD12	106.05887055526061	21.170549192153914	60	5000	120
ABCD13	105.85010996287208	21.023600607319157	40	5000	90
ABCD14	106.69169224937365	20.860874682822697	30	5000	80
ABCD15	106.6737304647156	20.861577759118152	70	3000	60
ABCD16	107.08117102238927	20.95126848059809	40	5000	120

4.1 M10's distribution supply chain by current method

Based on the weekly demand and the initial inventory levels at each retail point, the table below illustrates the transportation method and the quantity of goods transported to each retail point each week. An empty cell indicates that the retail point will not receive any shipments that week.

Table 5. Delivery matrix of current distribution method

Customer	Week 1	Week 2	Week 3	Week 4	Week 5
ABCD1		50	50	50	50
ABCD2			40	40	40
ABCD3		70	70	70	70
ABCD4				20	20
ABCD5	130	130	130	130	130
ABCD6					
ABCD7				60	60
ABCD8	80	80	80	80	80
ABCD9	110	110	110	110	110

ABCD10		80	80	80	80
ABCD11			40	40	40
ABCD12			60	60	60
ABCD13			40	40	40
ABCD14			30	30	30
ABCD15	70	70	70	70	70
ABCD16				40	40
Total delivered demand (unit)	390	590	800	920	920

From the table, we can observe the distribution plan for delivering goods to the retail points of the M10 fashion chain as follows:

Table 6. Distribution plan by current method

Customers	**Week 1**		**Week 2**		**Week 3**		**Week 4**		**Week 5**	
	Route 1	**Route 2**	**Route 1**	**Route 2**	**Route 1**	**Route 2**	**Route 1**	**Route 2**	**Route 1**	**Route 2**
ABCD0	x	x	x	x	x	x	x	x	x	x
ABCD1			x		x		x		x	
ABCD2					x		x		x	
ABCD3			x		x		x		x	
ABCD4							x		x	
ABCD5	x		x		x		x		x	
ABCD6										
ABCD7							x		x	
ABCD8	x		x		x		x		x	
ABCD9		x		x	x			x		x
ABCD10				x		x		x		x
ABCD11						x		x		x
ABCD12						x		x		x
ABCD13						x		x		x
ABCD14						x		x		x
ABCD15		x		x		x		x		x
ABCD16								x		x
Type of vehicle	200	200	500	200	500	500	500	500	500	500
Total distances (km)	87	77	204	176	394	317	541	474	541	474
Total distance (km)	3285									

Total transportation cost (VND)	49,275,000

In addition to transportation costs, the distribution network of the M10 fashion chain incurs inventory holding costs. The inventory level at each location is calculated as follows: Inventory Level = Initial Inventory - Demand + Incoming Goods. The table below will display the inventory levels and total inventory holding costs for each time period of the network:

Table 7. Inventory matrix of current distribution method

Customer	**Week 1**	**Week 2**	**Week 3**	**Week 4**	**Week 5**
ABCD0	1610	1520	1220	800	380
ABCD1	30	30	30	30	30
ABCD2	60	20	20	20	20
ABCD3	50	50	50	50	50
ABCD4	50	30	10	10	10
ABCD5	90	90	90	90	90
ABCD6	120	90	60	30	0
ABCD7	140	80	20	20	20
ABCD8	50	50	50	50	50
ABCD9	100	100	100	100	100
ABCD10	70	70	70	70	70
ABCD11	60	20	20	20	20
ABCD12	60	0	0	0	0
ABCD13	50	10	10	10	10
ABCD14	50	20	20	20	20
ABCD15	60	60	60	60	60
ABCD16	80	40	0	0	0
Total inventory	2730	2280	1830	1380	930
Total inventory cost (VND)	5,415,000	3,750,000	2,950,000	2,590,000	2,230,000
Total inventory cost for 5 weeks (VND)	16,935,000				

The total cost of M10's distribution chain by current method (including transportation cost and inventory cost) is 66,210,000 VND

4.2 Using ACO to solve IRP in M10's distribution supply chain.

In the pursuit of attaining global optimization within the framework of the Inventory Routing Problem (IRP), this study leverages a dual-pronged approach. Firstly, it employs a local search methodology to allocate quantities to retailers effectively. Subsequently, the Ant Colony Optimization (ACO) method is employed to ascertain the optimal distribution routes. To exemplify the cost-efficiency implications within the fashion product distribution network, this paper demonstrates the application of an Ant Colony Algorithm for solving the inventory routing problem using the Python programming language. The results are presented in the table below:

Table 8. Delivery matrix by using ACO - IRP

Customer	Week 1	Week 2	Week 3	Week 4	Week 5
ABCD1		100	50	50	
ABCD2		80	40		
ABCD3		140	70	70	
ABCD4		20	20		
ABCD5	260	130		130	130
ABCD6					
ABCD7				60	60
ABCD8	160	80		80	80
ABCD9	330	110			110
ABCD10	80	80	80		80
ABCD11		40	40		40
ABCD12		60	60		60
ABCD13		40	80		
ABCD14	30	30	30		
ABCD15	140	70		70	70
ABCD16				40	40
Total delivered demand	1000	980	470	500	670

From the Table 8, we could generate the distribution plan for M10's chain for each period by the following Table 9:

Table 9. Distribution plan by ACO - IRP

Customers	Week 1		Week 2		Week 3	Week 4	Week 5	
	Route 1	Route 2	Route 1	Route 2	Route 1	Route 1	Route 1	Route 2
ABCD0	x	x	x	x	x	x	x	x
ABCD1			x		x	x		
ABCD2				x	x			
ABCD3			x		x	x		
ABCD4			x		x			
ABCD5	x		x			x	x	
ABCD6								
ABCD7						x	x	
ABCD8	x			x		x		x
ABCD9		x	x					x

ABCD10	x			x	x			x
ABCD11				x	x			x
ABCD12				x	x			x
ABCD13				x	x			
ABCD14		x		x	x			
ABCD15		x		x		x		x
ABCD16						x		x
Type of vehicle	500	500	500	500	500	500	500	200
Total distances (Km)	87	119	262	404	605	389	153	452
Total distances (Km)	2471							
Total transportation cost (VND)	37,065,000							

Because the delivery method is different between the two methods, the amount of inventory at each point in each period is different, leading to a difference in total inventory costs. Inventory costs at each customer point per period will be shown in the table below:

Table 10. Inventory matrix by using ACO - IRP

Customers	**Week 1**	**Week 2**	**Week 3**	**Week 4**	**Week 5**
ABCD0	1000	520	550	550	380
ABCD1	30	80	80	80	30
ABCD2	60	100	100	60	20
ABCD3	50	120	120	120	50
ABCD4	50	50	50	30	10
ABCD5	220	220	90	90	90
ABCD6	120	90	60	30	0
ABCD7	140	80	20	20	20
ABCD8	130	130	50	50	50
ABCD9	320	320	210	100	100
ABCD10	150	150	150	70	70
ABCD11	60	60	60	20	20
ABCD12	60	60	60	0	0
ABCD13	50	50	90	50	10
ABCD14	80	80	80	50	20
ABCD15	130	130	60	60	60

ABCD16	80	40	0	0	0
Total inventory	2730	2280	1830	1380	930
Total inventory cost	7320000	7100000	5395000	3485000	223000
Total inventory cost for 5 weeks	25,530,000				

The total cost of M10's distribution chain by ACO – IRP (including transportation cost and inventory cost): 62,595,000 VND

With the above bases, the research team compares the efficiency of the traditional shipping method that M10's fashion is applying with the research team's algorithm. Average transportation cost: 15,000 VND/km including fuel costs, labor costs, road and bridge costs, vehicle depreciation costs (Source: ALS aviation logistics company).

Table 11. Comparison results

Criteria	Before	After
Total distance (km)	3285	2471
Total transportation cost (VND)	49,275,000	37,065,000
Total inventory cost (VND)	16,935,000	25,530,000
Total cost (VND)	66,210,000	62,595,000

From the above results, this research proved that the algorithm is efficient in distribution problem for the M10 fashion group. The application of the Ant Colony Algorithm to the Inventory Routing Problem (IRP) yielded notable improvements in cost efficiency. Prior to implementing the algorithm, the total cost for the inventory routing in Vietnamese Dong (VND) stood at 66,210,000. This represented the expenditure associated with distribution activities within the fashion product network. However, following the application of the Ant Colony Algorithm, a remarkable reduction in total cost was observed. The post-implementation evaluation revealed that the total cost had significantly decreased to 62,595,000 VND. It leads to reduce the total transportation cost and total logistics cost about 5.5 percent. This substantial reduction in cost is indicative of the algorithm's effectiveness in optimizing the routing and distribution process. The algorithm's ability to discover more efficient distribution routes and allocation strategies played a pivotal role in achieving this cost reduction. The optimization resulted in streamlined routes, reduced fuel consumption, and improved resource allocation, all contributing to the cost savings. Furthermore, the reduction in total cost is reflective of the algorithm's ability to find near-optimal solutions to the inventory routing problem, aligning with the primary objective of the research—to enhance cost-efficiency within the distribution network. These results underscore the potential of Ant Colony Algorithms as valuable tools in addressing complex logistical challenges, with tangible benefits in terms of cost reduction and resource optimization. This cost reduction is particularly significant as it can directly impact the bottom line of the company, leading to increased profitability and operational efficiency and reduce environmental impact.

5. CONCLUSION

The study provides a comprehensive overview of the theoretical foundation underlying the Inventory Routing Problem (IRP) and presents an application of Ant Colony Optimization (ACO) to address this problem. The research highlights the efficacy of ACO in optimizing complex problems, demonstrating its ability to yield solutions with improved accuracy and efficiency when compared to alternative algorithms. Moreover, the research team offers compelling evidence supporting the practical application of the IRP within distribution chains. Leveraging Vendor Managed Inventory (VMI) practices, the study illustrates the potential for substantial logistics cost savings by employing a flexible approach to product delivery from suppliers to retailers, thereby minimizing total costs. Notwithstanding these positive contributions, it is imperative to acknowledge several limitations. Firstly, the utilization of inventory routing software is essential for facilitating the implementation of ACO, necessitating software that is user-friendly and easily manipulated. Secondly, the practical effectiveness of the IRP is contingent upon various real-world parameters and constraints. Notably, this paper does not account for a range of actual requirements, including stochastic demand patterns, time window considerations, and the management of multiple products. Thirdly, the computational time required to solve the IRP is inherently linked to the complexity and size of the initial dataset, underscoring the significance of these factors in the practical application of the model.

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